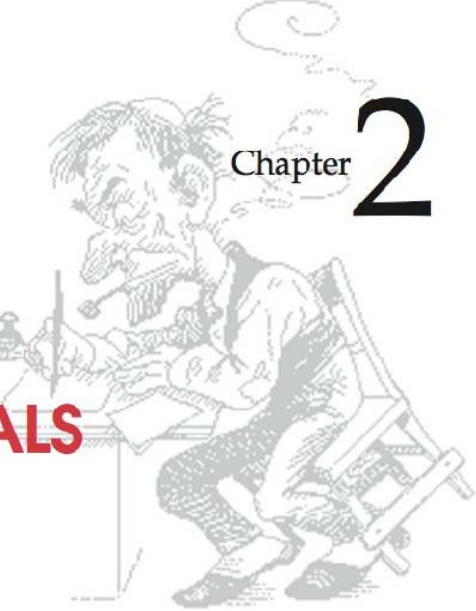


Chapter 2

KINEMATICS FUNDAMENTALS

Chance favors the prepared mind
PASTEUR

2.0 INTRODUCTION

This chapter will present definitions of a number of terms and concepts fundamental to the synthesis and analysis of mechanisms. It will also present some very simple but powerful analysis tools that are useful in the synthesis of mechanisms.

2.1 DEGREES OF FREEDOM (DOF) OR MOBILITY

A mechanical system's **mobility** (M) can be classified according to the number of **degrees of freedom** (*DOF*) that it possesses. The system's *DOF* is equal to *the number of independent parameters (measurements) that are needed to uniquely define its position in space at any instant of time*. Note that *DOF* is defined with respect to a selected frame of reference. Figure 2-1 shows a pencil lying on a flat piece of paper with an x, y coordinate system added. If we constrain this pencil to always remain in the plane of the paper, three parameters (*DOF*) are required to completely define the position of the pencil on the paper, two linear coordinates (x, y) to define the position of any one point on the pencil and one angular coordinate (θ) to define the angle of the pencil with respect to the axes. The minimum number of measurements needed to define its position is shown in the figure as x, y, and θ. This system of the pencil in a plane then has **three** *DOF*. Note that the particular parameters chosen to define its position are not unique. Any alternate set of three parameters could be used. There is an infinity of sets of parameters possible, but in this case there must be three parameters per set, **such as two lengths and an angle**, to define the system's position because *a rigid body in plane motion has three DOF*.

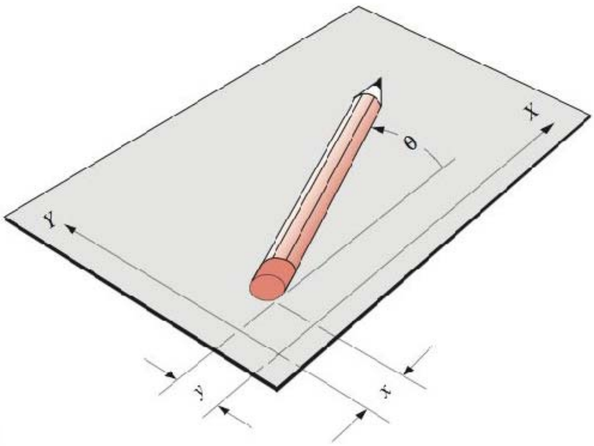


FIGURE 2-1
A rigid body in a plane has three *DOF*

Now allow the pencil to exist in a three-dimensional world. Hold it above your desktop and move it about. You now will need six parameters to define its **six** *DOF*. One possible set of parameters that could be used is **three lengths**, (x, y, z), plus **three angles** (θ, ϕ, ρ). *Any rigid body in three-space has six degrees of freedom.* Try to identify these six *DOF* by moving your pencil or pen with respect to your desktop.

The pencil in these examples represents a **rigid body**, or **link**, which for purposes of kinematic analysis we will assume to be incapable of deformation. This is merely a convenient fiction to allow us to more easily define the gross motions of the body. We can later superpose any deformations due to external or inertial loads onto our kinematic motions to obtain a more complete and accurate picture of the body's behavior. But remember, we are typically facing a *blank sheet of paper* at the beginning stage of the design process. We cannot determine deformations of a body until we define its size, shape, material properties, and loadings. Thus, at this stage we will assume, for purposes of initial kinematic synthesis and analysis, that *our kinematic bodies are rigid and massless.*

2.2 TYPES OF MOTION

A rigid body free to move within a reference frame will, in the general case, have **complex motion**, which is a simultaneous combination of **rotation** and **translation**. In three-dimensional space, there may be rotation about any axis (any skew axis or one of the three principal axes) and also simultaneous translation that can be resolved into components along three axes. In a plane, or two-dimensional space, complex motion becomes a combination of simultaneous rotation about one axis (perpendicular to the plane) and also translation resolved into components along two axes in the plane. For simplicity, we will limit our present discussions to the case of **planar (2-D) kinematic systems**. We will define these terms as follows for our purposes, in planar motion:

Pure rotation

The body possesses one point (center of rotation) that has no motion with respect to the "stationary" frame of reference. All other points on the body describe arcs about that center. A reference line drawn on the body through the center changes only its angular orientation.

Pure translation

All points on the body describe parallel (curvilinear or rectilinear) paths. A reference line drawn on the body changes its linear position but does not change its angular orientation.

Complex motion

A simultaneous combination of rotation and translation. Any reference line drawn on the body will change both its linear position and its angular orientation. Points on the body will travel nonparallel paths, and there will be, at every instant, a center of rotation, which will continuously change location.

Translation and **rotation** represent independent motions of the body. Each can exist without the other. If we define a 2-D coordinate system as shown in Figure 2-1 (p. 31), the x and y terms represent the translation components of motion, and the θ term represents the rotation component.

2.3 LINKS, JOINTS, AND KINEMATIC CHAINS

We will begin our exploration of the kinematics of mechanisms with an investigation of the subject of **linkage design**. Linkages are the basic building blocks of all mechanisms. We will show in later chapters that all common forms of mechanisms (cams, gears, belts, chains) are in fact variations on a common theme of linkages. Linkages are made up of links and joints.

A **link**, as shown in Figure 2-2, is an (assumed) rigid body that possesses at least two **nodes** that are *points for attachment to other links.*

Binary link - *one with two nodes.*

Ternary link - *one with three nodes.*

Quaternary link - *one with four nodes.*

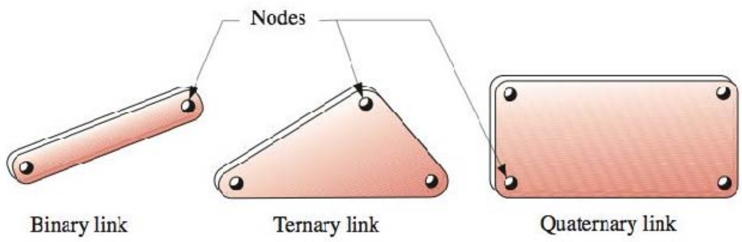


FIGURE 2-2

Links of different order

A **joint** is *a connection between two or more links (at their nodes), which allows some motion, or potential motion, between the connected links.* **Joints** (also called **kinematic pairs**) can be classified in several ways:

1 By the type of contact between the elements, line, point, or surface.

2 By the number of degrees of freedom allowed at the joint.

3 By the type of physical closure of the joint: either **force** or **form** closed.

4 By the number of links joined (order of the joint).

Reuleaux[1] coined the term **lower pair** to describe joints with surface contact (as with a pin surrounded by a hole) and the term **higher pair** to describe joints with point or line contact. However, if there is any clearance between pin and hole (as there must be for motion), so-called surface contact in the pin joint actually becomes line contact, as the pin contacts only one "side" of the hole. Likewise, at a microscopic level, a block sliding on a flat surface actually has contact only at discrete points, which are the tops of the surfaces' asperities. The main practical advantage of lower pairs over higher pairs is their better ability to trap lubricant between their enveloping surfaces. This is especially true for the rotating pin joint. The lubricant is more easily squeezed out of a higher pair, nonenveloping joint. As a result, the pin joint is preferred for low wear and long life, even over its lower pair cousin, the prismatic or slider joint.

Figure 2-3a (p. 34) shows the six possible lower pairs, their degrees of freedom, and their one-letter symbols. The revolute (R) and the prismatic (P) pairs are the only lower pairs usable in a planar mechanism. The screw (H), cylindric (C), spherical (S), and flat (F) lower pairs are all combinations of the revolute and/or prismatic pairs and are used in spatial (3-D) mechanisms. The R and P pairs are the basic building blocks of all other pairs that are combinations of those two as shown in Table 2-1.

A more useful means to classify joints (pairs) is by the number of degrees of freedom that they allow between the two elements joined. Figure 2-3 (p. 34) also shows examples of both one- and two-freedom joints commonly found in planar mechanisms. Figure 2-3b (p. 34) shows two forms of a planar, **one-freedom** joint (or pair), namely, a rotating (revolute) pin joint (R) and a translating (prismatic) slider joint (P). These are also referred to as **full joints** (i.e., full = 1 *DOF*) and are **lower pairs**. The pin joint allows one rotational *DOF*, and the slider joint allows one translational *DOF* between the joined links. These are both contained within (and each is a limiting case of) another common, one-freedom joint, the screw and nut (Figure 2-3a). Motion of either the nut or the screw with respect to the other results in helical motion. If the helix angle is made zero, the nut rotates without advancing and it becomes the pin joint. If the helix angle is made 90 degrees, the nut will translate along the axis of the screw, and it becomes the slider joint.

Figure 2-3c shows examples of two-freedom joints (higher pairs) that simultaneously allow two independent, relative motions, namely translation and rotation, between the joined links. Paradoxically, this **two-freedom joint** is sometimes referred to as a **"half joint,"** with its two freedoms placed in the denominator. The **half joint** is also called a **roll-slide joint** because it allows both rolling and sliding. A spherical, or ball-and-socket joint, (Figure 2-3a) is an example of a three-freedom joint, which allows three independent angular motions between the two links joined. This *joystick* or *ball joint* is typically used in a three-dimensional mechanism, one example being the ball joints in an automotive suspension system.

TABLE 2-1
The Six Lower Pairs

Name (Symbol)	*DOF*	Contains
Revolute (R)	1	R
Prismatic (P)	1	P
Helical (H)	1	RP
Cylindric (C)	2	RP
Spherical (S)	3	RRR
Planar (F)	3	RPP

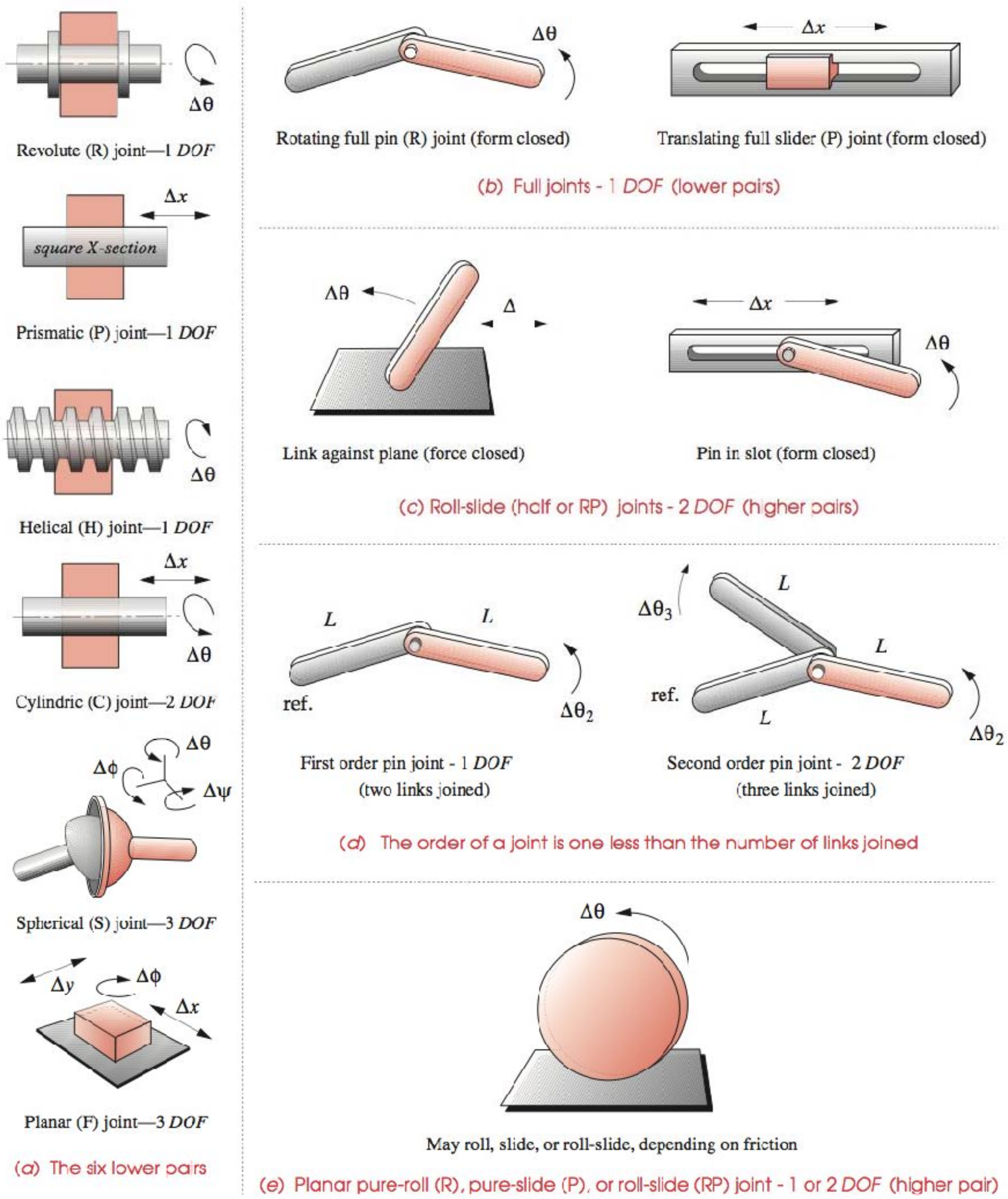


FIGURE 2-3

Joints (pairs) of various types

A joint with more than one freedom may also be a **higher pair** as shown in Figure 2-3c. Full joints (lower pairs) and half joints (higher pairs) are both used in planar (2-D), and in spatial (3-D) mechanisms. Note that if you do not allow the two links in Figure 2-3c connected by a roll-slide joint to slide, perhaps by providing a high friction coefficient between them, you can "lock out" the translating (Δx) freedom and make it behave as a full joint. This is then called a **pure rolling joint** and has rotational freedom ($\Delta\theta$) only. A common example of this type of joint is your automobile tire rolling against the road, as shown in Figure 2-3e. In normal use there is pure rolling and no sliding at this joint, unless, of course, you encounter an icy road or become too enthusiastic about accelerating or cornering. If you lock your brakes on ice, this joint converts to a pure sliding one like the slider block in Figure 2-3b. Friction determines the actual number of freedoms at this kind of joint. It can be **pure roll**, **pure slide**, or **roll-slide**.

To visualize the degree of freedom of a joint in a mechanism, it is helpful to "mentally disconnect" the two links that create the joint from the rest of the mechanism. You can then more easily see how many freedoms the two joined links have with respect to one another.

Figure 2-3c also shows examples of both **form-closed** and **force-closed** joints. A **form-closed** joint is kept together or *closed by its geometry*. A pin in a hole or a slider in a two-sided slot is form closed. In contrast, a **force-closed** joint, such as a pin in a half-bearing or a slider on a surface, *requires some external force to keep it together or closed.* This force could be supplied by gravity, a spring, or any external means. There can be substantial differences in the behavior of a mechanism due to the choice of force or form closure, as we shall see. The choice should be carefully considered. In linkages, form closure is usually preferred, and it is easy to accomplish. But for cam-follower systems, force closure is often preferred. This topic will be explored further in later chapters.

Figure 2-3d shows examples of joints of various orders, where **joint order** is defined as *the number of links joined minus one*. It takes two links to make a single joint; thus the simplest joint combination of two links has joint order one. As additional links are placed on the same joint, the joint order is increased on a one-for-one basis. Joint order has significance in the proper determination of overall degree of freedom for the assembly. We gave definitions for a **mechanism** and a **machine** in Chapter 1. With the kinematic elements of links and joints now defined, we can define those devices more carefully based on Reuleaux's classifications of the kinematic chain, mechanism, and machine. [1]

A kinematic chain is defined as:

An assemblage of links and joints, interconnected in a way to provide a controlled output motion in response to a supplied input motion.

A mechanism is defined as:

A kinematic chain in which at least one link has been "grounded," or attached, to the frame of reference (which itself may be in motion).

A machine is defined as:

A combination of resistant bodies arranged to compel the mechanical forces of nature to do work accompanied by determinate motions.

2

By Reuleaux's* definition[1] a machine is *a collection of mechanisms arranged to transmit forces and do work*. He viewed all energy or force transmitting devices as machines that utilize mechanisms as their building blocks to provide the necessary motion constraints.

We will now define a **crank** as *a link that makes a complete revolution and is pivoted to ground*, a **rocker** as *a link that has oscillatory (back and forth) rotation and is pivoted to ground*, and a **coupler** (or connecting rod) *as a link that has complex motion and is not pivoted to ground*. **Ground** is defined as *any link or links that are fixed* (nonmoving) with respect to the reference frame. Note that the reference frame may in fact itself be in motion.

2.4 DRAWING KINEMATIC DIAGRAMS

Analyzing the kinematics of mechanisms requires that we draw clear, simple, schematic kinematic diagrams of the links and joints of which they are made. Sometimes it can be difficult to identify the kinematic links and joints in a complicated mechanism. Beginning students of this topic often have this difficulty. This section defines one approach to the creation of simplified kinematic diagrams.

Real links can be of any shape, but a "kinematic" link, or link edge, is defined as a line between joints that allow relative motion between adjacent links. Joints can allow rotation, translation, or both between the links joined. The possible joint motions must be clear and obvious from the kinematic diagram. Figure 2-4 shows recommended schematic notations for binary, ternary, and higher-order links, and for movable and grounded joints of rotational and translational freedoms plus an example of their combination. Many other notations are possible, but whatever notation is used, it is critical that your diagram indicate which links or joints are grounded and which can move. Otherwise nobody will be able to interpret your design's kinematics. Shading or crosshatching should be used to indicate that a link is solid.

Figure 2-5a shows a photograph of a simple mechanism used for weight training called a leg press machine. It has six pin-jointed links labeled L_1 through L_6 and seven pin joints. The moving pivots are labeled *A* through *D*; O_2, O_4 and O_6 denote the grounded pivots of their respective link numbers. Even though its links are in parallel planes sepa-

* Reuleaux created a set of 220 models of mechanisms in the 19th century to demonstrate machine motions. Cornell University acquired the collection in 1892 and has now put images and descriptions of them on the web at: http://kmoddl.library.cornell.edu. The same site also has depictions of three other collections of machines and gear trains.

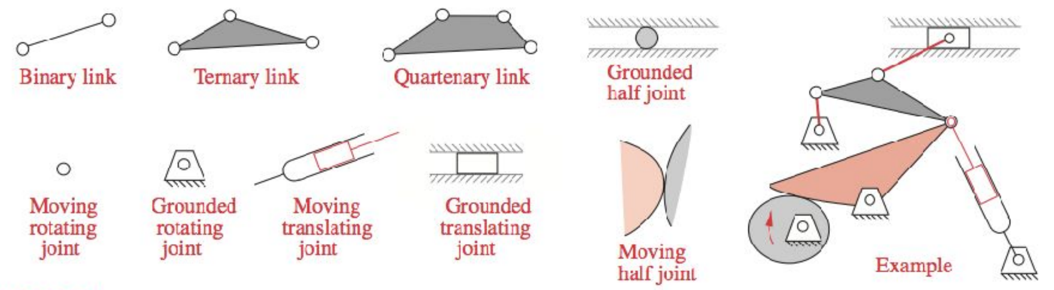
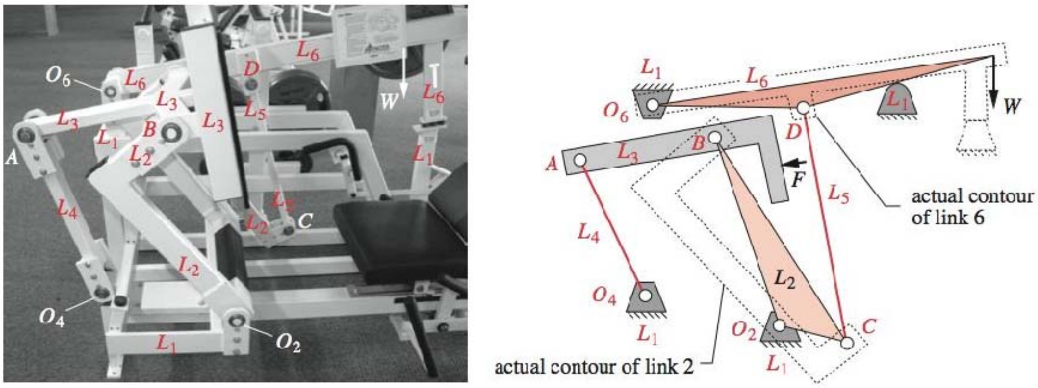


FIGURE 2-4

Schematic notation for kinematic diagrams



(a) Weight-training mechanism

(b) Kinematic diagram

FIGURE 2-5

A mechanism and its kinematic diagram

rated by some distance in the *z*-direction, it can still be analyzed kinematically as if all links were in a common plane.

To use the leg press machine, the user loads some weights on link 6 at top right, sits in the seat at lower right, places both feet against the flat surface of link 3 (a coupler) and pushes with the legs to lift the weights through the linkage. The linkage geometry is designed to give a variable mechanical advantage that matches the human ability to provide force over the range of leg motion. Figure 2-5b shows a kinematic diagram of its basic mechanism. Note that here all the links have been brought to a common plane. Link 1 is the ground. Links 2, 4, and 6 are rockers. Links 3 and 5 are couplers. The input force *F* is applied to link 3. The "output" resistance weight *W* acts on link 6. Note the difference between the actual and kinematic contours of links 2 and 6.

The next section discusses techniques for determining the mobility of a mechanism. That exercise depends on an accurate count of the number of links and joints in the mechanism. Without a proper, clear, and complete kinematic diagram of the mechanism, it will be impossible to get the count, and thus the mobility, correct.

2.5 DETERMINING DEGREE OF FREEDOM OR MOBILITY

The concept of **degree of freedom** (*DOF*) is fundamental to both the synthesis and analysis of mechanisms. We need to be able to quickly determine the *DOF* of any collection of links and joints that may be suggested as a solution to a problem. Degree of freedom (also called the **mobility** *M)* of a system can be defined as:

Degree of Freedom

the number of inputs that need to be provided in order to create a predictable output;

also:

the number of independent coordinates required to define its position.

2

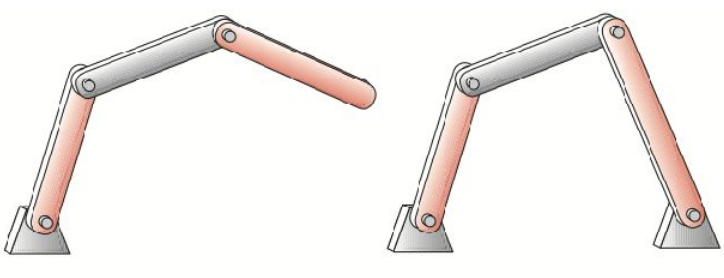
At the outset of the design process, some general definition of the desired output motion is usually available. The number of inputs needed to obtain that output may or may not be specified. Cost is the principal constraint here. Each required input will need some type of actuator, either a human operator or a "slave" in the form of a motor, solenoid, air cylinder, or other energy conversion device. (These devices are discussed in Section 2.19 on p. 74.) These multiple-input devices will have to have their actions coordinated by a "controller," which must have some intelligence. This control is now often provided by a computer but can also be mechanically programmed into the mechanism design. There is no requirement that a mechanism have only one *DOF*, although that is often desirable for simplicity. Some machines have many *DOF*. For example, picture the number of control levers or actuating cylinders on a bulldozer or crane. See Figure 1-1b (p. 7).

Kinematic chains or mechanisms may be either **open** or **closed**. Figure 2-6 shows both open and closed mechanisms. A closed mechanism will have no open attachment points or **nodes** and may have one or more degrees of freedom. An open mechanism of more than one link will always have more than one degree of freedom, thus requiring as many actuators (motors) as it has *DOF*. A common example of an open mechanism is an industrial robot. *An open kinematic chain of two binary links and one joint* is called a **dyad**. The sets of links shown in Figure 2-3b and c (p. 34) are **dyads**.

Reuleaux limited his definitions to closed kinematic chains and to mechanisms having only one *DOF*, which he called *constrained*.[1] The somewhat broader definitions above are perhaps better suited to current-day applications. A multi-*DOF* mechanism, such as a robot, will be constrained in its motions as long as the necessary number of inputs is supplied to control all its *DOF*.

Degree of Freedom (Mobility) in Planar Mechanisms

To determine the overall *DOF* of any mechanism, we must account for the number of links and joints, and for the interactions among them. The *DOF* of any assembly of links can be predicted from an investigation of the **Gruebler condition**.[2] Any link in a plane has 3 *DOF*. Therefore, a system of L unconnected links in the same plane will have $3L$



(*a*) Open mechanism chain (*b*) Closed mechanism chain

FIGURE 2-6

Mechanism chains

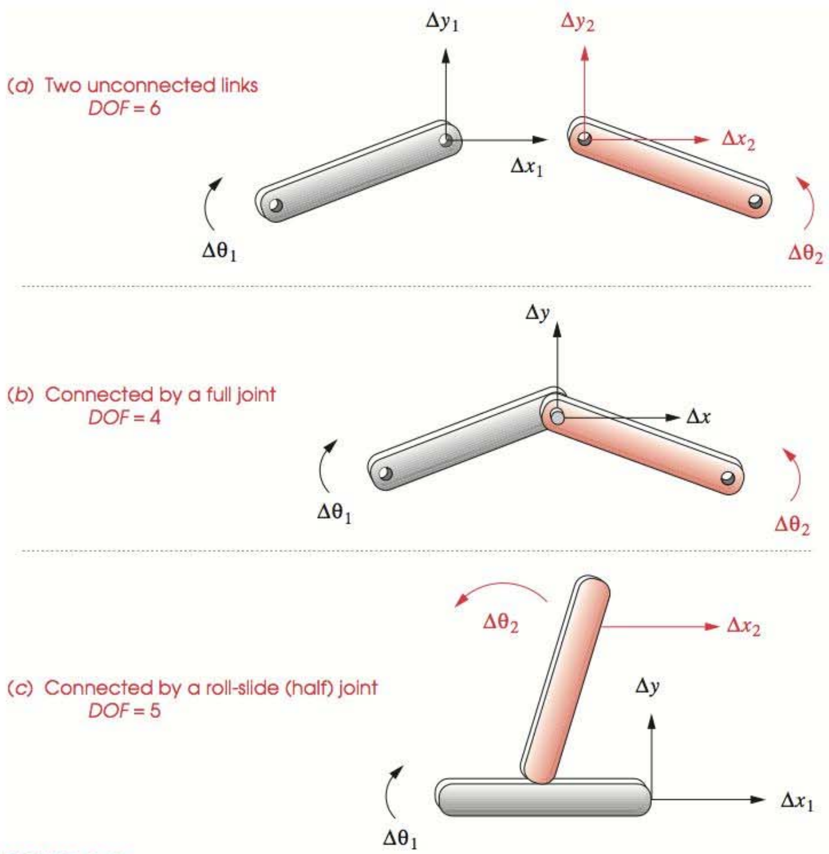


FIGURE 2-7

Joints remove degrees of freedom

DOF, as shown in Figure 2-7a where the two unconnected links have a total of six *DOF*. When these links are connected by a **full joint** in Figure 2-7b, Δy_1 and Δy_2 are combined as Δy, and Δx_1 and Δx_2 are combined as Δx. This removes two *DOF*, leaving four *DOF*. In Figure 2-7c the half joint removes only one *DOF* from the system (because a half joint has two *DOF*), leaving the system of two links connected by a half joint with a total of five *DOF*. In addition, when any link is grounded or attached to the reference frame, all three of its *DOF* will be removed. This reasoning leads to **Gruebler's equation**:

$$M = 3L - 2J - 3G \tag{2.1a}$$

where: M = *degree of freedom or mobility*
L = *number of links*
J = *number of joints*
G = *number of grounded links*

Note that in any real mechanism, even if more than one link of the kinematic chain is grounded, the net effect will be to create one larger, higher-order ground link, as there can be only one ground plane. Thus G is always one, and Gruebler's equation becomes:

$$M = 3(L-1) - 2J \tag{2.1b}$$

The value of J in equations 2.1a and 2.1b must reflect the value of all joints in the mechanism. That is, half joints count as 1/2 because they only remove one *DOF*. It is less confusing if we use **Kutzbach's** modification of Gruebler's equation in this form:

$$M = 3(L-1) - 2J_1 - J_2 \tag{2.1c}$$

where: M = *degree of freedom or mobility*
L = *number of links*
J_1 = *number of 1 DOF (full) joints*
J_2 = *number of 2 DOF (half) joints*

The value of J_1 and J_2 in these equations must still be carefully determined to account for all full, half, and multiple joints in any linkage. Multiple joints count as one less than the number of links joined at that joint and add to the "full" (J_1) category. The *DOF* of any proposed mechanism can be quickly ascertained from this expression before investing any time in more detailed design. It is interesting to note that this equation has no information in it about link sizes or shapes, only their quantity. Figure 2-8a shows a mechanism with one *DOF* and only full joints in it.

Figure 2-8b shows a structure with zero *DOF* that contains both half and multiple joints. Note the schematic notation used to show the ground link. The ground link need not be drawn in outline as long as all the grounded joints are identified. Note also the joints labeled **multiple** and **half** in Figure 2-8a and b. As an exercise, compute the *DOF* of these examples with **Kutzbach's** equation.

Degree of Freedom (Mobility) in Spatial Mechanisms

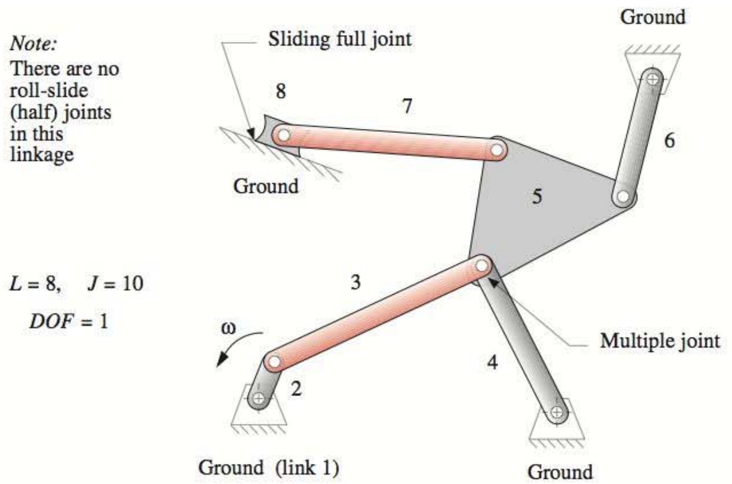
The approach used to determine the mobility of a planar mechanism can be easily extended to three dimensions. Each unconnected link in three-space has 6 *DOF*, and any one of the six lower pairs can be used to connect them, as can higher pairs with more freedom. A one-freedom joint removes 5 *DOF*, a two-freedom joint removes 4 *DOF*, etc. Grounding a link removes 6 *DOF*. This leads to the Kutzbach mobility equation for spatial linkages:

$$M = 6(L-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - J_5 \tag{2.2}$$

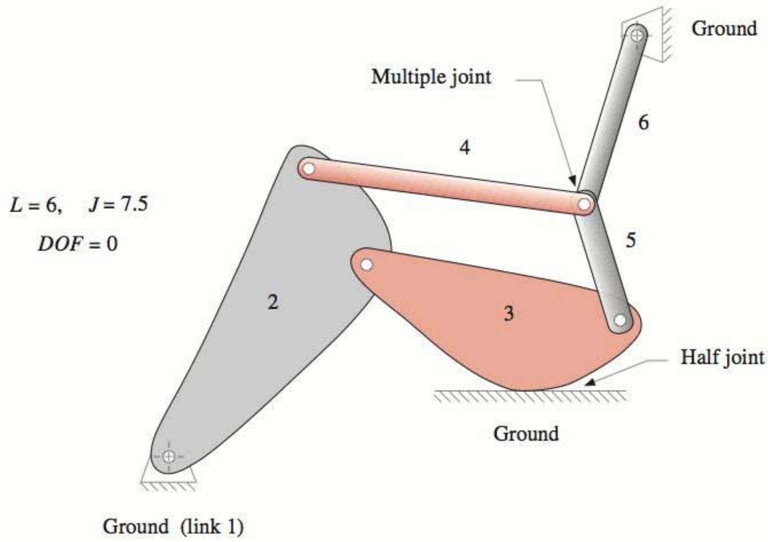
where the subscript refers to the number of freedoms of the joint. We will limit our study to 2-D mechanisms in this text.

2.6 MECHANISMS AND STRUCTURES

The degree of freedom of an assembly of links completely predicts its character. There are only three possibilities. *If the DOF is positive, it will be a* ***mechanism***, and the links will have relative motion. *If the DOF is exactly zero, then it will be a* ***structure***, and no motion is possible. *If the DOF is negative, then it is a* ***preloaded structure***, which means that no motion is possible and some stresses may also be present at the time of assembly. Figure 2-9 (p. 42) shows examples of these three cases. One link is grounded in each case.



(a) Linkage with full and multiple joints

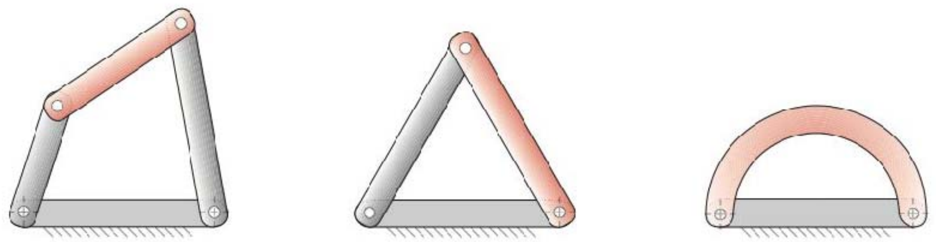


(b) Linkage with full, half, and multiple joints

FIGURE 2-8

Linkages containing joints of various types

2



(a) Mechanism—*DOF* = +1 (b) Structure—*DOF* = 0 (c) Preloaded structure—*DOF* = −1

FIGURE 2-9

Mechanisms, structures, and preloaded structures

Figure 2-9a shows four links joined by four full joints which, from the Gruebler equation, gives one *DOF*. It will move, and only one input is needed to give predictable results.

Figure 2-9b shows three links joined by three full joints. It has zero *DOF* and is thus a **structure**. Note that if the link lengths will allow connection,* all three pins can be inserted into their respective pairs of link holes (nodes) without straining the structure, as a position can always be found to allow assembly. This is called *exact constraint*.†

Figure 2-9c shows two links joined by two full joints. It has a *DOF* of minus one, making it a **preloaded structure**. In order to insert the two pins without straining the links, the center distances of the holes in both links must be exactly the same. Practically speaking, it is impossible to make two parts exactly the same. There will always be some manufacturing error, even if very small. Thus you may have to force the second pin into place, creating some stress in the links. The structure will then be preloaded. You have probably met a similar situation in a course in applied mechanics in the form of an indeterminate beam, one in which there were too many supports or constraints for the equations available. An indeterminate beam also has negative *DOF*, while a *simply supported* beam has zero *DOF*.

Both structures and preloaded structures are commonly encountered in engineering. In fact the true structure of zero *DOF* is rare in civil engineering practice. Most buildings, bridges, and machine frames are preloaded structures, due to the use of welded and riveted joints rather than pin joints. Even simple structures like the chair you are sitting in are often preloaded. Since our concern here is with mechanisms, we will concentrate on devices with positive *DOF* only.

2.7 NUMBER SYNTHESIS

The term **number synthesis** has been coined to mean *the determination of the number and order of links and joints necessary to produce motion of a particular DOF*. **Link order** in this context refers to the number of nodes per link,§ i.e., **binary, ternary, quaternary**, etc. The value of number synthesis is to allow the exhaustive determination of all possible

* If the sum of the lengths of any two links is less than the length of the third link, then their interconnection is impossible.

† The concept of *exact constraint* also applies to mechanisms with positive *DOF*. It is possible to provide redundant constraints to a mechanism (e.g., making its theoretical *DOF* = 0 when 1 is desired) yet still have it move because of particular geometry (see Section 2.8 *Paradoxes*). Non-exact constraint should be avoided in general as it can lead to unexpected mechanical behavior. For an excellent and thorough discussion of this issue see Blanding, D. L., *Exact Constraint: Machine Design Using Kinematic Principles*, ASME Press, 1999.

§ Not to be confused with "joint order" as defined earlier, which refers to the number of *DOF* that a joint possesses.

combinations of links that will yield any chosen *DOF*. This then equips the designer with a definitive catalog of potential linkages to solve a variety of motion control problems.

As an example we will now derive all the possible link combinations for one *DOF*, including sets of up to eight links, and link orders up to and including hexagonal links. For simplicity we will assume that the links will be connected with only single, full rotating joints (i.e., a pin connecting two links). We can later introduce half joints, multiple joints, and sliding joints through linkage transformation. First let's look at some interesting attributes of linkages as defined by the above assumption regarding full joints.

Hypothesis: If all joints are full joints, an odd number of *DOF* requires an even number of links and vice versa.

Proof: **Given:** All even integers can be denoted by $2m$ or by $2n$, and all odd integers can be denoted by $2m - 1$ or by $2n - 1$, where n and m are any positive integers. The number of joints must be a positive integer.

Let : L = number of links, J = number of joints, and $M = DOF = 2m$ (i.e., all even numbers)

Then: Rewriting Gruebler's equation 2.1b to solve for J,

$$J = \frac{3}{2}(L-1) - \frac{M}{2} \tag{2.3a}$$

Try: Substituting $M = 2m$, and $L = 2n$ (i.e., both any even numbers):

$$J = 3n - m - \frac{3}{2} \tag{2.3b}$$

This cannot result in J being a positive integer as required.

Try: $M = 2m - 1$ and $L = 2n - 1$ (i.e., both any odd numbers):

$$J = 3n - m - \frac{5}{2} \tag{2.3c}$$

This also cannot result in J being a positive integer as required.

Try: $M = 2m - 1$, and $L = 2n$ (i.e., odd-even):

$$J = 3n - m - 2 \tag{2.3d}$$

This is a positive integer for $m \geq 1$ and $n \geq 2$.

Try: $M = 2m$ and $L = 2n - 1$ (i.e., even-odd):

$$J = 3n - m - 3 \tag{2.3e}$$

This is a positive integer for $m \geq 1$ and $n \geq 2$.

So, for our example of one-*DOF* mechanisms, we can only consider combinations of 2, 4, 6, 8, . . . links. Letting the order of the links be represented by:

B = *number of binary links*
T = *number of ternary links*
Q = *number of quaternaries*
P = *number of pentagonals*
H = *number of hexagonals*

2

the total number of links in any mechanism will be:

$$L = B + T + Q + P + H + \cdots \tag{2.4a}$$

Since *two link nodes* are needed to make *one joint*:

$$J = \frac{nodes}{2} \tag{2.4b}$$

and

$$nodes = order\ of\ link \times no.\ of\ links\ of\ that\ order \tag{2.4c}$$

then

$$J = \frac{(2B + 3T + 4Q + 5P + 6H + \cdots)}{2} \tag{2.4d}$$

Substitute equations 2.4a and 2.4d into Gruebler's equation (2.1b, on p. 40)

$$\begin{aligned} M &= 3(B+T+Q+P+H-1) - 2\left(\frac{2B+3T+4Q+5P+6H}{2}\right) \\ M &= B - Q - 2P - 3H - 3 \end{aligned} \tag{2.4e}$$

Note what is missing from this equation! The ternary links have dropped out. The *DOF* is independent of the number of ternary links in the mechanism. But because each ternary link has three nodes, it can only create or remove 3/2 joints. So we must add or subtract ternary links in pairs to maintain an integer number of joints. *The addition or subtraction of ternary links in pairs will not affect the DOF of the mechanism.*

In order to determine all possible combinations of links for a particular *DOF*, we must combine equations 2.3a (p. 43) and 2.4d:*

$$\begin{aligned} \frac{3}{2}(L-1) - \frac{M}{2} &= \frac{2B + 3T + 4Q + 5P + 6H}{2} \\ 3L - 3 - M &= 2B + 3T + 4Q + 5P + 6H \end{aligned} \tag{2.5}$$

Now combine equation 2.5 with equation 2.4a to eliminate B:

$$L - 3 - M = T + 2Q + 3P + 4H \tag{2.6}$$

We will now solve equations 2.4a and 2.6 simultaneously (by progressive substitution) to determine all compatible combinations of links for $DOF = 1$, up to eight links. The strategy will be to start with the smallest number of links, and the highest-order link possible with that number, eliminating impossible combinations.

(Note: *L must be even for odd DOF.*)

CASE 1. $L = 2$

$$L - 4 = T + 2Q + 3P + 4H = -2 \tag{2.7a}$$

This requires a negative number of links, so $L = 2$ is impossible.

* Karunamoorthy [17] defines some useful rules for determining the number of possible combinations for any number of links with a given degree of freedom.

CASE 2. $L = 4$

$$L - 4 = T + 2Q + 3P + 4H = 0 \qquad \text{so: } T = Q = P = H = 0$$

$$L = B + 0 = 4 \qquad B = 4 \tag{2.7b}$$

The simplest one-*DOF* linkage is four binary links—the **fourbar linkage**.

CASE 3. $L = 6$

$$L - 4 = T + 2Q + 3P + 4H = 2 \qquad \text{so: } P = H = 0 \tag{2.7c}$$

T may only be 0, 1, or 2; Q may only be 0 or 1

If $Q = 0$ then T must be 2 and:

$$L = B + 2T + 0Q = 6 \qquad B = 4 \qquad T = 2 \tag{2.7d}$$

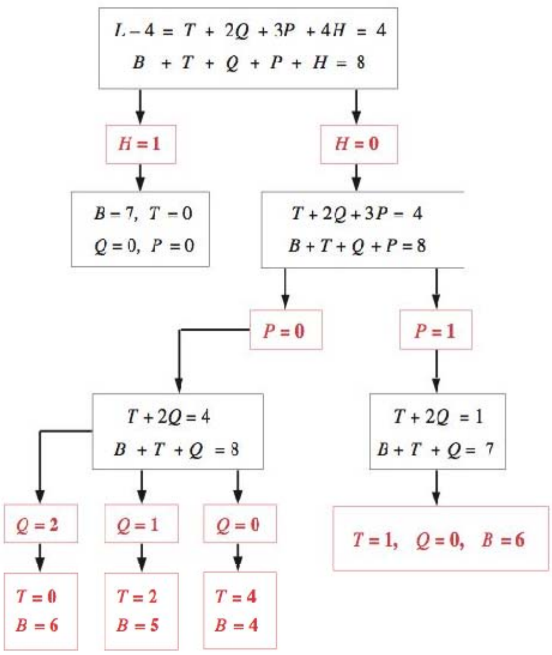
If $Q = 1$, then T must be 0 and:

$$L = B + 0T + 1Q = 6 \qquad B = 5 \qquad Q = 1 \tag{2.7e}$$

There are then two possibilities for $L = 6$. Note that one of them is in fact the simpler fourbar with two ternaries added as was predicted above.

CASE 4. $L = 8$

A tabular approach is needed with this many links:



(2.7f)

2

TABLE 2-2 1-*DOF* Planar Mechanisms with Revolute Joints and Up to 8 Links

Total Links	Link Sets				
	Binary	Ternary	Quaternary	Pentagonal	Hexagonal
4	4	0	0	0	0
6	4	2	0	0	0
6	5	0	1	0	0
8	7	0	0	0	1
8	4	4	0	0	0
8	5	2	1	0	0
8	6	0	2	0	0
8	6	1	0	1	0

From this analysis we can see that, for one *DOF*, there is only one possible four-link configuration, two six-link configurations, and five possibilities for eight links using binary through hexagonal links. Table 2-2 shows the so-called "link sets" for all the possible linkages for one *DOF* up to 8 links and hexagonal order.

2.8 PARADOXES

Because the Gruebler criterion pays no attention to link sizes or shapes, it *can give misleading results* in the face of unique geometric configurations. For example, Figure 2-10a shows a structure (*DOF* = 0) with the ternary links of arbitrary shape. This link arrangement is sometimes called the **"E-quintet,"** because of its resemblance to a capital **E** and the fact that it has five links, including the ground.* It is the next simplest **structural** building block to the **"delta triplet."**

Figure 2-10b shows the same E-quintet with the ternary links straight and parallel and with equispaced nodes. The three binaries are also equal in length. With this very unique geometry, you can see that it will move despite Gruebler's prediction to the contrary.

Figure 2-10c shows a very common mechanism that also disobeys Gruebler's criterion. The joint between the two wheels can be postulated to allow no slip, provided that sufficient friction is available. If no slip occurs, then this is a one-freedom, or full, joint that allows only relative angular motion ($\Delta\theta$) between the wheels. With that assumption, there are 3 links and 3 full joints, from which Gruebler's equation predicts zero *DOF*. However, this linkage does move (actual *DOF* = 1), because the center distance, or length of link 1, is exactly equal to the sum of the radii of the two wheels.

There are other examples of paradoxes that disobey the Gruebler criterion due to their unique geometry. The designer needs to be alert to these possible inconsistencies. Gogu† has shown that none of the simple mobility equations so far discovered (Gruebler, Kutzbach, etc.) are capable of resolving the many paradoxes that exist. A complete analysis of the linkage motions (as described in Chapter 4) is necessary to guarantee mobility.

* It is also called an Assur chain.

† Gogu, G. (2005), Mobility of Mechanisms: A Critical Review." *Mechanism and Machine Theory* (40) pp. 1068-1097.

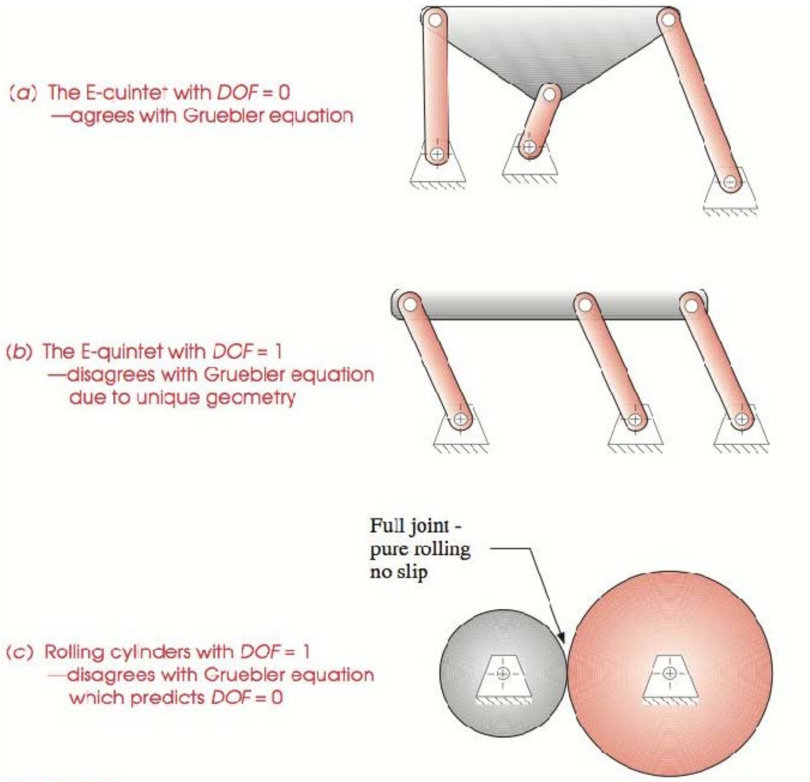


FIGURE 2-10

Gruebler paradoxes—linkages that do not behave as predicted by the Gruebler equation

2.9 ISOMERS

The word **isomer** is from the Greek and means *having equal parts*. Isomers in chemistry are compounds that have the same number and type of atoms but which are interconnected differently and thus have different physical properties. Figure 2-11a shows two hydrocarbon isomers, n-butane and isobutane. Note that each has the same number of carbon and hydrogen atoms (C_4H_{10}), but they are differently interconnected and have different properties.

Linkage isomers are analogous to these chemical compounds in that the **links** (like atoms) have various **nodes** (electrons) available to connect to other links' nodes. The assembled linkage is analogous to the chemical compound. Depending on the particular connections of available links, the assembly will have different motion properties. The number of isomers possible from a given collection of links (as in any row of Table 2-2) is far from obvious. In fact mathematically prediction of the number of isomers of all link combinations has been a long-unsolved problem. Many researchers have spent much

TABLE 2-3
Number of Valid Isomers

Links	Valid Isomers
4	1
6	2
8	16
10	230
12	6856

effort on this problem with some recent success. See references [3] through [7] for more information. Dharaṙipragada [6] presents a good historical summary of isomer research to 1994. Table 2-3 shows the number of valid isomers found for one-*DOF* mechanisms with revolute pairs, up to 12 links.

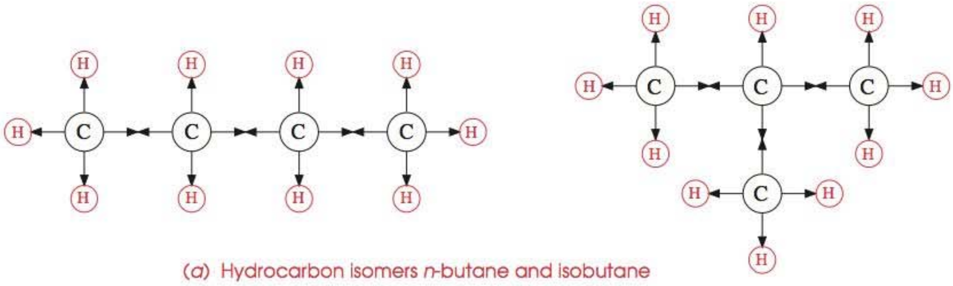
Figure 2-11b shows all the isomers for the simple cases of one *DOF* with 4 and 6 links. Note that there is only one isomer for the case of 4 links. An isomer is only unique if the interconnections between its types of links are different. That is, all binary links are considered equal, just as all hydrogen atoms are equal in the chemical analog. Link lengths and shapes do not figure into the Gruebler criterion or the condition of isomerism. The 6-link case of 4 binaries and 2 ternaries has only two valid isomers. These are known as **Watt's chain** and **Stephenson's chain** after their discoverers. Note the different interconnections of the ternaries to the binaries in these two examples. Watt's chain has the two ternaries directly connected, but Stephenson's chain does not.

There is also a third potential isomer for this case of six links, as shown in Figure 2-11c, but it fails the test of **distribution of degree of freedom**, which requires that the overall *DOF* (here 1) be uniformly distributed throughout the linkage and not concentrated in a subchain. Note that this arrangement (Figure 2-11c) has a **structural subchain** of *DOF* = 0 in the triangular formation of the two ternaries and the single binary connecting them. This creates a truss, or **delta triplet**. The remaining three binaries in series form a fourbar chain (*DOF* = 1) with the structural subchain of the two ternaries and the single binary effectively reduced to a structure that acts like a single link. Thus this arrangement has been reduced to the simpler case of the fourbar linkage despite its six bars. This is an **invalid isomer** and is rejected.

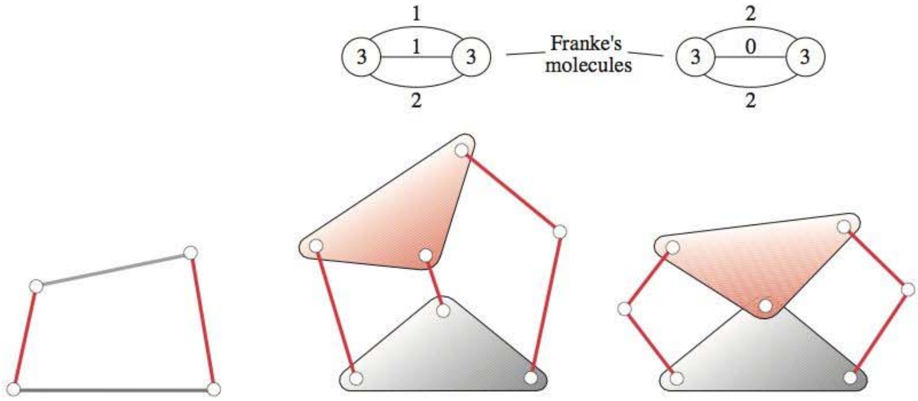
Franke's "Condensed Notation for Structural Synthesis" method can be used to help find the isomers of any collection of links that includes some links of higher order than binary. Each higher-order link is shown as a circle with its number of nodes (its valence) written in it as shown in Figure 2-11. These circles are connected with a number of lines emanating from each circle equal to its valence. A number is placed on each line to represent the quantity of binary links in that connection. This gives a "molecular" representation of the linkage and allows exhaustive determination of all the possible binary link interconnections among the higher links. Note the correspondence in Figure 2-11b between the linkages and their respective Franke molecules. The only combinations of 3 integers (including zero) that add to 4 are: (1, 1, 2), (2, 0, 2), (0, 1, 3), and (0, 0, 4). The first two are, respectively, Stephenson's and Watt's linkages; the third is the invalid isomer of Figure 2-11c. The fourth combination is also invalid as it results in a 2-DOF chain of 5 binaries in series with the 5th "binary" comprised of the two ternaries locked together at two nodes in a preloaded structure with a subchain *DOF* of −1. Figure 2-11d shows all 16 valid isomers of the eightbar 1-*DOF* linkage.

2.10 LINKAGE TRANSFORMATION

The number synthesis techniques described above give the designer a tool kit of basic linkages of particular *DOF*. If we now relax the arbitrary constraint that restricted us to only revolute joints, we can transform these basic linkages to a wider variety of mechanisms with even greater usefulness. There are several transformation techniques or rules that we can apply to planar kinematic chains.



(a) Hydrocarbon isomers n-butane and isobutane

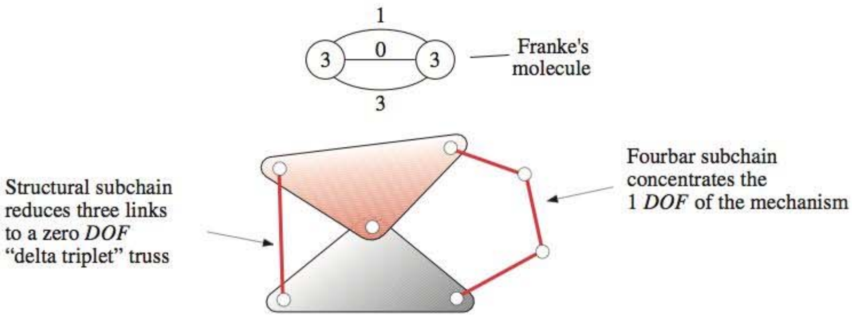


The only fourbar isomer

Stephenson's sixbar isomer

Watt's sixbar isomer

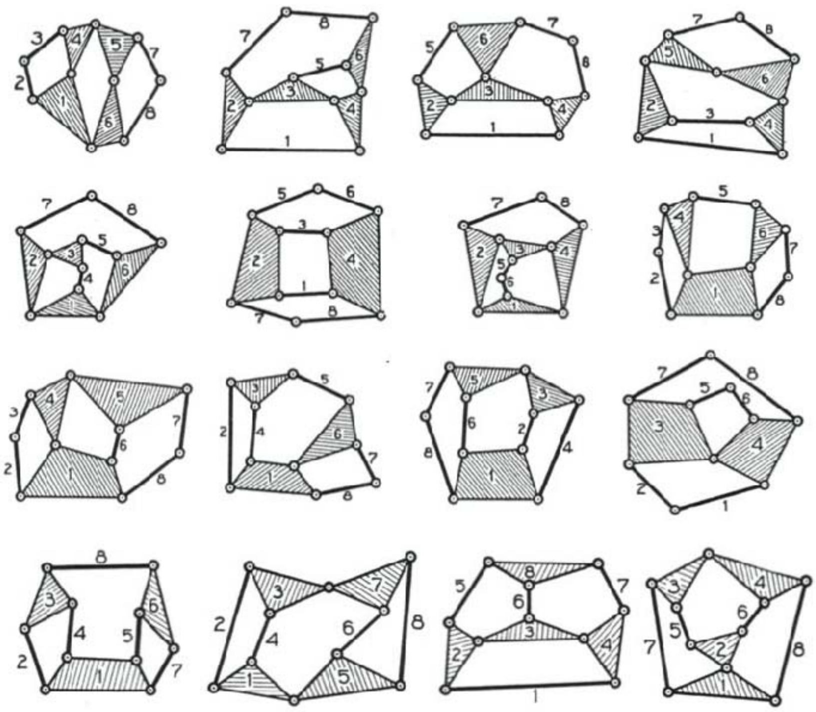
(b) All valid isomers of the fourbar and sixbar linkages



(c) An invalid sixbar isomer which reduces to the simpler fourbar

FIGURE 2-11 Part 1

Isomers of kinematic chains



(d) All the valid eightbar 1-*DOF* isomers

FIGURE 2-11 Part 2

Isomers of kinematic chains (*Source: Klein, A. W., 1917, Kinematics of Machinery, McGraw-Hill, NY.*)

1 Revolute joints in any loop can be replaced by prismatic joints with no change in *DOF* of the mechanism, provided that at least two revolute joints remain in the loop.*

2 Any full joint can be replaced by a half joint, but this will increase the *DOF* by one.

3 Removal of a link will reduce the *DOF* by one.

4 The combination of rules 2 and 3 above will keep the original *DOF* unchanged.

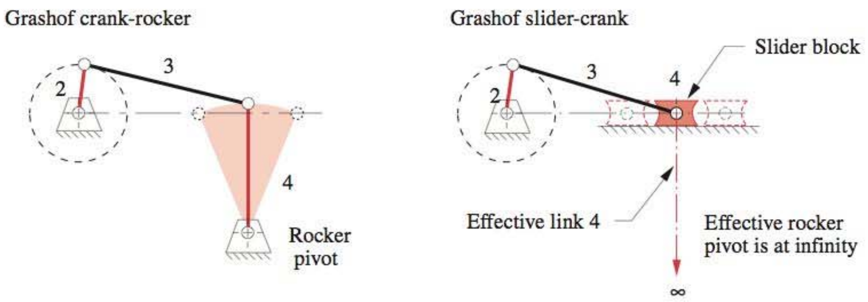
5 Any ternary or higher-order link can be partially "shrunk" to a lower-order link by coalescing nodes. This will create a multiple joint but will not change the *DOF* of the mechanism.

6 Complete shrinkage of a higher-order link is equivalent to its removal. A multiple joint will be created, and the *DOF* will be reduced.

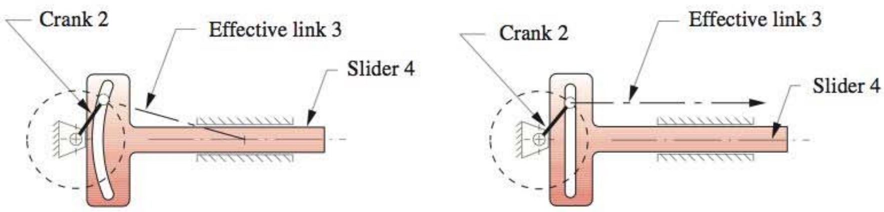
Figure 2-12a† shows a fourbar crank-rocker linkage transformed into the fourbar slider-crank by the application of rule #1. It is still a fourbar linkage. Link 4 has become a sliding block. Gruebler's equation is unchanged at one *DOF* because the slider block

* If all revolute joints in a fourbar linkage are replaced by prismatic joints, the result will be a two-*DOF* assembly. Also, if three revolute joints in a fourbar loop are replaced with prismatic joints, the one remaining revolute joint will not be able to turn, effectively locking the two pinned links together as one. This effectively reduces the assembly to a threebar linkage which should have zero *DOF*. But a delta triplet with three prismatic joints has one *DOF*—another Gruebler paradox.

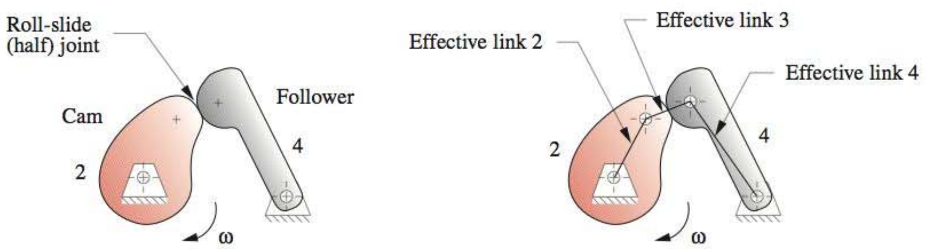
† This figure is provided as animated AVI and Working Model files on the DVD. Its filename is the same as the figure number.



(*a*) Transforming a fourbar crank-rocker to a slider-crank



(*b*) Transforming the slider-crank to the Scotch yoke



(*c*) The cam-follower mechanism has an effective fourbar equivalent

FIGURE 2-12

Linkage transformation

2

provides a full joint against link 1, as did the pin joint it replaces. Note that this transformation from a rocking output link to a slider output link is equivalent to increasing the length (radius) of rocker link 4 until its arc motion at the joint between links 3 and 4 becomes a straight line. Thus the slider block is equivalent to an infinitely long rocker link 4, which is pivoted at infinity along a line perpendicular to the slider axis as shown in Figure 2-12a (p. 51).*

Figure 2-12b* shows a fourbar slider-crank transformed via rule #4 by the substitution of a half joint for the coupler. The first version shown retains the same motion of the slider as the original linkage by use of a curved slot in link 4. The effective coupler is always perpendicular to the tangent of the slot and falls on the line of the original coupler. The second version shown has the slot made straight and perpendicular to the slider axis. The effective coupler now is "pivoted" at infinity. This is called a **Scotch yoke** and gives exact *simple harmonic motion* of the slider in response to a constant speed input to the crank.

Figure 2-12c shows a fourbar linkage transformed into a **cam-follower** linkage by the application of rule #4. Link 3 has been removed and a half joint substituted for a full joint between links 2 and 4. This still has one *DOF*, and the cam-follower is in fact a fourbar linkage in another disguise, in which the coupler (link 3) has become an effective link of *variable length*. We will investigate the fourbar linkage and these variants of it in greater detail in later chapters.

Figure 2-13a shows **Stephenson's sixbar chain** from Figure 2-11b (p. 49) transformed by *partial shrinkage* of a ternary link (rule #5) to create a multiple joint. It is still a one-*DOF* Stephenson sixbar. Figure 2-13b shows **Watt's sixbar chain** from Figure 2-11b (p. 49) with one ternary link *completely shrunk* to create a multiple joint. This is now a structure with *DOF* = 0. The two triangular subchains are obvious. Just as the fourbar chain is the basic building block of one-*DOF* mechanisms, this threebar triangle **delta triplet** is the *basic building block* of zero-*DOF* structures (trusses).

* This figure is provided as animated AVI and Working Model files on the DVD. Its filename is the same as the figure number.

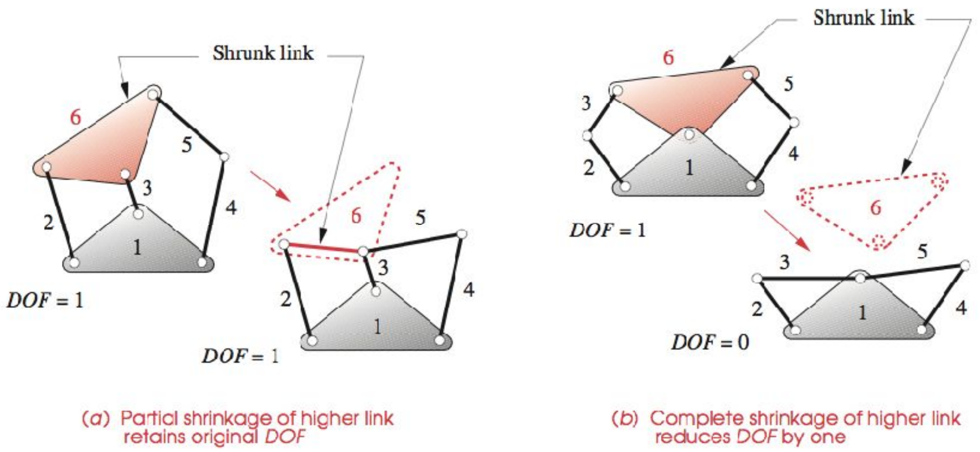


FIGURE 2-13
Link shrinkage

2.11 INTERMITTENT MOTION

Intermittent motion is *a sequence of motions and dwells*. A **dwell** is *a period in which the output link remains stationary while the input link continues to move*. There are many applications in machinery that require intermittent motion. The **cam-follower** variation on the fourbar linkage as shown in Figure 2-12c (p. 51) is often used in these situations. The design of that device for both intermittent and continuous output will be addressed in detail in Chapter 8. Other pure linkage **dwell mechanisms** are discussed in Chapter 3.

GENEVA MECHANISM A common form of intermittent motion device is the **Geneva mechanism** shown in Figure 2-14a (p. 54).* This is also a transformed fourbar linkage in which the coupler has been replaced by a half joint. The input crank (link 2) is typically motor driven at a constant speed. The **Geneva wheel** is fitted with at least three equispaced, radial slots. The crank has a pin that enters a radial slot and causes the Geneva wheel to turn through a portion of a revolution. When the pin leaves that slot, the Geneva wheel remains stationary until the pin enters the next slot. The result is intermittent rotation of the Geneva wheel.

The crank is also fitted with an arc segment, which engages a matching cutout on the periphery of the Geneva wheel when the pin is out of the slot. This keeps the Geneva wheel stationary and in the proper location for the next entry of the pin. The number of slots determines the number of "stops" of the mechanism, where *stop* is synonymous with *dwell*. A Geneva wheel needs a minimum of three stops to work. The maximum number of stops is limited only by the size of the wheel.

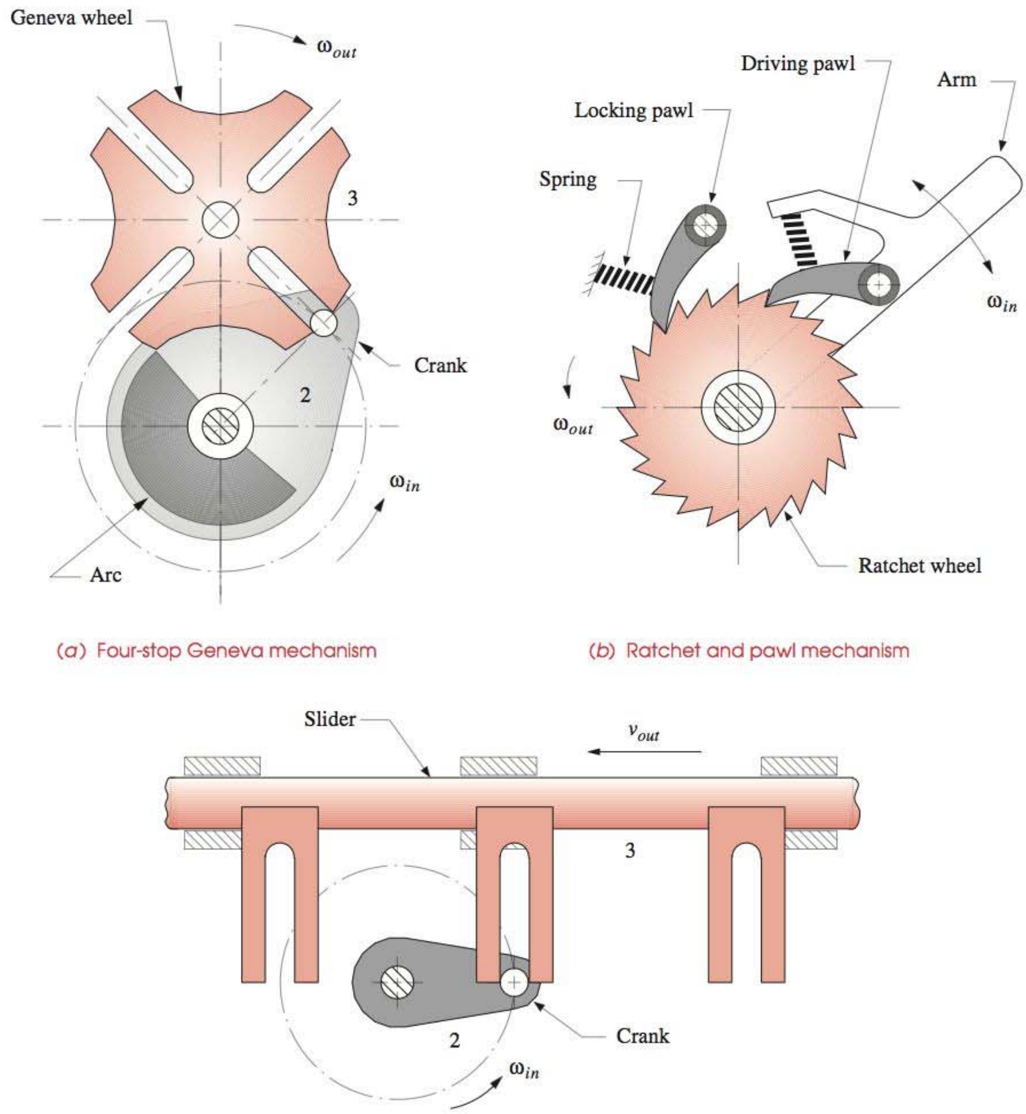
RATCHET AND PAWL Figure 2-14b* shows a ratchet and pawl mechanism. The **arm** pivots about the center of the toothed **ratchet wheel** and is moved back and forth to index the wheel. The **driving pawl** rotates the ratchet wheel (or **ratchet**) in the counterclockwise direction and does no work on the return (clockwise) trip. The **locking pawl** prevents the ratchet from reversing direction while the driving pawl returns. Both pawls are usually spring-loaded against the ratchet. This mechanism is widely used in devices such as "ratchet" wrenches, winches, etc.

LINEAR GENEVA MECHANISM There is also a variation of the Geneva mechanism that has linear translational output, as shown in Figure 2-14c.* This mechanism is analogous to an open Scotch yoke device with multiple yokes. It can be used as an intermittent conveyor drive with the slots arranged along the conveyor chain or belt. It also can be used with a reversing motor to get linear, reversing oscillation of a single slotted output slider.

2.12 INVERSION

It should now be apparent that there are many possible linkages for any situation. Even with the limitations imposed in the number synthesis example (one *DOF*, eight links, up to hexagonal order), there are eight linkage combinations shown in Table 2-2 (p. 46), and these together yield 19 valid isomers as shown in Table 2-3 (p. 48). In addition, we can introduce another factor, namely mechanism inversion. An **inversion** is *created by grounding a different link in the kinematic chain.* Thus there are as many inversions of a given linkage as it has links.

* This figure is provided as animated AVI and Working Model files on the DVD. Its filename is the same as the figure number.



(*a*) Four-stop Geneva mechanism

(*b*) Ratchet and pawl mechanism

(*c*) Linear intermittent motion "Geneva" mechanism

See also Figures P3-7 (p. 160) and P4-6 (p. 220) for other examples of linear intermittent motion mechanisms

FIGURE 2-14

Rotary and linear intermittent motion mechanisms

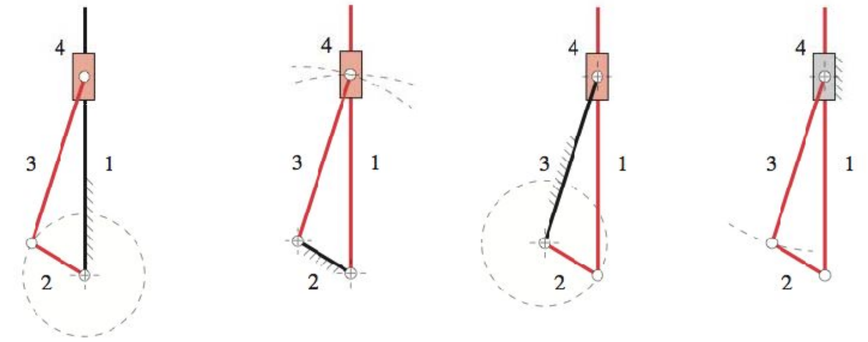


FIGURE 2-15

Four distinct inversions of the fourbar slider-crank mechanism (each black link is stationary—all red links move)

The motions resulting from each inversion can be quite different, but some inversions of a linkage may yield motions similar to other inversions of the same linkage. In these cases only some of the inversions may have distinctly different motions. We will denote the *inversions that have distinctly different motions* as **distinct inversions.**

Figure 2-15* shows the four inversions of the fourbar slider-crank linkage, all of which have distinct motions. Inversion #1, with link 1 as ground and its slider block in pure translation, is the most commonly seen and is used for **piston engines** and **piston pumps**. Inversion #2 is obtained by grounding link 2 and gives the **Whitworth** or **crank-shaper** quick-return mechanism, in which the slider block has complex motion. (Quick-return mechanisms will be investigated further in the Chapter 3.) Inversion #3 is obtained by grounding link 3 and gives the slider block pure rotation. Inversion #4 is obtained by grounding the slider link 4 and is used in hand-operated, **well pump** mechanisms, in which the handle is link 2 (extended) and link 1 passes down the well pipe to mount a piston on its bottom. (It is upside down in the figure.)

Watt's sixbar chain has two distinct inversions, and **Stephenson's sixbar** has three distinct inversions, as shown in Figure 2-16.† The pin-jointed fourbar has four distinct inversions: the crank-rocker, double-crank, double-rocker, and triple-rocker which are shown in Figures 2-17* (p. 57) and 2-18 (p. 58).*

2.13 THE GRASHOF CONDITION§

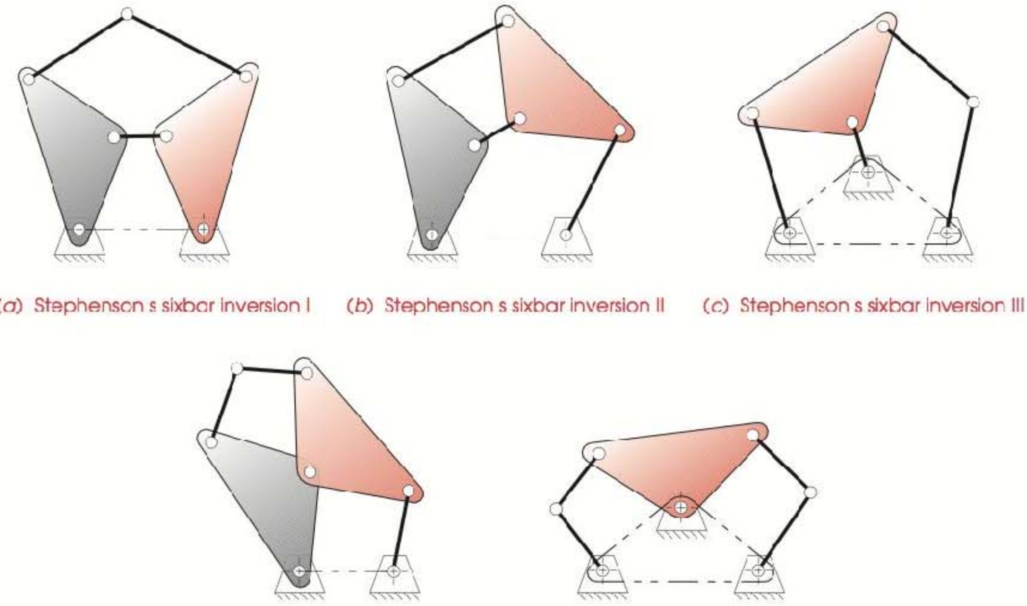
The **fourbar linkage** has been shown above to be the *simplest possible pin-jointed mechanism* for single-degree-of-freedom controlled motion. It also appears in various disguises such as the **slider-crank** and the **cam-follower**. It is in fact the most common and ubiquitous device used in machinery. It is also extremely versatile in terms of the types of motion that it can generate.

* These figures are provided as animated AVI and Working Model files on the DVD. Its filename is the same as the figure number.

§ The Watt I is the only sixbar that has a floating binary link separated from ground by two links at each node, so it is good for long-reach applications and as a parallel motion generator. The Watt II is good for amplifying force or motion and is often used for function generation. The Stephenson III is often used to improve transmission angles by connecting a driven dyad to its coupler. It is also stable due to its three fixed pivots (as is the Watt II). The other two Stephenson inversions are not as often used.

§ A video on *The Grashof Condition* is included on the book's DVD.

2



(a) Stephenson's sixbar inversion I (b) Stephenson's sixbar inversion II (c) Stephenson's sixbar inversion III

(d) Watt's sixbar inversion I (e) Watt's sixbar inversion II

FIGURE 2-16

All distinct inversions of the sixbar linkage

Simplicity is one mark of good design. The fewest parts that can do the job will usually give the least expensive and most reliable solution. Thus the **fourbar linkage** should be among the first solutions to motion control problems to be investigated. The **Grashof condition**[8] is a very simple relationship that predicts the *rotation behavior* or **rotatability** of a fourbar linkage's inversions based only on the link lengths.

Let: S = length of shortest link

L = length of longest link

P = length of one remaining link

Q = length of other remaining link

Then if:

$$S + L \le P + Q \tag{2.8}$$

the linkage is **Grashof** and at least one link will be capable of making a full revolution with respect to the ground plane. This is called a **Class I** kinematic chain. If the inequality is not true, then the linkage is **non-Grashof** and *no* link will be capable of a complete revolution relative to any other link.*† This is a **Class II** kinematic chain.

Note that the above statements apply regardless of the order of assembly of the links. That is, the determination of the Grashof condition can be made on a set of unassembled links. Whether they are later assembled into a kinematic chain in S, L, P, Q or S, P, L, Q, or any other order, will *not* change the Grashof condition.

* According to Hunt[18] (p. 84), Waldron proved that in a Grashof linkage, no two of the links other than the crank can rotate more than 180° with respect to one another, but in a non-Grashof linkage (which has no crank) links can have more than 180° of relative rotation.

† See footnote on opposite page.

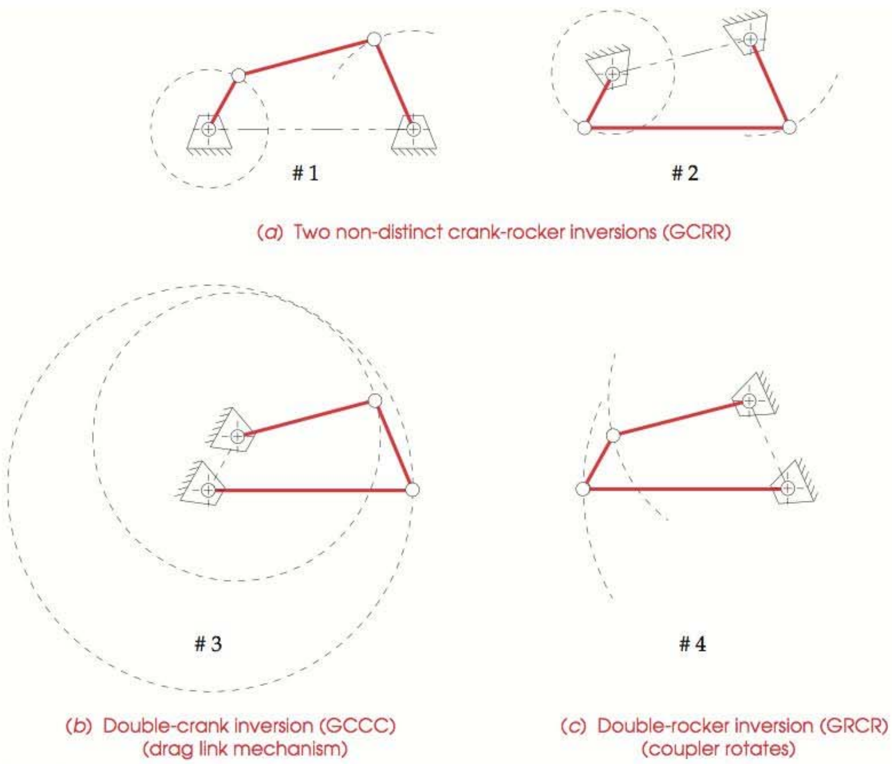


FIGURE 2-17

All inversions of the Grashof fourbar linkage

The motions possible from a fourbar linkage will depend on both the Grashof condition and the **inversion** chosen. The inversions will be defined with respect to the shortest link. The motions are:

For the Class I case, $S + L < P + Q$:

Ground either link adjacent to the shortest and you get a **crank-rocker**, in which the shortest link will fully rotate and the other link pivoted to ground will oscillate.

Ground the shortest link and you will get a **double-crank**, in which both links pivoted to ground make complete revolutions as does the coupler.

Ground the link opposite the shortest and you will get a **Grashof double-rocker**, in which both links pivoted to ground oscillate and only the coupler makes a full revolution.

For the Class II case, $S + L > P + Q$:

All inversions will be **triple-rockers** [9] in which no link can fully rotate.

For the Class III case, $S + L = P + Q$:

† The fourbar slider is a special case. Because two of its links are effectively infinite in length (the effective slider and the effective ground link are parallel and "meet" at infinity), the Grashof condition for a fourbar slider is always true, provided that the link lengths are such that they can physically connect. If so, $S + \infty$ is always <= $P + \infty$.

2

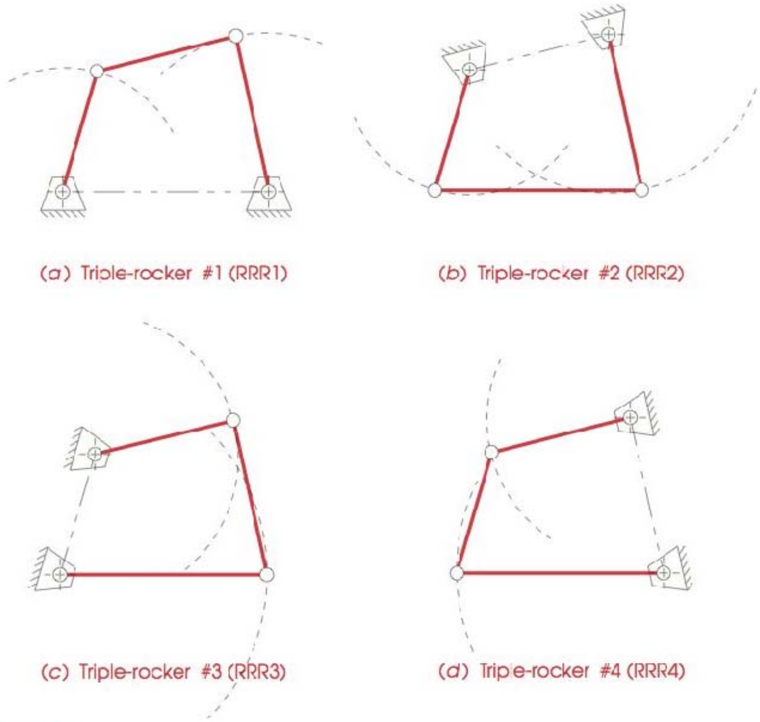


FIGURE 2-18

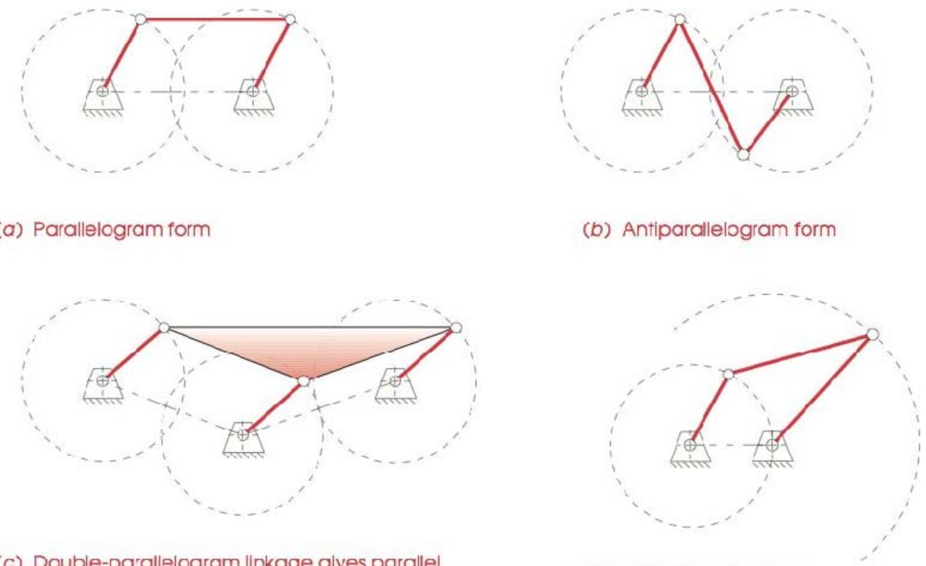
All inversions of the non-Grashof fourbar linkage are triple rockers

Referred to as **special-case Grashof** and also as a **Class III** kinematic chain, all inversions will be either **double-cranks** or **crank-rockers** but will have **"change points"** twice per revolution of the input crank when the links all become colinear. At these change points the output behavior will become indeterminate. Hunt[18] calls these **"uncertainty configurations."** At these colinear positions, the linkage behavior is unpredictable as it may assume either of two configurations. Its motion must be limited to avoid reaching the change points or an additional, out-of-phase link must be provided to guarantee a "carry through" of the change points. (See Figure 2-19c.)

Figure 2-17* (p. 57) shows the four possible inversions of the **Grashof case**: two crank-rockers, a double-crank (also called a drag link), and a double-rocker with rotating coupler. The two crank-rockers give similar motions and so are not distinct from one another. Figure 2-18* shows four nondistinct inversions, all triple-rockers, of a **non-Grashof linkage**.

Figure 2-19a and b shows the **parallelogram** and **antiparallelogram** configurations of the **special-case Grashof** linkage. The parallelogram linkage is quite useful as it exactly duplicates the rotary motion of the driver crank at the driven crank. One common use

* This figure is provided as animated AVI and Working Model files on the DVD. Its filename is the same as the figure number.



(*a*) Parallelogram form

(*b*) Antiparallelogram form

(*c*) Double-parallelogram linkage gives parallel motion (pure curvilinear translation) to coupler and also carries through the change points

(*d*) Deltoid or kite form

FIGURE 2-19

Some forms of the special-case Grashof linkage

is to couple the two windshield wiper output rockers across the width of the windshield on an automobile. The coupler of the parallelogram linkage is in curvilinear translation, remaining at the same angle while all points on it describe identical circular paths. It is often used for this parallel motion, as in truck tailgate lifts and industrial robots.

The antiparallelogram linkage (also called "butterfly" or "bow-tie") is also a double-crank, but the output crank has an angular velocity different from the input crank. Note that the change points allow the linkage to switch unpredictably between the parallelogram and antiparallelogram forms every 180 degrees unless some additional links are provided to carry it through those positions. This can be achieved by adding an out-of-phase companion linkage coupled to the same crank, as shown in Figure 2-19c. A common application of this double parallelogram linkage was on steam locomotives, used to connect the drive wheels together. The change points were handled by providing the duplicate linkage, 90 degrees out of phase, on the other side of the locomotive's axle shaft. When one side was at a change point, the other side would drive it through.

The **double-parallelogram** arrangement shown in Figure 2-19c is quite useful as it gives a translating coupler that remains horizontal in all positions. The two parallelogram stages of the linkage are out of phase so each carries the other through its change points. Figure 2-19d shows the **deltoid** or **kite** configuration that is a double-crank in which the shorter crank makes two revolutions for each one made by the long crank. This is also called an **isoceles** linkage or a **Galloway** mechanism after its discoverer.

2

There is nothing either bad or good about the Grashof condition. Linkages of all three persuasions are equally useful in their place. If, for example, your need is for a motor driven windshield wiper linkage, you may want a non-special-case Grashof crank-rocker linkage in order to have a rotating link for the motor's input, plus a special-case parallelogram stage to couple the two sides together as described above. If your need is to control the wheel motions of a car over bumps, you may want a non-Grashof triple-rocker linkage for short stroke oscillatory motion. If you want to exactly duplicate some input motion at a remote location, you may want a special-case Grashof parallelogram linkage, as used in a drafting machine. In any case, this simply determined condition tells volumes about the behavior to be expected from a proposed fourbar linkage design prior to any construction of models or prototypes.*

Classification of the Fourbar Linkage

Barker[10] has developed a classification scheme that allows prediction of the type of motion that can be expected from a fourbar linkage based on the values of its link ratios. A linkage's angular motion characteristics are independent of the absolute values of its link lengths. This allows the link lengths to be normalized by dividing three of them by the fourth to create three dimensionless ratios that define its geometry.

Let the link lengths be designated r_1, r_2, r_3, and r_4 (all positive and nonzero), with the subscript 1 indicating the ground link, 2 the driving link, 3 the coupler, and 4 the remaining (output) link. The link ratios are then formed by dividing each link length by r_2 giving: $\lambda_1 = r_1 / r_2$, $\lambda_3 = r_3 / r_2$, $\lambda_4 = r_4 / r_2$.

Each link will also be given a letter designation based on its type of motion when connected to the other links. If a link can make a full revolution with respect to the other links, it is called a crank (C), and if not, a rocker (R). The motion of the assembled linkage based on its Grashof condition and inversion can then be given a letter code such as GCRR for a Grashof crank-rocker or GCCC for a Grashof double-crank (drag link) mechanism. The motion designators C and R are always listed in the order of input link, coupler, output link. The prefix G indicates a Grashof linkage, S a special-case Grashof (change point), and no prefix a non-Grashof linkage.

Table 2-4 shows Barker's 14 types of fourbar linkage based on this naming scheme. The first four rows are the Grashof inversions, the next four are the non-Grashof triple-rockers, and the last six are the special-case Grashof linkages. He gives unique names to each type based on a combination of their Grashof condition and inversion. The traditional names for the same inversions are also shown for comparison and are less specific than Barker's nomenclature. Note his differentiation between the Grashof crank-rocker (subclass -2) and rocker-crank (subclass -4). To drive a GRRC linkage from the rocker requires adding a flywheel to the crank as is done with the internal combustion engine's slider-crank mechanism (which is a GPRC linkage). See Figure 2-12a (p. 51).

Barker also defines a *solution space* whose axes are the link ratios λ_1, λ_3, λ_4 as shown in Figure 2-20. These ratios' values theoretically extend to infinity, but for any practical linkages the ratios can be limited to a reasonable value.

In order for the four links to be assembled, the longest link must be shorter than the sum of the other three links,

$$L < (S + P + Q) \tag{2.9}$$

* See the video "The Grashof Condition" on the book's DVD for a more detailed and complete exposition of this topic.

TABLE 2-4 Barker's Complete Classification of Planar Fourbar Mechanisms

Adapted from ref. (10). s = shortest link, l = longest link, Gxxx = Grashof, RRRx = non-Grashof, Sxx = Special case

Type	$s + l$ vs. $p + q$	Inversion	Class	Barker's Designation	Code	Also Known As
1	<	$L_1 = s$ = ground	I-1	Grashof crank-crank-crank	GCCC	Double-crank
2	<	$L_2 = s$ = input	I-2	Grashof crank-rocker-rocker	GCRR	Crank-rocker
3	<	$L_3 = s$ = coupler	I-3	Grashof rocker-crank-rocker	GRCR	Double-rocker
4	<	$L_4 = s$ = output	I-4	Grashof rocker-rocker-crank	GRRC	Rocker-crank
5	>	$L_1 = l$ = ground	II-1	Class 1 rocker-rocker-rocker	RRR1	Triple-rocker
6	>	$L_2 = l$ = input	II-2	Class 2 rocker-rocker-rocker	RRR1	Triple-rocker
7	>	$L_3 = l$ = coupler	II-3	Class 3 rocker-rocker-rocker	RRR3	Triple-rocker
8	>	$L_4 = l$ = output	II-4	Class 4 rocker-rocker-rocker	RRR4	Triple-rocker
9	=	$L_1 = s$ = ground	III-1	Change-point crank-crank-crank	SCCC	SC* double-crank
10	=	$L_2 = s$ = input	III-2	Change-point crank-rocker-rocker	SCRR	SC crank-rocker
11	=	$L_3 = s$ = coupler	III-3	Change-point rocker-crank-rocker	SRCR	SC double-rocker
12	=	$L_4 = s$ = output	III-4	Change-point rocker-rocker-crank	SRRC	SC rocker-crank
13	=	Two equal pairs	III-5	Double change point	S2X	Parallelogram or deltoid
14	=	$L_1 = L_2 = L_3 = L_4$	III-6	Triple change point	S3X	Square

SC = special case

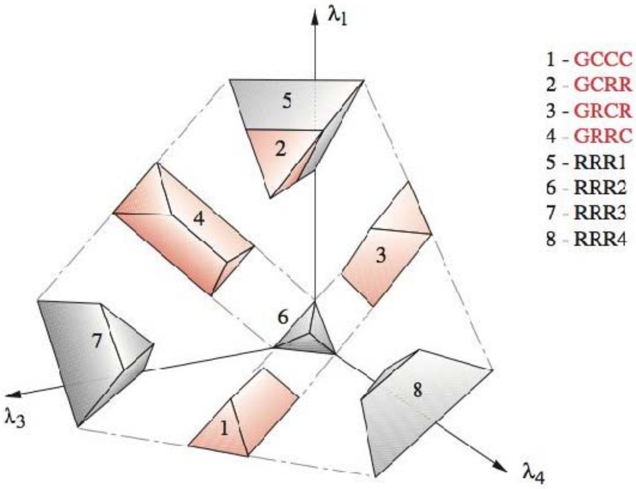


FIGURE 2-20

Barker's solution space for the fourbar linkage *Adapted from reference (10).*

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If $L = S + P + Q$, then the links can be assembled but will not move, so this condition provides a criterion to separate regions of no mobility from regions that allow mobility within the solution space. Applying this criterion in terms of the three link ratios defines four planes of zero mobility that provide limits to the solution space.

$$\begin{aligned} 1 &= \lambda_1 + \lambda_3 + \lambda_4 \\ \lambda_3 &= \lambda_1 + 1 + \lambda_4 \\ \lambda_4 &= \lambda_1 + 1 + \lambda_3 \\ \lambda_1 &= 1 + \lambda_3 + \lambda_4 \end{aligned} \tag{2.10}$$

Applying the $S + L = P + Q$ Grashof condition (in terms of the link ratios) defines three additional planes on which the change-point mechanisms all lie.

$$\begin{aligned} 1 + \lambda_1 &= \lambda_3 + \lambda_4 \\ 1 + \lambda_3 &= \lambda_1 + \lambda_4 \\ 1 + \lambda_4 &= \lambda_1 + \lambda_3 \end{aligned} \tag{2.11}$$

The positive octant of this space, bounded by the λ_1–λ_3, λ_1–λ_4, λ_3–λ_4 planes and the four zero-mobility planes (equation 2.10), contains eight volumes that are separated by the change-point planes (equation 2.11). Each volume contains mechanisms unique to one of the first eight classifications in Table 2-4 (p. 61). These eight volumes are in contact with one another in the solution space, but to show their shapes, they have been "exploded" apart in Figure 2-20 (p. 61). The remaining six change-point mechanisms of Table 2-4 (p. 61) exist only in the change-point planes that are the interfaces between the eight volumes. For more details on this solution space and Barker's classification system than space permits here, see reference [10].

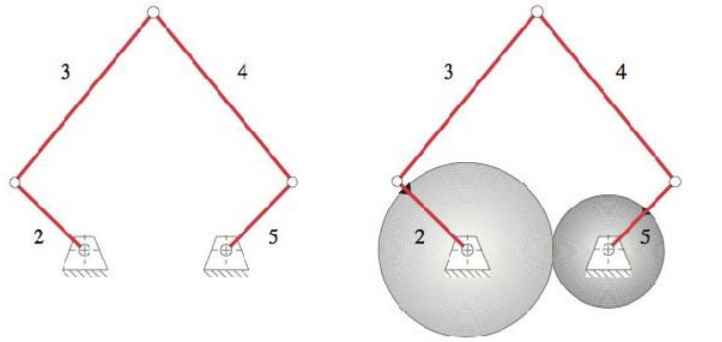
2.14 LINKAGES OF MORE THAN FOUR BARS

Geared Fivebar Linkages

We have seen that the simplest one-*DOF* linkage is the fourbar mechanism. It is an extremely versatile and useful device. Many quite complex motion control problems can be solved with just four links and four pins. Thus in the interest of simplicity, designers should always first try to solve their problems with a fourbar linkage. However, there will be cases when a more complicated solution is necessary. Adding one link and one joint to form a fivebar (Figure 2-21a) will increase the *DOF* by one, to two. By adding a pair of gears to tie two links together with a new half joint, the *DOF* is reduced again to one, and the **geared fivebar mechanism (GFBM)** of Figure 2-21b* is created.

The geared fivebar mechanism provides more complex motions than the fourbar mechanism at the expense of the added link and gearset as can be seen in Appendix E. The reader may also observe the dynamic behavior of the linkage shown in Figure 2-21b by running the program FIVEBAR provided with this text and opening the data file F02-21b.5br. See Appendix A for instructions on running the program. Accept all the default values, and animate the linkage.

* This figure is provided as animated AVI and Working Model files on the DVD. Its filename is the same as the figure number.



(a) Fivebar linkage—2 DOF (b) Geared fivebar linkage—1 DOF

FIGURE 2-21

Two forms of the fivebar linkage

Sixbar Linkages

We already met Watt's and Stephenson's sixbar mechanisms. See Figure 2-16 (p. 56). **Watt's sixbar** can be thought of as *two fourbar linkages connected in series* and sharing two links in common. **Stephenson's sixbar** can be thought of as *two fourbar linkages connected in parallel* and sharing two links in common. Many linkages can be designed by the technique of combining multiple fourbar chains as *basic building blocks* into more complex assemblages. Many real design problems will require solutions consisting of more than four bars. Some Watt's and Stephenson's linkages are provided as built-in examples to the program SIXBAR supplied with this text. You may run that program to observe these linkages dynamically. Select any example from the menu, accept all default responses, and animate the linkages.

Grashof-Type Rotatability Criteria for Higher-Order Linkages

Rotatability is defined as *the ability of at least one link in a kinematic chain to make a full revolution with respect to the other links* and defines the chain as Class I, II, or III. **Revolvability** refers to *a specific link in a chain and indicates that it is one of the links that can rotate*.

ROTATABILITY OF GEARED FIVEBAR LINKAGES Ting[11] has derived an expression for rotatability of the geared fivebar linkage that is similar to the fourbar's Grashof criterion. Let the link lengths be designated L_1 through L_5 in order of increasing length,

then if: $$L_1 + L_2 + L_5 < L_3 + L_4 \qquad (2.12)$$

the two shortest links can revolve fully with respect to the others and the linkage is designated a **Class I** kinematic chain. If this inequality is *not* true, then it is a **Class II** chain and may or may not allow any links to fully rotate depending on the gear ratio and phase angle between the gears. If the inequality of equation 2.12 is replaced with an equal sign,

2

the linkage will be a **Class III** chain in which the two shortest links can fully revolve but it will have change points like the special-case Grashof fourbar.

Reference [11] describes the conditions under which a Class II geared fivebar linkage will and will not be rotatable. In practical design terms, it makes sense to obey equation 2.12 in order to guarantee a Grashof condition. It also makes sense to avoid the Class III change-point condition. Note that if one of the short links (say L_2) is made zero, equation 2.12 reduces to the Grashof formula of equation 2.8 (p. 56).

In addition to the linkage's rotatability, we would like to know about the kinds of motions that are possible from each of the five inversions of a fivebar chain. Ting[11] describes these in detail. But if we want to apply a gearset between two links of the fivebar chain (to reduce its *DOF* to 1), we really need it to be a double-crank linkage, with the gears attached to the two cranks. A Class I fivebar chain will be a **double-crank** mechanism if *the two shortest links are among the set of three links that comprise the mechanism's ground link and the two cranks pivoted to ground.* [11]

ROTATABILITY OF *N*-BAR LINKAGES Ting et al. [12], [13] have extended rotatability criteria to all single-loop linkages of *N*-bars connected with revolute joints and have developed general theorems for **linkage rotatability** and the **revolvability** of individual links based on link lengths. Let the links of an *N*-bar linkage be denoted by L_i (i = 1, 2, . . . , N), with $L_1 \le L_2 \le \cdots \le L_N$. The links need not be connected in any particular order as rotatability criteria are independent of that factor.

A single-loop, revolute-jointed linkage of *N* links will have (*N* – 3) *DOF*. The necessary and sufficient condition for the **assemblability** of an *N*-bar linkage is:

$$L_N \le \sum_{k=1}^{N-1} L_k \tag{2.13}$$

A link *K* will be a so-called *short* link if

$$\{K\}_{k=1}^{N-3} \tag{2.14a}$$

and a so-called *long* link if

$$\{K\}_{k=N-2}^{N} \tag{2.14b}$$

There will be three long links and (*N* – 3) short links in any linkage of this type.

A single-loop *N*-bar kinematic chain containing only first-order revolute joints will be a Class I, Class II, or Class III linkage depending on whether the sum of the lengths of its longest link and its (*N* – 3) shortest links is, respectively, less than, greater than, or equal to the sum of the lengths of the remaining two long links:

$$\begin{aligned} &\text{Class I:} && L_N + (L_1 + L_2 + \cdots + L_{N-3}) < L_{N-2} + L_{N-1} \\ &\text{Class II:} && L_N + (L_1 + L_2 + \cdots + L_{N-3}) > L_{N-2} + L_{N-1} \\ &\text{Class III:} && L_N + (L_1 + L_2 + \cdots + L_{N-3}) = L_{N-2} + L_{N-1} \end{aligned} \tag{2.15}$$

and, for a Class I linkage, there must be one and only one long link between two noninput angles. These conditions are necessary and sufficient to define the rotatability.

The **revolvability** of any link L_i is defined as its ability to rotate fully with respect to the other links in the chain and can be determined from:

$$L_i + L_N \leq \sum_{k=1,\, k \neq i}^{N-1} L_k \tag{2.16}$$

Also, if L_i is a revolvable link, any link that is not longer than L_i will also be revolvable.

Additional theorems and corollaries regarding limits on link motions can be found in references [12] and [13]. Space does not permit their complete exposition here. Note that the rules regarding the behavior of geared fivebar linkages and fourbar linkages (the Grashof law) stated above are consistent with, and contained within, these general rotatability theorems.

2.15 SPRINGS AS LINKS

We have so far been dealing only with rigid links. In many mechanisms and machines, it is necessary to counterbalance the static loads applied to the device. A common example is the hood hinge mechanism on your automobile. Unless you have the (cheap) model with the strut that you place in a hole to hold up the hood, it will probably have either a fourbar or sixbar linkage connecting the hood to the body on each side. The hood may be the coupler of a non-Grashof linkage whose two rockers are pivoted to the body. A spring is fitted between two of the links to provide a force to hold the hood in the open position. The spring in this case is an additional link of variable length. As long as it can provide the right amount of force, it acts to reduce the *DOF* of the mechanism to zero, and holds the system in static equilibrium. However, you can force it to again be a one-*DOF* system by overcoming the spring force when you pull the hood shut.

Another example, which may now be right next to you, is the ubiquitous adjustable arm desk lamp, shown in Figure 2-22.* This device has two springs that counterbalance the weight of the links and lamp head. If well designed and made, it will remain stable over a fairly wide range of positions despite variation in the overturning moment due to the lamp head's changing moment arm. This is accomplished by careful design of the geometry of the spring-link relationships so that, as the spring force changes with increasing length, its moment arm also changes in a way that continually balances the changing moment of the lamp head.

A linear spring can be characterized by its spring constant, $k = F / x$, where F is force and x is spring displacement. Doubling its deflection will double the force. Most coil springs of the type used in these examples are linear.

FIGURE 2-22
A spring-balanced linkage mechanism

2.16 COMPLIANT MECHANISMS

The mechanisms so far described in this chapter all consist of discrete elements in the form of rigid links or springs connected by joints of various types. Compliant mechanisms can provide similar motions with fewer parts and fewer (even zero) physical joints. Compliance is the opposite of stiffness. A member or "link" that is compliant is capable of significant deflection in response to load. An ancient example of a compliant mechanism is the bow and arrow, in which the bow's deflection in response to the archer pulling back

* This figure is provided as animated AVI and Working Model files on the DVD. Its filename is the same as the figure number.

2

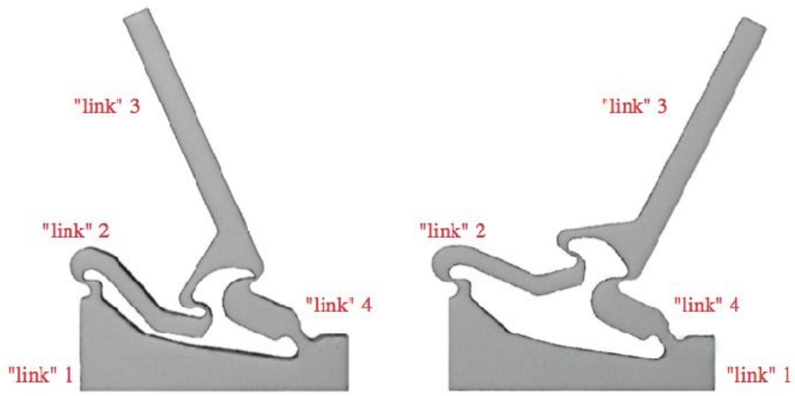
the bowstring stores elastic strain energy in the flexible (compliant) bow, and that energy launches the arrow.

FIGURE 2-23

A tackle box with "living hinge" *Courtesy of Penn Plastics Inc, Bridgeport, CT*

The bow and bowstring comprise two parts, but in its purest form a compliant mechanism consists of a single link whose shape is carefully designed to provide areas of flexibility that serve as pseudo joints. Probably the most commonly available example of a simple compliant mechanism is the ubiquitous plastic tackle box or toolbox made with a "living hinge" as shown in Figure 2-23. This is a dyad or two-link mechanism (box and cover) with a thin section of material connecting the two. Certain thermoplastics, such as polypropylene, allow thin sections to be flexed repeatedly without failure. When the part is removed from the mold, and is still warm, the hinge must be flexed once to align the material's molecules. Once cooled, it can withstand millions of open-close cycles without failure. Figure 2-24 shows a prototype of a fourbar linkage toggle switch made in one piece of plastic as a compliant mechanism. It moves between the on and off positions by flexure of the thin hinge sections that serve as pseudo joints between the "links." The case study discussed in Chapter 1 describes the design of a compliant mechanism that is also shown in Figure 6-13 (p. 307).

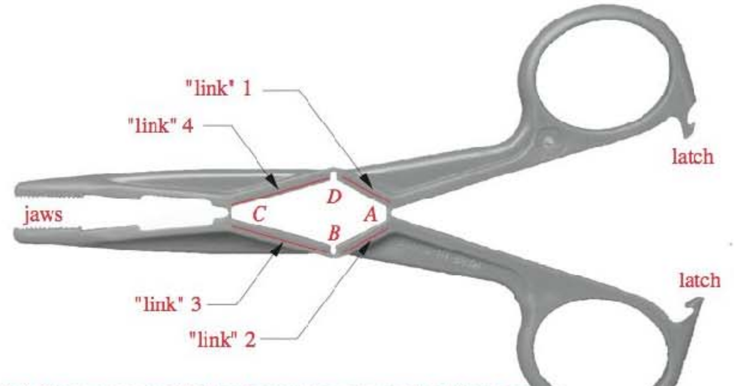
Figure 2-25a shows a forceps designed as a one-piece compliant mechanism. Instead of the conventional two pieces connected by a pin joint, this forceps has small cross sections designed to serve as pseudo joints. It is injection molded of polypropylene thermoplastic with "living hinges." Note that there is a fourbar linkage 1, 2, 3, 4 at the center whose "joints" are the compliant sections of small dimension at A, B, C, and D. The compliance of the material in these small sections provides a built-in spring effect to hold it open in the rest condition. The other portions of the device such as the handles and jaws are designed with stiffer geometry to minimize their deflections. When the user closes the jaws, the hooks on the handles latch it closed, clamping the gripped item. Figure 2-25b shows a two-piece snap hook that uses the compliance of the spring closure that results from either ear of the wire spring being pivoted at different locations A_1 and A_2.



(*a*) Switch on (*b*) Switch off

FIGURE 2-24

One-piece compliant switch *Courtesy of Professor Larry L. Howell, Brigham Young University*



(a) One-piece molded polypropylene compliant forceps
(Nalge Nunc International, Sweden)

(b) Compliant snap hook ™
(Wichard USA, Portsmouth, RI)

FIGURE 2-25
Compliant mechanisms

These examples show some of the advantages of compliant mechanisms over conventional ones. No assembly operations are needed, as there is only one part. The needed spring effect is built in by control of geometry in local areas. The finished part is ready to use as it comes out of the mold. These features all reduce cost.

Compliant mechanisms have been in use for a long time (e.g., the bow and arrow, fingernail clipper, paper clips), but found new applications in the late 20th century due in part to the availability of new materials and modern manufacturing processes. Some of their advantages over conventional mechanisms are the reduction of number of parts, elimination of joint clearances, inherent spring loading, and potential reductions in cost, weight, wear, and maintenance compared to conventional mechanisms. They are, however, more difficult to design and analyze because of their relatively large deflections that preclude the use of conventional small-deflection theory. This text will consider only the design and analysis of noncompliant (i.e., assumed rigid) links and mechanisms with physical joints. For information on the design and analysis of compliant mechanisms see reference [16].

2.17 MICRO ELECTRO-MECHANICAL SYSTEMS (MEMS)*

Recent advances in the manufacture of microcircuitry such as computer chips have led to a new form of mechanism known as micro electro-mechanical systems or MEMS. These devices have features measured in micrometers, and micromachines range in size from a few micrometers to a few millimeters. They are made from the same silicon wafer material that is used for integrated circuits or microchips. The shape or pattern of the desired device (mechanism, gear, etc.) is computer generated at large scale and then photographically reduced and projected onto the wafer. An etching process then removes the silicon material where the image either did or did not alter the photosensitive coating on the silicon (the process can be set to do either). What remains is a tiny reproduction

* More information on MEMS can be found at: http://www.sandia.gov/ and http://www.memsnet.org/mems/

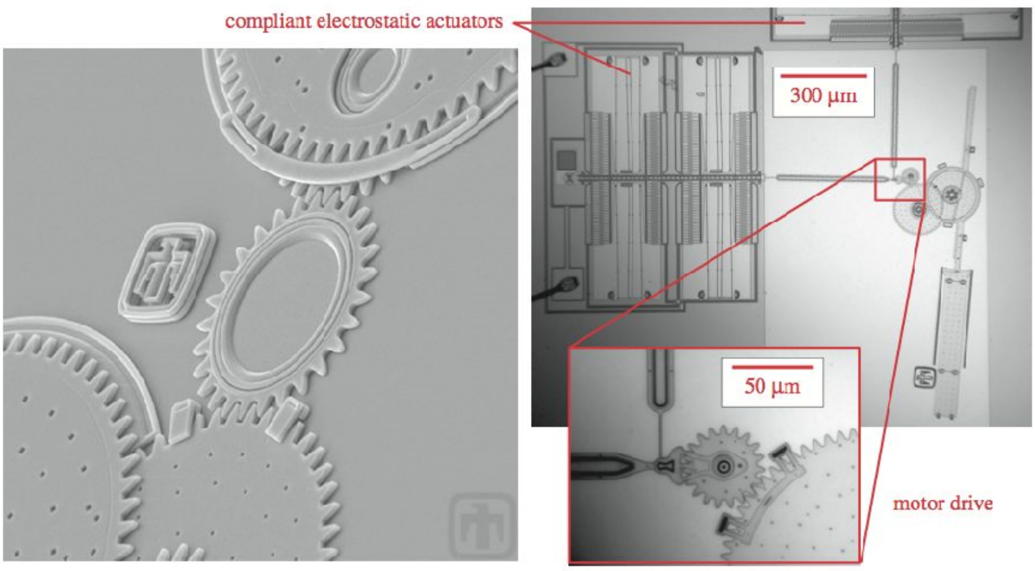
of the original geometric pattern in silicon. Figure 2-26a shows silicon microgears made by this method. They are only a few micrometers in diameter.

Compliant mechanisms are very adaptable to this manufacturing technique. Figure 2-26b shows a micromotor that uses the gears of Figure 2-26a and is smaller than a few millimeters overall. The motor drive mechanism is a series of compliant linkages that are oscillated by an electrostatic field to drive the crank shown in the enlarged view of Figure 2-26b. Two of these electrostatic actuators operate on the same crank, 90° out of phase to carry it through the dead center positions. This motor is capable of continuous speeds of 360 000 rpm and short bursts to a million rpm before overheating from friction at that high speed.

Figure 2-27 shows "a compliant bistable mechanism (known as the Young mechanism) in its two stable positions. Thermal actuators amplify thermal expansion to snap the device between its two positions. It can be used as a microswitch or a microrelay. Because it is so small, it can be actuated in a few hundred microseconds."†

Applications for these micro devices are just beginning to be found. Microsensors made with this technology are currently used in automobile airbag assemblies to detect sudden deceleration and fire the airbag inflator. MEMS blood pressure monitors that can be placed in a blood vessel have been made. MEMS pressure sensors are being fitted to automobile tires to continuously monitor tire pressure. Many other applications are being and will be developed to utilize this technology in the future.

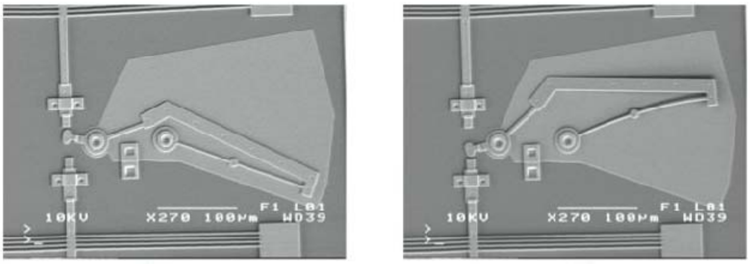
† Professor Larry L. Howell (2002), personal communication.



(a) Microgears (b) Micromotor and gear train

FIGURE 2-26

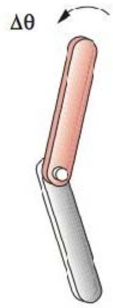
MEMS of etched silicon (a) microgears *Courtesy of Sandia National Laboratories* (b) micromotor by Sandia Labs *SEM photos courtesy of Professor Cosme Furlong, Worcester Polytechnic Institute*



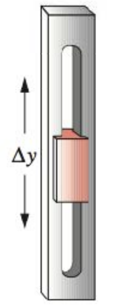
(a) (b)

FIGURE 2-27

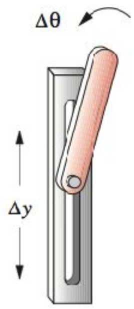
Compliant bistable silicon micromechanism in two positions *Courtesy of Professor Larry L. Howell, Brigham Young University*



(a) Pin joint



(b) Slider joint



(c) Half joint

FIGURE 2-28

Joints of various types

2.18 PRACTICAL CONSIDERATIONS

There are many factors that need to be considered to create good-quality designs. Not all of them are contained within the applicable theories. A great deal of art based on experience is involved in design as well. This section attempts to describe a few such practical considerations in machine design.

Pin Joints versus Sliders and Half Joints

Proper material selection and good lubrication are the key to long life in any situation, such as a joint, where two materials rub together. Such an interface is called a **bearing**. Assuming the proper materials have been chosen, the choice of joint type can have a significant effect on the ability to provide good, clean lubrication over the lifetime of the machine.

REVOLUTE (PIN) JOINTS The simple revolute or pin joint (Figure 2-28a) is the clear winner here for several reasons. It is relatively easy and inexpensive to design and build a good-quality pin joint. In its pure form—a so-called **sleeve** or **journal** bearing—the geometry of pin-in-hole traps a lubricant film within its annular interface by capillary action and promotes a condition called *hydrodynamic lubrication* in which the parts are separated by a thin film of lubricant as shown in Figure 2-29 (p. 70). Seals can easily be provided at the ends of the hole, wrapped around the pin, to prevent loss of the lubricant. Replacement lubricant can be introduced through radial holes into the bearing interface, either continuously or periodically, without disassembly.

A convenient form of bearing for linkage pivots is the commercially available **spherical rod end** shown in Figure 2-30 (p. 70). This has a spherical, sleeve-type bearing that *self-aligns* to a shaft that may be out of parallel. Its body threads onto the link, allowing links to be conveniently made from round stock with threaded ends that allow adjustment of link length.

Relatively inexpensive **ball** and **roller bearings** are commercially available in a large variety of sizes for revolute joints as shown in Figure 2-31 (p. 70). Some of these

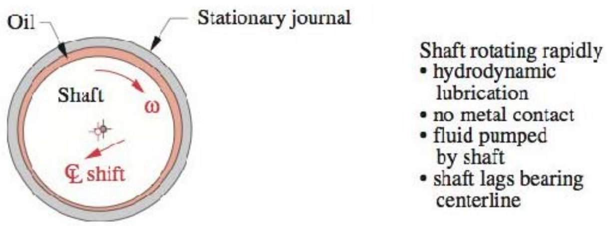
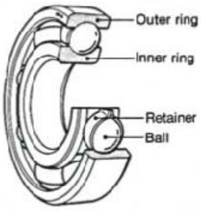
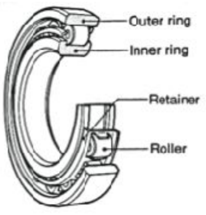


FIGURE 2-29

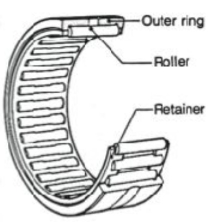
Hydrodynamic lubrication in a sleeve bearing—clearance and motions exaggerated



(a) Ball bearing



(b) Roller bearing



(c) Needle bearing

FIGURE 2-31

Ball, roller, and needle bearings for revolute joints *Courtesy of NTN Corporation, Japan*

bearings (principally ball type) can be obtained prelubricated and with end seals. Their rolling elements provide low-friction operation and good dimensional control. Note that *rolling-element bearings* actually contain higher-joint interfaces (half joints) at each ball or roller, which is potentially a problem as noted below. However, the ability to trap lubricant within the roll cage (by end seals) combined with the relatively high rolling speed of the balls or rollers promotes elastohydrodynamic lubrication and long life. For more detailed information on bearings and lubrication, see reference [15].

For revolute joints pivoted to ground, several commercially available bearing types make the packaging easier. **Pillow blocks** and **flange-mount bearings** (Figure 2-32, p. 71) are available fitted with either rolling-element (ball, roller) bearings or sleeve-type journal bearings. The pillow block allows convenient mounting to a surface parallel to the pin axis, and flange mounts fasten to surfaces perpendicular to the pin axis.

PRISMATIC (SLIDER) JOINTS require a carefully machined and straight slot or rod (Figure 2-28b, p. 69). The bearings often must be custom made, though linear ball bearings (Figure 2-33, p. 71) are commercially available but must be run over hardened and ground shafts. Lubrication is difficult to maintain in any sliding joint. The lubricant is not geometrically captured, and it must be resupplied either by running the joint in an oil bath or by periodic manual regreasing. An open slot or shaft tends to accumulate airborne dirt particles that can act as a grinding compound when trapped in the lubricant. This will accelerate wear.

FIGURE 2-30

Spherical rod end *Courtesy of Emerson Power Transmission, Ithaca, NY*

(a) Pillow-block bearing

(b) Flange-mount bearing

FIGURE 2-32

Pillow block and flange-mount bearing units. *Courtesy of Emerson Power Transmission, Ithaca, NY.*

FIGURE 2-33

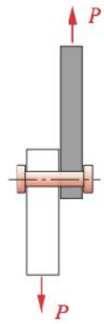
Linear ball bushing
Courtesy of Thomson Industries, Port Washington, NY

Higher (Half) Joints such as a round pin in a slot (Figure 2-28c, p. 69) or a cam-follower joint (Figure 2-12c, p. 51) suffer even more acutely from the slider's lubrication problems, because they typically have two oppositely curved surfaces in line contact, which tend to squeeze any lubricant out of the joint. This type of joint needs to be run in an oil bath for long life. This requires that the assembly be housed in an expensive, oil-tight box with seals on all protruding shafts.

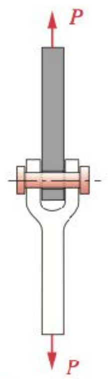
These joint types are all used extensively in machinery with great success. As long as the proper *attention to engineering detail* is paid, the design can be successful. Some common examples of all three joint types can be found in an automobile. The windshield wiper mechanism is a pure pin-jointed linkage. The pistons in the engine cylinders are true sliders and are bathed in engine oil. The valves in the engine are opened and closed by cam-follower (half) joints that are drowned in engine oil. You probably change your engine oil fairly frequently. When was the last time you lubricated your windshield wiper linkage? Has this linkage (not the motor) ever failed?

Cantilever or Straddle Mount?

Any joint must be supported against the joint loads. Two basic approaches are possible as shown in Figure 2-34. A cantilevered joint has the pin (journal) supported only, as a cantilever beam. This is sometimes necessary as with a crank that must pass over the coupler and cannot have anything on the other side of the coupler. However, a cantilever beam is inherently weaker (for the same cross section and load) than a straddle-mounted (simply supported) beam. The straddle mounting can avoid applying a bending moment to the links by keeping the forces in the same plane. The pin will feel a bending moment in both cases, but the straddle-mounted pin is in double shear—two cross sections are sharing the load. A cantilevered pin is in single shear. It is good practice to use straddle-mounted joints (whether revolute, prismatic, or higher) wherever possible. If a cantilevered pin must be used, then a commercial shoulder screw that has a hardened and ground shank as shown in Figure 2-35 (p. 72) can sometimes serve as a pivot pin.



(a) Cantilever mount —single shear



(b) Straddle mount —double shear

FIGURE 2-34

Cantilever, and straddle-mounted pin joints

2

FIGURE 2-35

Shoulder screw
Courtesy of Cordova Bolt Inc., Buena Park, CA

Short Links

It sometimes happens that the required length of a crank is so short that it is not possible to provide suitably sized pins or bearings at each of its pivots. The solution is to design the link as an **eccentric crank**, as shown in Figure 2-36. One pivot pin is enlarged to the point that it, in effect, contains the link. The outside diameter of the circular crank becomes the journal for the moving pivot. The fixed pivot is placed a distance *e* from the center of this circle equal to the required crank length. The distance *e* is the crank's eccentricity (the crank length). This arrangement has the advantage of a large surface area within the bearing to reduce wear, though keeping the large-diameter journal lubricated can be difficult.

Bearing Ratio

The need for straight-line motion in machinery requires extensive use of linear translating slider joints. There is a very basic geometrical relationship called *bearing ratio*, which if ignored or violated will invariably lead to problems.

The **bearing ratio** (BR) is defined as *the effective length of the slider over the effective diameter of the bearing*: $BR = L / D$. For smooth operation **this ratio should be greater than 1.5, and never less than 1**. The larger it is, the better. **Effective length** is defined as *the distance over which the moving slider contacts the stationary guide*. There need not be continuous contact over that distance. That is, two short collars, spaced far apart, are effectively as long as their overall separation plus their own lengths and are kinematically equivalent to a long tube. **Effective diameter** is *the largest distance across the stationary guides*, in any plane perpendicular to the sliding motion.

If the slider joint is simply a rod in a bushing, as shown in Figure 2-37a, the effective diameter and length are identical to the actual dimensions of the rod diameter and bushing length. If the slider is a platform riding on two rods and multiple bushings, as shown in Figure 2-37b, then the effective diameter and length are the overall width and length, respectively, of the platform assembly. It is this case that often leads to poor bearing ratios.

A common example of a device with a poor bearing ratio is a drawer in an inexpensive piece of furniture. If the only guides for the drawer's sliding motion are its sides running against the frame, it will have a bearing ratio less than 1, since it is wider than it

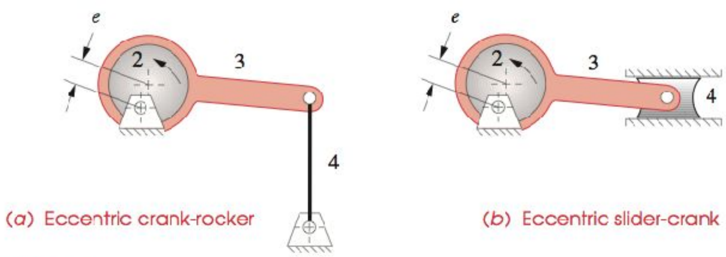


FIGURE 2-36

Eccentric cranks

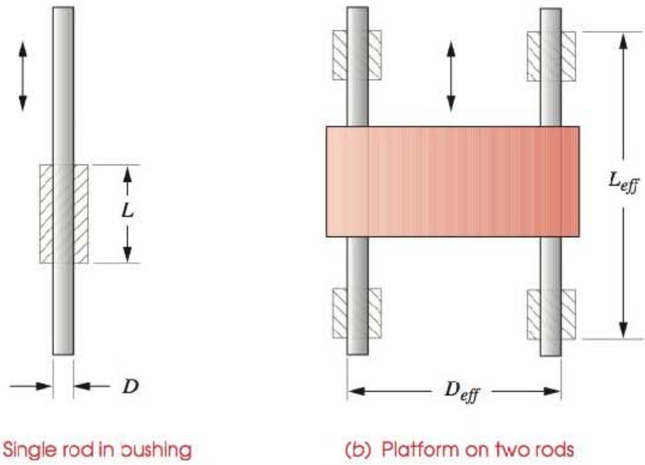


FIGURE 2-37
Bearing ratio

is deep. You have probably experienced the sticking and jamming that occurs with such a drawer. A better-quality chest of drawers will have a center guide with a large L / D ratio under the bottom of the drawer and will slide smoothly.

Commercial Slides

Many companies provide standard linear slides that can be used for slider crank linkages and cam-follower systems with translating followers. These are available with linear ball bearings that ride on hardened steel ways giving very low friction. Some are preloaded to eliminate clearance and backlash error. Others are available with plain bearings. Figure 2-38 shows an example of a ball-bearing linear slide with two cars riding on a single rail. Mounting holes are provided for attaching the rail to the ground plane and in the cars for attaching the elements to be guided.

Linkages versus Cams

The pin-jointed linkage has all the advantages of revolute joints listed above. The cam-follower mechanism (Figure 2-12c, p. 51) has all the problems associated with the half joint listed above. But both are widely used in machine design, often in the same machine and in combination (cams driving linkages). So why choose one over the other?

The "pure" pin-jointed linkage with good bearings at the joints is a potentially superior design, all else equal, and it should be the first possibility to be explored in any machine design problem. However, there will be many problems in which the need for a straight, sliding motion or the exact dwells of a cam-follower are required. Then the practical limitations of cam and slider joints will have to be dealt with accordingly.

FIGURE 2-38
Ball bearing linear slide
Courtesy of THK America Inc., Schaumberg, IL

Linkages have the disadvantage of relatively large size compared to the output displacement of the working portion; thus they can be somewhat difficult to package. Cams tend to be compact in size compared to the follower displacement. Linkages are relatively difficult to synthesize, and cams are relatively easy to design (as long as a computer program such as DYNACAM is available). But linkages are much easier and cheaper to manufacture to high precision than cams. Dwells are easy to get with cams, and difficult with linkages. Linkages can survive very hostile environments, with poor lubrication, whereas cams cannot, unless sealed from environmental contaminants. Linkages have better high-speed dynamic behavior than cams, are less sensitive to manufacturing errors, and can handle very high loads, but cams can match specified motions better.

So the answer is far from clear-cut. It is another *design trade-off situation* in which you must weigh all the factors and make the best compromise. Because of the potential advantages of the pure linkage it is important to consider a linkage design before choosing a potentially easier design task but an ultimately more expensive solution.

2.19 MOTORS AND DRIVERS

Unless manually operated, a mechanism will require some type of driver device to provide the input motion and energy. There are many possibilities. If the design requires a continuous rotary input motion, such as for a Grashof linkage, a slider-crank, or a cam-follower, then a motor or engine* is the logical choice. Motors come in a wide variety of types. The most common energy source for a motor is electricity, but compressed air and pressurized hydraulic fluid are also used to power air and hydraulic motors. Gasoline or diesel engines are another possibility. If the input motion is translation, as is common in earth-moving equipment, then a hydraulic or pneumatic cylinder is usually needed.

Electric Motors

Electric motors are classified both by their function or application and by their electrical configuration. Some functional classifications (described below) are **gearmotors, servomotors,** and **stepping motors.** Many different electrical configurations as shown in Figure 2-39 are also available, independent of their functional classifications. The main electrical configuration division is between **AC** and **DC** motors, though one type, the **universal motor,** is designed to run on either AC or DC.

AC and **DC** refer to *alternating current* and *direct current* respectively. AC is typically supplied by the power companies and, in the United States, will be alternating sinusoidally at 60 hertz (Hz), at about ±120, ±240, or ±480 volts (V) peak. Many other countries supply AC at 50 Hz. Single-phase AC provides a single sinusoid varying with time, and 3-phase AC provides three sinusoids at 120° phase angles. DC is constant with time, supplied from generators or battery sources and is most often used in vehicles, such as ships, automobiles, aircraft, etc. Batteries are made in multiples of 1.5 V, with 6, 12, and 24 V being the most common. Electric motors are also classed by their rated power as shown in Table 2-5. Both AC and DC motors are designed to provide continuous rotary output. While they can be stalled momentarily against a load, they cannot tolerate a full-current, zero-velocity stall for more than a few minutes without overheating.

* The terms *motor* and *engine* are often used interchangeably, but they do not mean the same thing. Their difference is largely semantic, but the "purist" reserves the term *motor* for electrical, hydraulic and pneumatic motors and the term *engine* for thermodynamic devices such as external combustion (steam, stirling) engines and internal combustion (gasoline, diesel) engines. Thus, a conventional automobile is powered by a gasoline or diesel engine, but its windshield wipers and window lifts are run by electric motors. The newest hybrid automobiles have one or more electric motors to drive the wheels plus an engine to charge the battery and supply auxiliary power directly to the wheels. Diesel-electric locomotives are hybrids also, using electric motors at the wheels for direct drive and diesel engines running generators to supply the electricity. Modern commercial ships use a similar arrangement with diesel engines driving generators and electric motors turning the propellers.

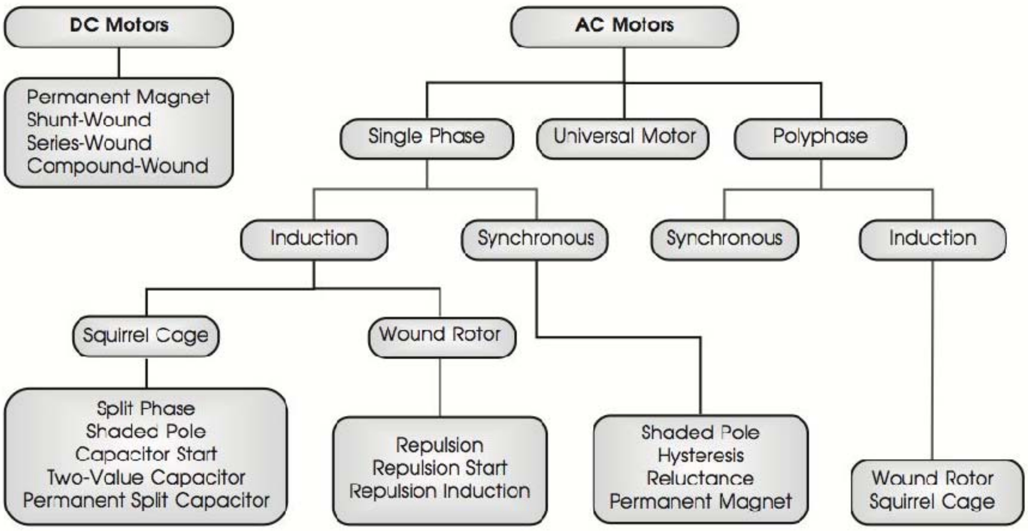


FIGURE 2-39

Types of electric motors *Source: Reference (14)*

DC MOTORS are made in different electrical configurations, such as *permanent magnet (PM), shunt-wound, series-wound, and compound-wound.* The names refer to the manner in which the rotating armature coils are electrically connected to the stationary field coils—in parallel (shunt), in series, or in combined series-parallel (compound). Permanent magnets replace the field coils in a PM motor. Each configuration provides different *torque-speed* characteristics. The *torque-speed* curve of a motor describes how it will respond to an applied load and is of great interest to the mechanical designer as it predicts how the mechanical-electrical system will behave when the load varies dynamically with time.

PERMANENT MAGNET DC MOTORS Figure 2-40a (p. 76) shows a torque-speed curve for a permanent magnet (PM) motor. Note that its torque varies greatly with speed, ranging from a maximum (stall) torque at zero speed to zero torque at maximum (no-load) speed. This relationship comes from the fact that *power = torque* X *angular velocity*. Since the power available from the motor is limited to some finite value, an increase in torque requires a decrease in angular velocity and vice versa. Its torque is maximum at stall (zero velocity), which is typical of all electric motors. This is an advantage when starting heavy loads: e.g., an electric-motor-powered vehicle needs no clutch, unlike one powered by an internal combustion engine that cannot start from stall under load. An engine's torque increases rather than decreases with increasing angular velocity.

Figure 2-40b (p. 76) shows a family of **load lines** superposed on the *torque-speed* curve of a PM motor. These load lines represent a time-varying load applied to the driven mechanism. The problem comes from the fact that *as the required load torque increases, the motor must reduce speed to supply it.* Thus, the input speed will vary in

TABLE 2-5
Motor Power Classes

Class	HP
Subfractional	< 1/20
Fractional	1/20 – 1
Integral	> 1