Chapter 18

Brakes and Clutches

A truck brake drum with cooling fins around the periphery for extended life and improved performance. *Source:* Courtesy of Webb Wheel Products, Inc.

Nothing has such power to broaden the mind as the ability to investigate systematically and truly all that comes under thy observation in life.

Marcus Aurelius, Roman Emperor

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This chapter deals with clutches and brakes, two machine elements that are very similar in function and appearance. Brakes convert mechanical energy to heat, and are widely used to bring all types of machinery to rest. Examples are the thrust pad and long-shoe, internal expanding (drum) brakes on automobiles, and external expanding and pivot shoe brakes in machinery. Clutches serve to bring one shaft to the same speed as another shaft, and are available in a wide variety of sizes. Both brakes and clutches use opposing surfaces, and rely on friction to fulfill their function. Friction causes heat generation, and it is important that brakes and clutches be designed properly to ovoid overheating. For this reason, clutches for high power applications are often operated while submerged in a fluid (wet clutches). However, thermal and material considerations are paramount for effective brake and clutch designs. The chapter begins by analyzing thrust brakes and clutches, as well as the related cone clutches. Block or short-shoe brakes introduce the concept of self-energizing brake shoes, which is further examined with drum brakes and clutches. Band brakes, pivot-shoe brakes, and slip clutches are also discussed.

Machine elements in this chapter: Brakes and clutches of all kinds.
Typical applications: Automobiles, aircraft, vehicles of all kinds; shafts on any machinery; transmissions.
Competing machine elements: Shaft couplings (Ch. 11), springs (Ch. 17).

Symbols

A	area, m^2; constant
b	cone or face width, m
C	cost
C_p	specific heat, J/(kg°C)
c	constant
D	largest diameter of cone, m
d	smallest diameter of cone, m
d_1-d_{10}	distances used for brakes, m
F	friction force, N
F_1	pin reaction force, N
F_2	actuating force, N
h_p	work or energy conversion rate, W
M	moment, N-m
m_a	mass, kg
N	number of sets of disks
n_s	safety factor
P	normal force, N
p	contact pressure, Pa
Q	energy, J
p_o	uniform pressure, Pa
R	reaction force, N
r	radius, m
T	torque, N-m
$\bar{T}$	dimensionless torque, $\dfrac{T}{(2\mu P r_o)}$
t_m	temperature, °C
u	sliding velocity, m/s
W	actuating force, N
α	half-cone angle, deg
β	radius ratio, $\dfrac{r_i}{r_o}$
β_o	optimum radius ratio
γ	extent of brake pad
θ	circumferential coordinate, deg
θ_a	angle where $p = p_{\max}$, deg
θ_1	location where shoe begins, deg
θ_2	location where shoe ends, deg
μ	coefficient of friction
ϕ	wrap angle, deg
ω	angular velocity, rad/s

Subscripts

c	conduction
d	deenergizing
F	friction force
f	friction
h	convection
i	inner
m	mean
o	outer
P	normal force
p	uniform pressure
s	self-energizing, storage
w	uniform wear

18.1 Introduction

Brakes and clutches are examples of machine elements that use friction in a useful way. Clutches are required when shafts must be frequently connected and disconnected. The function of a clutch is twofold: first, to provide a gradual increase in the angular velocity of the driven shaft, so that its speed can be brought up to that of the driving shaft without shock; second, when the two shafts are rotating at the same angular velocity, to act as a coupling without slip or loss of speed in the driving shaft. A **brake** is a device used to bring a moving system to rest, to slow its speed, or to control its speed to a certain value. The function of the brake is to turn mechanical energy into heat. The design of brakes and clutches is subjected to uncertainties in the value of the coefficient of friction that must necessarily be used. Material selection topics from Sections 3.5 and 3.7, as well as friction and wear covered in Sections 8.8 and 8.9 will be used in this chapter.

Figure 18.1 illustrates selected brakes and clutches that are covered in this chapter. These include a **rim type** that has internal expanding and external contracting shoes, a **band brake**, a **thrust disk**, and a **cone disk**. This figure also shows the actuating forces being applied to each brake or clutch. Table 18.1 summarizes the types of brakes and clutches as well as some typical applications.

Brakes and clutches are similar, but different from other machinery elements in that they are tribological systems where friction is intended to be high. Therefore, much effort has been directed toward identifying and developing materials that result in simultaneous high coefficients of friction and low wear so that a reasonable combination of performance and service life can be achieved. In previous years, brake and clutch materials were asbestos-fiber-containing composites, but the wear particles associated with these materials resulted in excessive health hazards to maintenance personnel. Modern brakes and clutches use "semimetallic" materials (i.e., metals produced using powder metallurgy techniques) in the tribological interface, even though longer life could be obtained by using the older asbestos-based linings. This substitution is a good example of multidisciplinary design, in that a consideration totally outside of mechanical engineering has eliminated a class of materials from consideration.

Typical brake and clutch design also involves selecting components of sufficient size and capacity to attain reasonable service life. Many of the problems are solid mechanics oriented; the associated theory is covered in Chapters 4 to 7 and is not repeated here. Because this chapter is mainly concerned with the performance of brake and clutch systems, the focus here is on the actuating forces and resultant torques. Components of brake systems, such as springs, rivets, etc., are considered elsewhere and will not be repeated here. However, it should be noted that a brake or clutch consists of a number of components integrated into a system.

18.2 Thermal Considerations

A critical consideration in the design of brake and clutch components is temperature. Whenever brakes or clutches are activated, one high-friction material slides over another under a large normal force. The associated energy is converted into heat which always results in elevated temperatures in the lining material. While all brakes encounter wear, thermal effects can lead to accelerated wear and can compromise performance and life. Obviously, such circumstances can only be discovered through periodic inspection. Therefore, regular maintenance of brake and clutch systems is essential, and their service lives are often much lower than those of other machinery elements.

As mentioned above, thermal effects are important in braking and clutch systems. If temperatures become too high, damage to components could result, which could compromise the useful life or performance of brake and clutch systems. Thermally induced damage often takes the following forms:

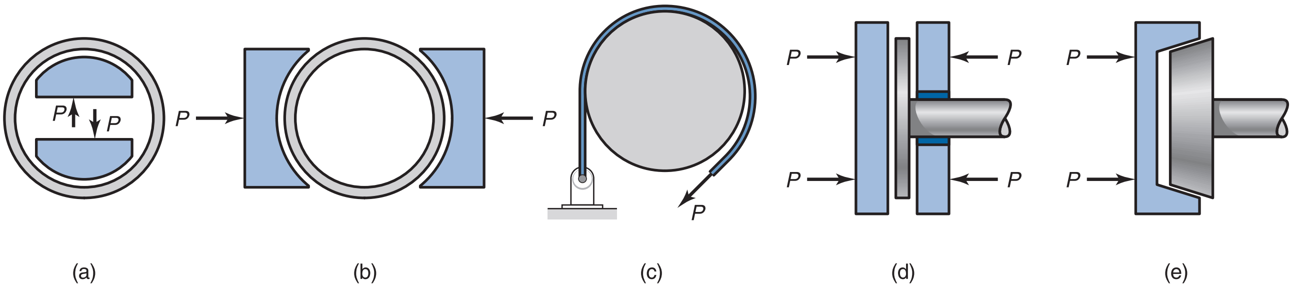

Figure 18.1: Five types of brake and clutch. (a) Internal, expanding rim type; (b) external contracting rim type; (c) band brake; (d) thrust disk; (e) cone disk.

Table 18.1: Typical applications of clutches and brakes.

Type	Application notes
Thrust pad (disc)	Extremely common and versatile arrangement; can be wet or dry; wide variety of materials including carbon-carbon composites for aircraft brakes; preferred for front axles of vehicles because of superior convective cooling; cannot self-lock.
Cone	Higher pressure and torque for the same sized clutch compared to thrust pad due to wedging action of cone; common for lower speed applications with little sliding such as washing machines or extractors, or high-performance applications such as vehicle racing.
Block or short-shoe	Available in a wide variety of configurations and capacities; commonly applied to roller coasters, industrial equipment and positioning devices.
Long-shoe (drum)	Widely applied in vehicles on rear axles; self-locking promotes “parking brake” function; economical and reliable; limited heat dissipation capability.
Pivot-shoe	Used for low-torque applications in architecture, fishing equipement; higher torque applications include hoists and cranes; difficult to properly locate pivot.
Band brakes	Simple, compact, and rugged, widely applied to chain saws, go-karts, motorcycles, and some bicycles; susceptible to chatter or grabbing.
Slip clutches	Used to prevent excessive torque transfer to machinery; available in a wide variety of sizes and capacities; applied to machinery to prevent overload, some garage door operators, cranes as an anti-two blocking device; torque is difficult to control.

- **Warped components**, such as out-of-round drums or non-planar rotors, result from excessive heating and are often associated with recovery of residual stresses. Such conditions can also result from improper machining of components during service or repair, or from mishandling or improper installation.

- **Heat checks**, shown in Fig. 18.2, are small cracks caused by repeated heating and cooling of brake surfaces, and could result in high wear rates and compromised performance. Light heat checking is a normal condition, and the small cracks form and are worn away during normal operation; the component can be machined to remove the damaged surface during service, as long as manufacturer’s tolerances are maintained. Excessive heat checks can be caused by an operator using the brakes excessively, damaged brake components (such as a worn return spring or bushing) or by out-of round or warped drums or rotors.

- **Glazed lining surfaces** are associated with excessive heat that can be attributable to a number of causes, and result in lower friction and diminished performance. Unless the lining is significantly worn, the glazed surfaces can be removed with an abrasive such as emery cloth to re-establish proper performance.

- **Hard spots** in the brake surface (Fig. 18.3) are caused by highly localized heating and cooling cycles, and result in chatter or pulsation during brake action, with the ultimate result of compromised performance and life. Hard spots are often associated with warped or out-of-round components.

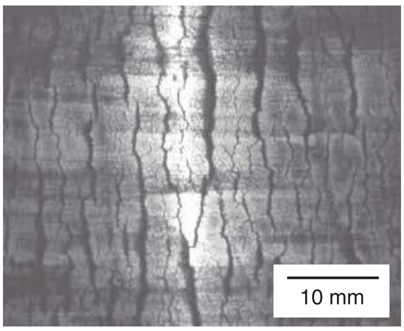

Figure 18.2: Brake drum surface showing a high level of heat checking. *Source:* Courtesy Webb Wheel Products, Inc.

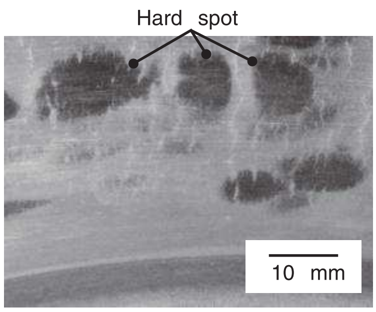

Figure 18.3: Hard spot on a brake drum. *Source:* Courtesy Webb Wheel Products, Inc.

Predicting temperatures of brake and clutch systems is extremely difficult in practice because they are operated under widely varying conditions. Neglecting radiation, the first law of thermodynamics requires that

$$Q_f = Q_c + Q_h + Q_s, \tag{18.1}$$

where

Q_f = energy input into brake or clutch system from friction between sliding elements

Q_c = heat transferred by conduction through machinery elements

Q_h = heat transferred by convection to surrounding environment

Q_s = energy stored in brake and clutch components, resulting in temperature increase

If conduction and convection are negligible, the temperature rise in the brake or clutch material is given by

$$\Delta t_m = \frac{Q_f}{C_p m_a}, \tag{18.2}$$

where C_p is the specific heat of the material, and m_a is the mass. This equation is useful for determining the instantaneous temperature rise in a brake or clutch pad, since the frictional energy is dissipated directly on the contacting surfaces and does not have time to be conducted or conveyed away. Brake pads and clutches usually have an area in contact that then moves out of contact (i.e., heat is conveyed) and can cool. The actual maximum operating temperature is therefore a complicated function of heat input and cooling rates.

The main difficulty in predicting brake system temperatures is that the heat conducted and the heat conveyed depend on the machine ambient temperature and the brake or clutch geometry, and can vary widely. In previous circumstances in this text, a worst-case analysis would be performed, which in this case quickly reduces to circumstances where brakes and clutches become obviously overheated. This result is not incorrect: most brake and clutch systems are overheated when abused and can sustain serious damage as a result. The alternative is to use such massive brake and clutch systems as to make the economic burden unbearable to responsible users. It is far more reasonable to use brake systems that require periodic maintenance and can be damaged through abuse than to incur the economic costs of surviving worst-case analyses. This differs from previous circumstances, where a worst-case analysis still resulted in a reasonable product.

Some clutches are intended to be used with a fluid (**wet clutches**) to aid in cooling the clutch. Similarly, some pads or shoes will include grooves so that air or fluid can be better entrained, and increased flow and convective heat transfer result. Predicting wet clutch temperatures is a complex problem and requires numerical, usually finite element, methods.

Obviously, the proper size of brake components is extremely difficult to determine with certainty. For the purposes of this text the values given by Juvinall and Marshek [2006] for the product of brake shoe or pad contact pressure and sliding velocity, pu, can be used to estimate component sizes (Table 18.2). As discussed in Section 18.3.2, pu is proportional to the power dissipated. Most manufacturers rely heavily on experimental verification of designs; the application of these numbers in the absence of experimental verification requires extreme caution, but is useful for evaluating designs and estimating component sizes.

Table 18.2: Product of contact pressure and sliding velocity for brakes and clutches. *Source:* Adapted from Juvinall, R.C., and Marshek, K.M. [2006].

Operating condition	***pu*, (kPa)(m/s)**
Continuous: poor heat dissipation	1050
Occasional: poor heat dissipation	2100
Continuous: good heat dissipation as in oil bath	3000

18.3 Thrust Pad Clutches and Brakes

A thrust disk has its axis of rotation perpendicular to the contacting surfaces, as shown in Fig. 18.1d; Fig. 18.4 illustrates the components of an automotive disk brake. Basically, a rotor or disk is attached to the vehicle's axle, and a caliper that is mounted on the automobile body contains two brake pads. The pads consist of a friction material supported by a backing plate. Pressurized brake fluid hydraulically actuates the brake cylinder, causing brake pads to bear against the rotor on opposite sides. The pressure applied determines the contacting pressure, friction, and torque, as will be shown below.

Figure 18.5 shows the various radii of the thrust disk clutch or brake. A typical design task is to obtain the axial force, P, needed to produce a certain torque, T, and the resulting contact pressure, p, and wear depth, δ. For some elemental area,

$$dA = (r\,d\theta)\;dr, \tag{18.3}$$

the normal force and the torque can be expressed as

$$dP = p\,dA = pr\,d\theta\,dr, \tag{18.4}$$

and

$$T = \int r\,dF = \int \mu r\,dP = \int\int \mu p r^2\,dr\,d\theta. \tag{18.5}$$

Only a single set of disks will be analyzed in the following sections, but the torque for a single set of disks is multiplied by N to get the torque for N sets of disks.

18.3.1 Uniform Pressure Model

For new, accurately flat, and aligned disks, the pressure will be uniform, or $p = p_o$. Substituting this into Eqs. (18.4) and (18.5) gives

$$P_p = \pi p_o \left(r_o^2 - r_i^2\right), \tag{18.6}$$

$$T_p = \frac{2\pi\mu p_o}{3}\left(r_o^3 - r_i^3\right) = \frac{2\mu P_p \left(r_o^3 - r_i^3\right)}{3\left(r_o^2 - r_i^2\right)}. \tag{18.7}$$

Thus, expressions for the normal load and torque can be determined from the uniform pressure, the geometry (r_o and r_i), and the coefficient of friction, μ.

18.3.2 Uniform Wear Model

If the mating surfaces of the clutch are sufficiently rigid, it can be assumed that uniform wear will occur. This assumption generally holds true after some initial running in. For example, consider the Archard wear law of Eq. (8.39). For a thrust disk clutch, the sliding distance per revolution is proportional to the radius; that is, the outside of the disk sees a

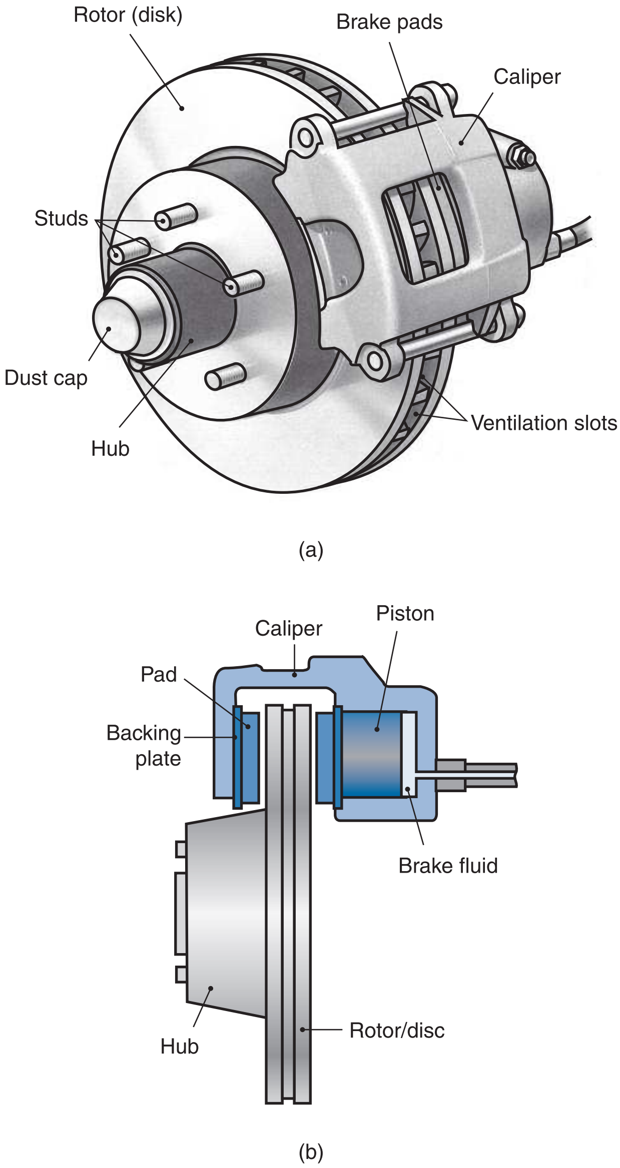

Figure 18.4: Thrust brake terminology and operation. (a) Illustration of a thrust brake, with wheel removed for clarity. Note that the caliper shown has a window to allow observation of the brake pad thickness, a feature that is not always present. (b) Section view of the disk brake, showing the caliper and brake cylinder.

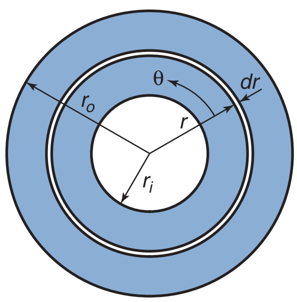

Figure 18.5: Thrust disk clutch surface with various radii.

larger sliding distance than the inner radius. If the pressure is uniform, and the hardness of a brake pad or clutch disk is constant, then more wear will occur on the outside of the disc. Of course, this results in a redistribution of pressure, so that after initial wear has taken place, uniform wear becomes possible. Thus, for the disk shown in Fig. 18.5 the wear is constant over the surface area $r_i \leq r \leq r_o$ and around the circumference of the disk.

The Archard wear law suggests that the rate of wear is proportional to the product of force and velocity, as can be seen by taking the derivative with respect to time of Eq. (8.39). Since the product of force and speed is power,

$$h_p = Fu = \mu Pu = \mu pAu, \tag{18.8}$$

where

F = friction force, N
μ = coefficient of friction
P = normal force, N
u = velocity, m/s
A = area, m^2

If the brake cylinder actuates linearly, and tolerances are reasonably tight, any initial uniform pressure will need to adjust. Where the surfaces wear the most, the pressure decreases the most. Thus, multiplying pressure and velocity will produce a constant work or energy conversion, implying that the wear should be uniform at any radius. Then, μpAu remains constant; and if μA is constant, p is inversely proportional to u, and for any radius, r,

$$p = \frac{c}{r}. \tag{18.9}$$

Substituting Eq. (18.9) into Eq. (18.4) gives

$$P_w = 2\pi c \left(r_o - r_i\right). \tag{18.10}$$

Since $p = p_{\max}$ at $r = r_i$, it follows from Eq. (18.9) that

$$c = p_{\max} r_i. \tag{18.11}$$

Substituting Eq. (18.11) into Eq. (18.10) results in

$$P_w = 2\pi p_{\max} r_i \left(r_o - r_i\right), \tag{18.12}$$

and inserting Eq. (18.9) into Eq. (18.5) produces

$$T_w = \mu c \int\int r\, dr\, d\theta = \frac{2\pi\mu c}{2}\left(r_o^2 - r_i^2\right). \tag{18.13}$$

Substituting Eq. (18.11) into Eq. (18.13) gives

$$T_w = \pi\mu r_i p_{\max}\left(r_o^2 - r_i^2\right). \tag{18.14}$$

Substituting Eq. (18.12) into Eq. (18.14) gives

$$T_w = F_w r_m = \frac{\mu P_w \left(r_o + r_i\right)}{2}. \tag{18.15}$$

By coincidence, Eq. (18.15) gives the same result as if the torque was obtained by multiplying the mean radius $r_m = (r_o + r_i)/2$ by the friction force, F.

Equations (18.7) and (18.15) can be expressed as the dimensionless torque for uniform pressure, $\bar{T}_p$, and for uniform wear, $\bar{T}_w$, by the following equations:

$$\bar{T}_w = \frac{T_w}{2\mu P_w r_o} = \frac{(1+\beta)}{4}, \tag{18.16}$$

$$\bar{T}_p = \frac{T_p}{2\mu P_p r_o} = \frac{(1-\beta^3)}{3(1-\beta^2)}, \tag{18.17}$$

Table 18.3: Representative properties of contacting materials operating dry, when rubbing against smooth cast iron or steel.

Friction material	Coefficient of friction, μ	Maximum contact pressure,[a] $p_{\max}$ kPa	Maximum bulk temperature, $t_{m,\ \max}$ $^\circ$C
Molded	0.25–0.45	1030–2070	204–260
Woven	0.25–0.45	345–690	204–260
Sintered metal	0.15–0.45	1030–2070	204–677
Cork	0.30–0.50	55–95	82
Wood	0.20–0.30	345–620	93
Cast iron; hard steel	0.15–0.25	690–1720	260

[a] Use of lower values will give longer life.

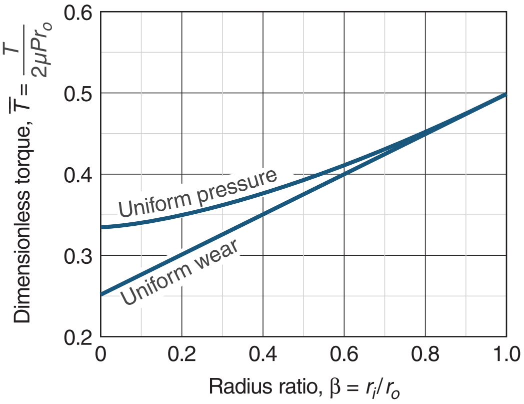

Figure 18.6: Effect of radius ratio on dimensionless torque for uniform pressure and uniform wear models.

where

$$\beta = \frac{r_i}{r_o}. \tag{18.18}$$

Figure 18.6 shows the effect of radius ratio, β, on dimensionless torque for the uniform pressure and uniform wear models. The largest difference between these models occurs at a radius ratio of zero, and the smallest difference occurs at a radius ratio of 1. Also, for the same dimensionless torque the uniform wear model requires a larger radius ratio than does the uniform pressure model. This larger radius ratio implies that a smaller area is predicted by the uniform wear model. Thus, the uniform wear model may be viewed as the safer approach, although the two approaches yield similar torque predictions. Usually a brake or clutch is analyzed using both conditions to make sure a design is robust over its life. The rationale is that, when new, the pressure can be uniform, but as the brake wears, the pressure will adjust until wear is uniform across the pad area.

Table 18.3 gives the coefficient of friction for several materials rubbing against smooth cast iron or steel under dry conditions. It also gives the maximum contact pressure and the maximum bulk temperature for these materials. Table 18.4 gives the coefficient of friction for several materials, including those in Table 18.3, rubbing against smooth cast iron or steel in oil. As would be expected, the coefficients of friction are much smaller in oil than under dry conditions.

Equations (18.6) and (18.7) for the uniform pressure model and Eqs. (18.12) and (18.15) for the uniform wear model, which are applicable for thrust disk clutches, are also applicable for thrust disk brakes provided that the disk shape is similar to that shown in Fig. 18.5. A detailed analysis of disk brakes gives equations that result in slightly larger torques than those resulting from the clutch equations. This text assumes that the brake and clutch equations are identical.

Table 18.4: Coefficient of friction for contacting materials operating in oil when rubbing against steel or cast iron.

Friction material	Coefficient of friction, μ
Molded	0.06–0.09
Woven	0.08–0.10
Sintered metal	0.05–0.08
Paper	0.10–0.14
Graphitic	0.12 (avg.)
Polymeric	0.11 (avg.)
Cork	0.15–0.25
Wood	0.12–0.16
Cast iron; hard steel	0.03–0.16

18.3.3 Partial Contact

It should be noted that many brakes do not use a full circle of contact, as shown in Fig. 18.4. A common practice is to assume that a pad can be approximated as a sector or wedge, so that it covers a fraction of the rotor or disc. This has to be accommodated in the equations for the actuating force and torque developed for the uniform pressure and uniform wear models. This can be easily accomplished by defining γ as the percentage of the rotor covered by the pad. Then, for the uniform pressure model, Eqs. (18.6) and (18.7) become

$$P_p = \gamma\pi p_o\left(r_o^2 - r_i^2\right), \tag{18.19}$$

$$T_p = \frac{2\gamma\pi\mu p_o}{3}\left(r_o^3 - r_i^3\right), \tag{18.20}$$

and for the uniform wear model, Eq. (18.12) and (18.14) become

$$P_w = 2\pi\gamma p_{\max} r_i\left(r_o - r_i\right), \tag{18.21}$$

$$T_w = \pi\gamma\mu r_i p_{\max}\left(r_o^2 - r_i^2\right). \tag{18.22}$$

Example 18.1: Optimum Size of a Thrust Disk Clutch

Given: A single set of thrust disk clutches is to be designed for use in an engine with a maximum torque of 150 N-m. A woven fiber reinforced polymer will contact steel in a dry environment. A safety factor of 1.5 is assumed in order to account for slippage at full engine torque. The outside diameter should be as small as possible.

Find: Determine the appropriate values for r_o, r_i, and P.

Solution: For a woven material in contact with steel in a dry environment, Table 18.3 gives the coefficient of friction as $\mu = 0.35$ and the maximum contact pressure as $p_{\max} = 345$ kPa = 0.345 MPa. The average coefficient of friction has been used to estimate average performance; however, the smallest pressure has been selected for a long life. Making use of the above and Eq. (18.14) gives

$$\begin{aligned} r_i\left(r_o^2 - r_i^2\right) &= \frac{n_s T_w}{\pi \mu p_{\max}} \\ &= \frac{(1.5)(150)}{\pi (0.35)\left(0.345 \times 10^6\right)} \\ &= 5.931 \times 10^{-4}\ \mathrm{m}^3. \end{aligned}$$

Solving for the outside radius, r_o,

$$r_o = \sqrt{\frac{5.931 \times 10^{-4}}{r_i} + r_i^2}. \qquad (a)$$

The minimum outside radius is obtained by taking the derivative of the outside radius with respect to the inside radius and setting it equal to zero:

$$\frac{dr_o}{dr_i} = \frac{0.5}{\sqrt{\frac{5.931 \times 10^{-4}}{r_i} + r_i^2}} \left(-\frac{5.931 \times 10^{-4}}{r_i^2} + 2r_i \right) = 0,$$

which is numerically solved as $r_i = 66.69$ mm. Therefore, Eq. (a) gives

$$r_o = \sqrt{\frac{5.931 \times 10^{-4}}{0.06669} + 0.06669^2} = 0.1155\ \mathrm{m} = 115.5\ \mathrm{mm}.$$

The radius ratio is

$$\beta = \frac{r_i}{r_o} = \frac{66.69}{115.5} = 0.5774.$$

The radius ratio required to maximize the torque capacity is the same as the radius ratio required to minimize the outside radius for a given torque capacity. Thus, the radius ratio for maximizing the torque capacity or for minimizing the outside radius is

$$\beta_o = \sqrt{\frac{1}{3}} = 0.5774.$$

From Eq. (18.15), the maximum normal force that can be applied to the clutch without exceeding the pad pressure constraint is

$$P = \frac{2 n_s T_w}{\mu (r_o + r_i)} = \frac{2(1.5)(150)}{(0.35)(0.1155 + 0.06669)} = 7057\ \mathrm{N}.$$

18.4 Cone Clutches and Brakes

Cone clutches use wedging action to increase the normal force on the lining, thus increasing the friction force and the torque. Usually the half cone angle, α, is above 4° to avoid jamming, and is usually between 6° and 8°. Figure 18.7 shows a conical surface with forces acting on an element. The area of the element and the normal force on the element are

$$dA = (r\, d\theta)\left(\frac{dr}{\sin\alpha}\right), \qquad (18.23)$$

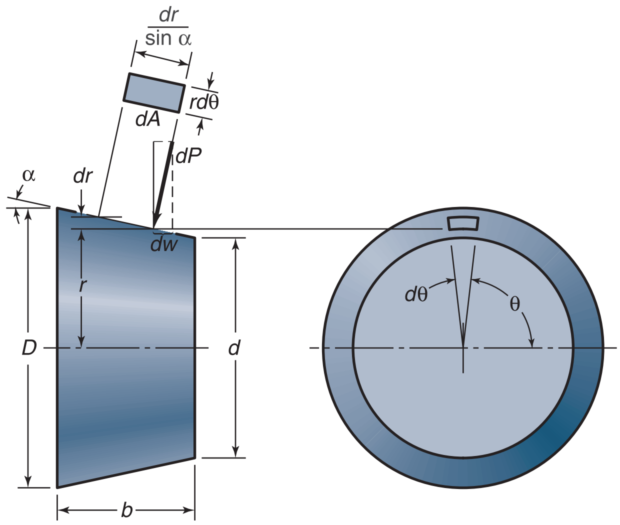

Figure 18.7: Forces acting on elements of a cone clutch.

$$dP = p\, dA. \qquad (18.24)$$

The actuating force is the thrust component, dW, of the normal force, dP, or

$$dW = dP \sin\alpha = p\, dA \sin\alpha = pr\, dr d\theta.$$

Using Eq. (18.4) gives the actuating force as

$$W = \int\int pr\, dr\, d\theta = 2\pi \int_{d/2}^{D/2} pr\, dr. \qquad (18.25)$$

Similarly, Eq. (18.5) gives the torque as

$$T = \int \mu r\, dP = \frac{2\pi}{\sin\alpha} \int_{d/2}^{D/2} \mu p r^2\, dr. \qquad (18.26)$$

18.4.1 Uniform Pressure Model

As was discussed in Section 18.3.1, the pressure for a new thrust disk clutch is assumed to be uniform over the surfaces, or $p = p_o$. Using this uniform pressure model for a cone clutch gives the actuating force as

$$W = \frac{\pi p_o}{4}\left(D^2 - d^2\right). \qquad (18.27)$$

Similarly, the torque is

$$T = \frac{2\pi p_o \mu}{3 \sin\alpha}\left(\frac{1}{8}\right)\left(D^3 - d^3\right) = \frac{\pi p_o \mu}{12 \sin\alpha}\left(D^3 - d^3\right). \qquad (18.28)$$

Making use of Eq. (18.27) enables Eq. (18.28) to be rewritten as

$$T = \frac{\mu W \left(D^3 - d^3\right)}{3 \sin\alpha \left(D^2 - d^2\right)}. \qquad (18.29)$$

18.4.2 Uniform Wear Model

Substituting Eq. (18.9) into Eq. (18.25) gives the actuating force as

$$W = 2\pi c \int_{d/2}^{D/2} dr = \pi c\,(D - d). \qquad (18.30)$$

Similarly, substituting Eq. (18.9) into Eq. (18.26) gives the torque as

$$T = \frac{2\pi\mu c}{\sin\alpha}\int_{d/2}^{D/2} r\,dr = \frac{\pi\mu c}{4\sin\alpha}\left(D^2 - d^2\right). \tag{18.31}$$

Making use of Eq. (18.30) enables Eq. (18.31) to be rewritten as

$$T = \frac{\mu W}{4\sin\alpha}\left(D + d\right). \tag{18.32}$$

Example 18.2: Cone Clutch

Given: A cone clutch similar to that shown in Fig. 18.7 has the following dimensions: D = 330 mm, d = 306 mm, and b = 60 mm. The clutch uses sintered metal on steel, with a coefficient of friction of 0.26, and the torque transmitted is 200 N-m.

Find: Determine the minimum required actuating force and the associated contact pressure by using the uniform pressure and uniform wear models.

Solution:

1. *Uniform wear:* From Fig. 18.7, the half-cone angle of the cone clutch is

$$\tan\alpha = \frac{D-d}{2b} = \frac{165-153}{60} = \frac{12}{60} = 0.200,$$

$$\alpha = 11.31^\circ.$$

The pressure is a maximum when $r = d/2$. Thus, making use of Eqs. (18.31) and (18.9) gives

$$T = \frac{\pi\mu d p_{\max}}{8\sin\alpha}\left(D^2 - d^2\right),$$

and

$$\begin{aligned} p_{\max} &= \frac{8T\sin\alpha}{\pi\mu d\left(D^2 - d^2\right)} \\ &= \frac{8(200)\sin 11.31^\circ}{\pi(0.26)(0.306)(0.330^2 - 0.306^2)} \\ &= 82.25 \text{ kPa}. \end{aligned}$$

From Eq. (18.32), the actuating force can be written as

$$W = \frac{4T\sin\alpha}{\mu(D+d)} = \frac{4(200)\sin 11.31^\circ}{(0.26)(0.330+0.306)} = 948.8 \text{ N}.$$

2. *Uniform pressure:* From Eq. (18.29), the actuating force can be expressed as

$$\begin{aligned} W &= \frac{3T\sin\alpha\left(D^2 - d^2\right)}{\mu\left(D^3 - d^3\right)} \\ &= \frac{3(200)\sin 11.31^\circ\left(0.330^2 - 0.306^2\right)}{(0.26)(0.330^3 - 0.306^3)} \\ &= 948.4 \text{ N}. \end{aligned}$$

From Eq. (18.27), the uniform pressure required is

$$\begin{aligned} p_{\max} &= p_o = \frac{4W}{\pi\left(D^2 - d^2\right)} \\ &= \frac{4(948.4)}{\pi(0.330^2 - 0.306^2)} \text{ Pa} \\ &= 79.11 \text{ kPa}. \end{aligned}$$

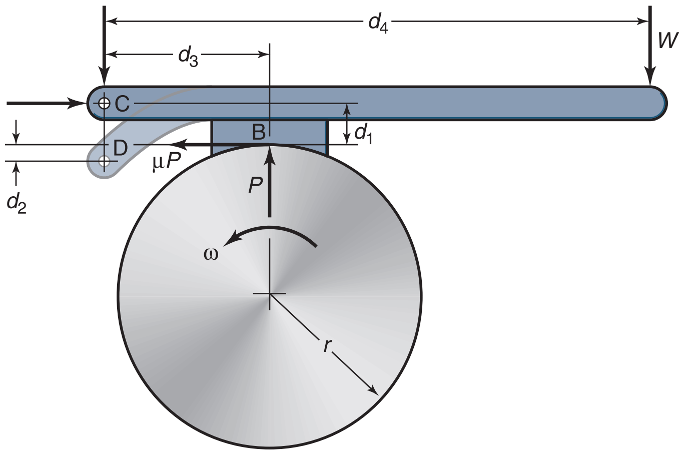

Figure 18.8: Block, or short-shoe brake, with two configurations.

18.5 Block or Short-Shoe Brakes

A **block**, or **short-shoe**, **brake** can be configured to move radially against a cylindrical drum or plate, as shown in Fig. 18.8, although other configurations exist. A normal force, P, develops a friction force, $F = \mu P$, on the drum or plate, where μ is the coefficient of friction. The actuating force, W, is also shown in Fig. 18.8 along with critical hinge pin dimensions d_1, d_2, d_3, and d_4. The normal force, P, and the friction force, μP, are the forces acting on the brake. For block or short-shoe brakes, a constant pressure is assumed over the pad surface. As long as the pad is short relative to the circumference of the drum, this assumption is reasonably accurate.

A brake is considered to be **self-energizing** if the friction moment assists the actuating moment in applying the brake. This implies that the signs of the friction and actuating moments are the same. **Deenergizing** effects occur if the friction moment counteracts the actuating moment in applying the brake. Figure 18.8 can be used to illustrate self-energizing and deenergizing effects. Note that merely changing the cylinder's direction of rotation will change a self-energizing shoe into a deenergizing shoe.

Summing the moments about the hinge at C (see Fig. 18.8) and setting equal to zero results in

$$d_4 W + \mu P d_1 - d_3 P = 0.$$

Since the signs of the friction and actuating moments are the same, the brake hinged at C is self-energizing. Solving for the normal force gives

$$P = \frac{d_4 W}{d_3 - \mu d_1}. \tag{18.33}$$

The braking torque at C is

$$T = Fr = \mu r P = \frac{\mu r d_4 W}{d_3 - \mu d_1}, \tag{18.34}$$

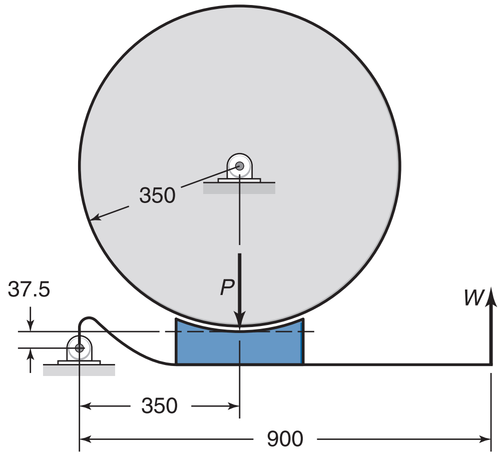

Figure 18.9: Short-shoe brake used in Example 18.3. Dimensions in millimeters.

where r is the radius of the brake drum. Summing the moments about the hinge at D (see Fig. 18.8) and setting the sum equal to zero results in

$$-Wd_4 + \mu P d_2 + d_3 P = 0.$$

Since the signs of the friction and actuating moments are opposite, the brake hinged at D is deenergizing. Solving for the normal force gives

$$P = \frac{Wd_4}{d_3 + \mu d_2}. \tag{18.35}$$

The torque of the brake hinged at D in Fig. 18.8 is

$$T = \frac{\mu d_4 r W}{d_3 + \mu d_2}. \tag{18.36}$$

A brake is considered to be **self-locking** if the actuating force (W in Fig. 18.8) equals zero for a non-zero torque. Self-locking brake geometries are avoided, since they seize or grab, thus operating unsatisfactorily or even dangerously.

Block brakes such as shown in Fig. 18.8 reduce an angular velocity, and in vehicle applications require a high friction force to exist between a wheel and road to affect braking. This does not exist for some conveyor designs, or for vehicles such as roller coasters that ride on rails where the friction between the wheel and rail is very low. In such circumstances, a common brake uses a vehicle-mounted rectangular pad or fin that bears against a flange or fin on the moving component. This is discussed in Case Study 18.1.

With such a block brake, when a plate or fin approaches the brake system, the fins bear against the brake pads and thereby develop a normal force. Not surprisingly, the leading edge of the brake pad will wear more than other sections of the brake. Clearly, the brake does not encounter uniform pressure. However, from a tribological standpoint, there is little difference between a concentrated load and a distributed load in generating a friction force. As long as the real area of contact between the pad and the fin remains small, a linear relationship exists between the friction and the normal force. For all practical purposes, this approximation is reasonable for the entire time of brake-fin contact.

Example 18.3: Short-Shoe Brake

Given: A 350-mm-radius brake drum contacts a single short shoe, as shown in Fig. 18.9, and sustains 225 Nm of torque at 500 rpm. Assume that the coefficient of friction for the drum and shoe combination is $\mu = 0.30$.

Find: Determine the following:

(*a*) Normal force acting on the shoe

(*b*) Required actuating force, W, when the drum has clockwise rotation

(*c*) Required actuating force, W, when the drum has counterclockwise rotation

(*d*) Required change in the 37.5 mm dimension (Fig. 18.9) for self-locking to occur if the other dimensions do not change

Solution:

(*a*) The torque of the brake is

$$T = rF = r\mu P,$$

where

$$P = \frac{T}{\mu r} = \frac{225}{(0.3)(0.350)} = 2140 \text{ N},$$

and

$$\mu P = (0.3)(2140) = 643 \text{ N}.$$

(*b*) For clockwise rotation, summing the moments about the hinge pin and setting the sum equal to zero yields

$$(0.0375)(643) + (0.900)W - (0.350)(2140) = 0, \quad (a)$$

which is solved as $W = 805$ N. Since the signs of the friction and actuating moments are the same, the brake is self-energizing.

(*c*) For counterclockwise rotation, summing the moments about the hinge pin and setting the sum equal to zero gives

$$(0.0375)(643) - (0.900)W + (0.350)(2140) = 0,$$

or $W = 859$ N. Since the signs of the friction and actuating moments are not the same, the brake is deenergizing.

(*d*) If, in Eq. (*a*), $W = 0$,

$$x(643) + (0.900)W - (0.350)(2140) = 0,$$

or, solving for x,

$$x = \frac{(0.350)(2140)}{643} = 1165 \text{ mm}$$

Therefore, self-locking will occur if the distance of 37.5 mm in Fig. 18.9 is changed to 1.165 m. Since self-locking is not a desirable effect in a brake and 37.5 mm is quite different from 1.165 m, we would not expect the brake to self-lock.

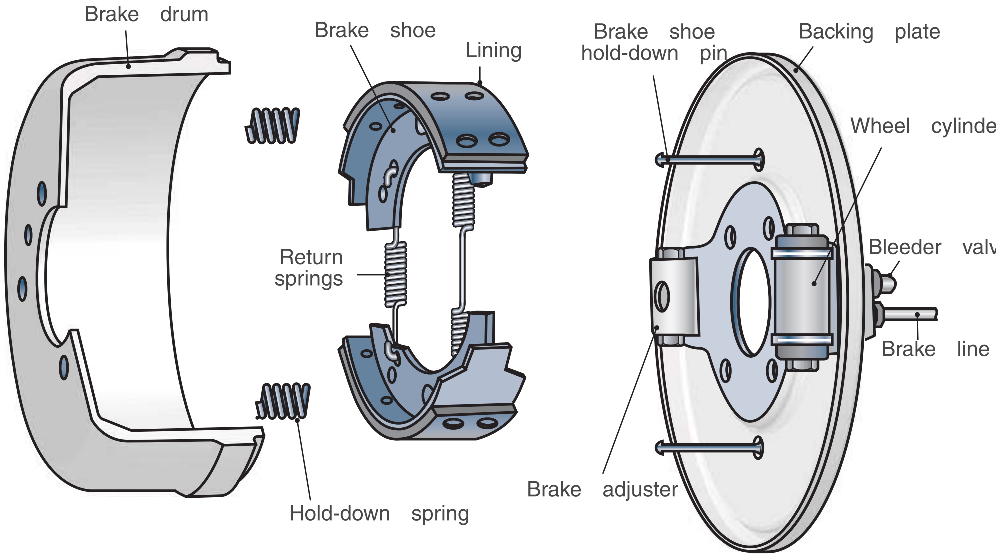

Figure 18.10: A typical automotive long-shoe, internal, expanding rim brake, commonly called a drum brake.

18.6 Long-Shoe, Internal, Expanding Rim Brakes

Figure 18.10 shows a long-shoe, internal, expanding rim brake with two **pads** or **shoes**, often referred to as **drum brakes**. They are widely used, especially in automobiles, where they are commonly applied for rear axle braking. As mentioned previously, disc brakes are commonly used for front brakes because they promote more convective heat transfer than drum designs, and the front axle encounters more air flow over the brake or disc. This can be accentuated with grooves or holes in the rotor, as shown in this chapter's opening illustration. Drum brakes are used because they have a self-energizing shoe, which is very easy to incorporate into a parking brake feature; disc brakes do not have self-energizing shoes, and when all-wheel disc brakes are used, it is common that a drum brake is built into the rear disc brakes to provide a parking brake feature.

Figure 18.11 illustrates the shoes and the drum, and defines important dimensions. The hinge pin for the right shoe is at A in Fig. 18.11. The *heel* of the pad is the region closest to the hinge pin, and the *toe* is the region closest to the actuating force, W. A major difference between a short shoe (Fig. 18.8) and a long shoe (Fig. 18.11) is that the pressure can be considered constant for a short shoe but not for a long shoe. In a long shoe, little if any pressure is applied at the heel, and the pressure increases toward the toe. This sort of pressure variation suggests that the pressure may vary sinusoidally. Thus, a relationship of the contact pressure, p, in terms of the maximum pressure, $p_{\max}$, may be written as

$$p = p_{\max}\left(\frac{\sin\theta}{\sin\theta_a}\right), \tag{18.37}$$

where θ_a is the angle where pressure is at a maximum value. Observe from Eq. (18.37) that $p = p_{\max}$ when $\theta = 90°$. If the shoe has an angular extent less than 90° (such as is the case in Fig. 18.11 if θ_2 was less than 90°), $p = p_{\max}$ when $\theta_a = \theta_2$.

Observe also in Fig. 18.11 that the distance d_6 is perpendicular to the actuating force W. Figure 18.12 shows the forces and critical dimensions of a long-shoe, internal, expanding rim brake. In Fig. 18.12, the θ-coordinate begins with a line drawn from the center of the drum to the center of the hinge pin. Also, the shoe lining does not begin at $\theta = 0°$, but at some θ_1 and extends until θ_2. At any angle, θ, of the lining, the differential normal force dP is

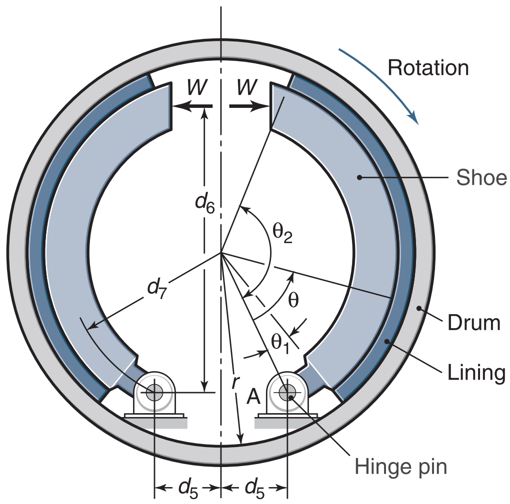

Figure 18.11: Long-shoe, internal, expanding rim brake with two shoes.

$$dP = pbr\,d\theta, \tag{18.38}$$

where b is the face width of the shoe (the distance perpendicular to the paper). Substituting Eq. (18.37) into Eq. (18.38) gives

$$dP = \frac{p_{\max}br\sin\theta\,d\theta}{\sin\theta_a}. \tag{18.39}$$

From Eq. (18.39), the normal force has a moment arm of $d_7\sin\theta$ so that its associated moment is

$$\begin{aligned} M_P &= \int d_7\sin\theta\,dP = \frac{d_7brp_{\max}}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin^2\theta\,d\theta \\ &= \frac{brd_7p_{\max}}{4\sin\theta_1}\left[2\left(\theta_2-\theta_1\right)\frac{\pi}{180°} - \sin 2\theta_2 + \sin 2\theta_1\right], \end{aligned} \tag{18.40}$$

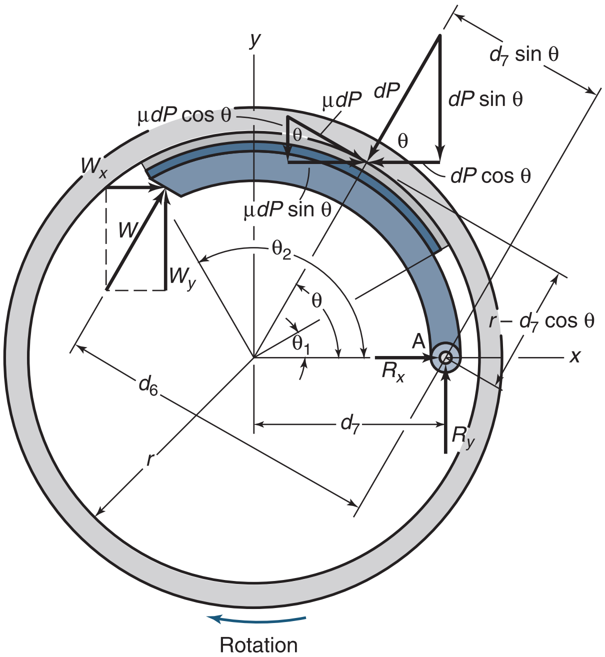

Figure 18.12: Forces and dimensions of long-shoe, internal expanding rim brake.

where θ_1 and θ_2 are in degrees. From Eq. (18.39), the friction force has a moment arm of $r - d_7 \cos\theta$, so that its moment is

$$\begin{aligned} M_F &= \int (r - d_7 \cos\theta)\, \mu\, dP \\ &= \frac{\mu p_{\max} b r}{\sin\theta_1} \int_{\theta_1}^{\theta_2} (r - d_7 \cos\theta) \sin\theta\, d\theta, \end{aligned}$$

or

$$M_F = -\frac{\mu p_{\max} b r}{\sin\theta_a}\left[r\left(\cos\theta_2 + \cos\theta_1\right) - \frac{d_7}{2}\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right] \qquad (18.41)$$

Long-shoe brakes can be designed so that all, some, or none of the shoes are self-energizing. Each type of shoe needs to be analyzed separately.

18.6.1 Self-Energizing Shoe

Setting the sum of the moments about the hinge pin equal to zero results in

$$-W d_6 - M_F + M_P = 0. \qquad (18.42)$$

Since the actuating and friction moments have the same sign in Eq. (18.42), the shoe shown in Fig. 18.12 is self-energizing. It can also be concluded just from Fig. 18.12 that the shoe is self-energizing because W_x and $\mu\, dP \sin\theta$ are in the same direction. Solving for the actuating force in Eq. (18.42) gives

$$W = \frac{M_P - M_F}{d_6}. \qquad (18.43)$$

From Eqs. (8.5) and (18.39), the braking torque is

$$\begin{aligned} T &= \int r \mu\, dP \\ &= \frac{\mu p_{\max} b r^2}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta \\ &= \frac{\mu p_{\max} b r^2 \left(\cos\theta_1 - \cos\theta_2\right)}{\sin\theta_a}. \end{aligned} \qquad (18.44)$$

Figure 18.12 shows the reaction forces as well as the friction force and the normal force. Summing the forces in the x-direction and setting the sum equal to zero results in

$$R_{xs} + W_x - \int \cos\theta\, dP + \int \mu \sin\theta\, dP = 0. \qquad (18.45)$$

Substituting Eq. (18.39) into Eq. (18.45) gives the reaction force in the x-direction for a self-energizing shoe as

$$\begin{aligned} R_{xs} &= \frac{p_{\max,s} b r}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin\theta \cos\theta\, d\theta \\ &\quad - \frac{\mu p_{\max,s} b r}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta - W_x, \end{aligned}$$

or

$$\begin{aligned} R_{xs} &= -W_x + \frac{p_{\max,s} b r}{4 \sin\theta_a}\left\{2\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right\} \\ &\quad - \frac{\mu p_{\max} b r}{4 \sin\theta_a}\left[2\left(\theta_2 - \theta_1\right)\frac{\pi}{180^\circ} - \sin 2\theta_2 + \sin 2\theta_1\right], \end{aligned} \qquad (18.46)$$

where θ_1 and θ_2 are in degrees. From Fig. 18.12, summing the forces in the y-direction and setting the sum equal to zero gives

$$R_{ys} + W_y - \int \mu\, dP \cos\theta - \int dP \sin\theta = 0, \qquad (18.47)$$

and

$$\begin{aligned} R_{ys} &= -W_y + \frac{2\mu p_{\max,s} b r}{4 \sin\theta_a}\left(\sin^2\theta_2 - \sin^2\theta_1\right) \\ &\quad + \frac{p_{\max,s} b r}{4 \sin\theta_a}\left[2\left(\theta_2 - \theta_1\right)\frac{\pi}{180^\circ} - \sin 2\theta_2 + \sin 2\theta_1\right]. \end{aligned} \qquad (18.48)$$

In Eqs. (18.45) to (18.48), the reference system has its origin at the center of the drum. The positive x-axis is taken to be through the hinge pin. The positive y-axis is in the direction of the shoe.

18.6.2 Deenergizing Shoe

If, in Fig. 18.12, the direction of rotation is changed from clockwise to counterclockwise, the friction forces change direction. Thus, summing the moments about the hinge pin and setting the sum equal to zero,

$$-W d_6 + M_F + M_P = 0. \qquad (18.49)$$

The only difference between Eq. (18.42) and Eq. (18.49) is the sign of the friction moment. Solving for the actuating force in Eq. (18.49) gives

$$W = \frac{M_P + M_F}{d_6}. \qquad (18.50)$$

For a deenergizing shoe, the only changes from the equations derived for the self-energizing shoe are that in Eqs. (18.46) and (18.48) a sign change occurs for terms containing the coefficient of friction, μ, resulting in the following:

$$\begin{aligned} R_{xd} &= -W_x + \frac{p_{\max,d} b r}{4 \sin\theta_a}\left[2\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right] \\ &\quad + \frac{\mu p_{\max,d} b r}{4 \sin\theta_a}\left[2\left(\theta_2 - \theta_1\right)\frac{\pi}{180^\circ} - \sin 2\theta_2 + \sin 2\theta_1\right], \end{aligned} \qquad (18.51)$$

$$R_{yd} = -W_y - \frac{\mu p_{\max,d} b r}{2 \sin\theta_a}\left(\sin^2\theta_2 - \sin^2\theta_1\right) + \frac{p_{\max,d} b r}{4 \sin\theta_a}\left[2\left(\theta_2 - \theta_1\right)\frac{\pi}{180^\circ} - \sin 2\theta_2 + \sin 2\theta_1\right]. \quad (18.52)$$

The maximum contact pressure used in evaluating a self-energizing shoe is taken from Table 18.3. The maximum contact pressure used in evaluating a deenergizing shoe is less than that for the self-energizing shoe, since the actuating force is the same for both types of shoe.

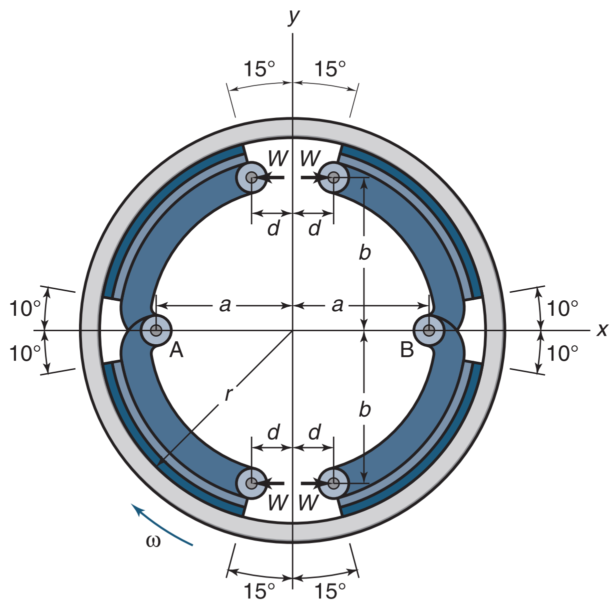

Figure 18.13: Four-long-shoe, internal expanding rim brake used in Example 18.4.

Design Procedure 18.1: Long-Shoe, Internal, Expanding Brake Analysis

It will be assumed that the application has a known velocity, and that the physical dimensions of the brake are known. This Design Procedure outlines the method used to obtain the maximum allowable brake force (which can be controlled by design of the hydraulic or pneumatic actuators) and braking torque.

1. Select a brake material. A reasonable starting point is to assume the drum is made of steel, using sintered metal lining material. Table 18.2 then allows estimation of maximum allowable contact pressure and friction coefficient. Table 18.3 also recommends a maximum pressure, but based on thermal conditions. The lower of the two contact pressures should be used for further analysis.

2. Draw a free body diagram of the brake shoes, paying special attention to the force that acts on the shoes due to friction. Identify which of the shoes, if any, are self-energizing or deenergizing. In a self-energizing shoe, the moment due to frictional force applied to the shoe will have the same sign as the moment due to the applied force. If it is not clear that a shoe is self-energizing or deenergizing, then assume the brake is self-energizing in order to be conservative regarding maximum shoe pressure. In any case, if the friction moment is close to zero, then the braking torque will be similar whether the brake was assumed to be self-energizing or deenergizing.

3. Evaluate M_P and M_F from Eqs. (18.40) and (18.41), respectively. Note that one or more terms may be unknown, but they can be treated as variables.

4. Consider the self-energizing shoe first. The self-energizing shoe will encounter a higher pressure than the deenergizing shoe, so that the limiting pressure determined above can be used to evaluate M_P and M_F.

5. Equation (18.43) can be used to determine the maximum braking force. Note that a lower braking force can be applied, but a higher braking force would exceed the allowable stress of the lining material, leading to plastic deformation or compromised brake life. If the braking force was prescribed, then Eq. (18.43) can be used to obtain the pressure in the shoe, which can be compared to the maximum allowable pressure obtained previously.

6. Equation (18.44) can be used to obtain the torque for the self-energizing shoe.

7. Equations (18.46) and (18.48) can be used to obtain the hinge pin reaction forces.

8. In most brakes, the force applied to the self-energizing and deenergizing shoes are the same. However, the maximum pressure on the deenergizing shoe will be lower than the self-energizing one. Thus, the applied force and pressure can be taken from the self-energizing shoe analysis, as this will reflect the higher stress.

9. Equation (18.50) allows calculation of the maximum pressure on the deenergizing shoe.

10. The torque can be obtained from Eq. (18.45) using the maximum pressure for the deenergizing shoe.

11. Equations (18.51) and (18.52) allow calculation of the hinge pin reaction forces.

Example 18.4: Long-Shoe Internal Brake

Given: Figure 18.13 shows four long shoes in an internal, expanding rim brake. The brake drum has a 400-mm inner diameter. Each hinge pin (A and B) supports a pair of shoes. The actuating mechanism is to be arranged to produce the same actuating force W on each shoe. The shoe face width is 75 mm. The material of the shoe and drum produces a coefficient of friction of 0.24 and a maximum contact pressure of 1 MPa. Additional dimensions for use in Fig. 18.13 are as follows: $d = 50$ mm, $b = 165$ mm, and $a = 150$ mm.

Find:

(*a*) Which shoes are self-energizing and which are deenergizing?

(*b*) What are the actuating forces and total torques for the four shoes?

(*c*) What are the hinge pin reactions as well as the resultant reaction?

Solution:

(*a*) With the drum rotating in the clockwise direction (Fig. 18.13) the top-right and bottom-left shoes have their actuating and friction moments acting in the same direction. Thus, they are self-energizing shoes. The top-left and bottom-right shoes have their actuating and friction moments acting in opposite directions. Thus, they are deenergizing shoes.

(*b*) The dimensions given in Fig. 18.13 correspond to the dimensions given in Figs. 18.11 and 18.12 as $d_5 = 50$ mm, $d_6 = 165$ mm, and $d_7 = 150$ mm. Also, since $\theta_2 < 90^\circ$, then $\theta_2 = \theta_a$. Because the hinge pins in Fig. 18.13 are at A and B, $\theta_1 = 10^\circ$ and $\theta_2 = \theta_a = 75^\circ$.

Self-energizing shoes: Making use of the above and Eq. (18.40) gives the normal force moment as

$$\begin{aligned} M_{Ps} &= \frac{brd_7 p_{\max,s}}{4\sin\theta_a} \\ &\quad \times \left[2\left(\theta_2-\theta_1\right)\frac{\pi}{180^\circ} - \sin 2\theta_2 + \sin 2\theta_1\right] \\ &= \frac{(0.075)(0.2)(0.15)\left(1\times 10^6\right)}{4\sin 75^\circ} \\ &\quad \times \left[2\left(75-10\right)\frac{\pi}{180^\circ} - \sin 150^\circ + \sin 20^\circ\right] \\ &= 1229 \text{ N-m.} \end{aligned}$$

From Eq. (18.41), the friction moment is

$$\begin{aligned} M_{Fs} &= -\frac{\mu p_{\max,s} br}{\sin\theta_a}\left[r\left(\cos\theta_2 - \cos\theta_1\right)\right. \\ &\qquad \left. + \frac{d_7}{2}\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right] \\ &= 288.8 \text{ N-m.} \end{aligned}$$

From Eq. (18.43), the actuating force for both the self-energizing and deenergizing shoes is

$$W_s = W_d = \frac{M_P - M_F}{d_6} = \frac{1229 - 288.8}{0.165} = 5698 \text{ N.}$$

From Eq. (18.44), the braking torque for each self-energizing shoe is

$$\begin{aligned} T_s &= \frac{\mu p_{\max,s} br^2\left(\cos\theta_1 - \cos\theta_2\right)}{\sin\theta_a} \\ &= \frac{(0.24)\left(0.075\times 10^6\right)(0.2)^2(\cos 10^\circ - \cos 75^\circ)}{\sin 75^\circ} \\ &= 541.2 \text{ N-m.} \end{aligned}$$

Deenergizing shoes: The only change in the calculation of the normal and friction moments for the deenergizing shoes is the maximum pressure. This is unknown, but using Eqs. (18.40), (18.41) and (18.50) will allow its determination. M_{Pd} and M_{Fd} are obtained as

$$\begin{aligned} M_{Pd} &= \frac{brd_7 p_{\max,d}}{4\sin\theta_a} \\ &\quad \times \left[2\left(\theta_2-\theta_1\right)\frac{\pi}{180^\circ} - \sin 2\theta_2 + \sin 2\theta_1\right] \\ &= \frac{(0.075)(0.2)(0.15)p_{\max,d}}{4\sin 75^\circ} \\ &\quad \times \left[2\left(75^\circ-10^\circ\right)\frac{\pi}{180^\circ} - \sin 150^\circ + \sin 20^\circ\right] \\ &= 0.001229 p_{\max,d}, \end{aligned}$$

$$\begin{aligned} M_{Fd} &= \frac{\mu p_{\max,d} br}{\sin\theta_a}\left[-r\left(\cos\theta_2 - \cos\theta_1\right)\right. \\ &\qquad \left. - \frac{d_7}{2}\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right] \\ &= 0.0002888 p_{\max,d}. \end{aligned}$$

From Eq. (18.50) the actuating load for the deenergizing shoes is

$$\begin{aligned} W_d &= W_s = 5698 \text{ N} = \frac{M_{Pd} + M_{Fd}}{d_6} \\ &= \left(\frac{0.001229 + 0.0002888}{0.165}\right) p_{\max,d}. \end{aligned}$$

Solving for the maximum pressure yields $p_{\max,d} = 0.6194$ MPa. The braking torque for the deenergizing shoes is

$$T_d = T_s\left(\frac{p_{\max,d}}{p_{\max,s}}\right) = 541.2\left(\frac{0.6194}{1.000}\right) = 335.2 \text{ N-m.}$$

The total braking torque of the four shoes, two of which are self-energizing and two of which are deenergizing, is

$$T = 2\left(T_s + T_d\right) = 2(541.2 + 335.2) = 1753 \text{ N-m.}$$

(*c*) The hinge reactions are obtained from Eqs. (18.46) and (18.48) for the self-energizing and deenergizing shoes, respectively. First, for the self-energizing shoes:

From Eq. (18.46),

$$\begin{aligned} R_{xs} &= -W_x + \frac{p_{\max,s} br}{4\sin\theta_a}\left\{2\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right\} \\ &\quad -\mu\frac{\mu p_{\max} br}{4\sin\theta_a} \\ &\quad \times \left[2\left(\theta_2-\theta_1\right)\frac{\pi}{180^\circ} - \sin 2\theta_2 + \sin 2\theta_1\right], \\ &= -654.6 \text{ N} \end{aligned}$$

From Eq. (18.48)

$$\begin{aligned} R_{ys} &= \left[\frac{(1\times 10^6)(0.075)(0.2)}{4\sin 75^\circ}\right] \\ &\quad \times \left[2(75^\circ - 10^\circ)\frac{\pi}{180^\circ} - \sin 150^\circ + \sin 20^\circ\right. \\ &\qquad \left. + 2(0.24)\left(\sin^2 75^\circ - \sin^2 10^\circ\right)\right] \\ &= 9878 \text{ N.} \end{aligned}$$

Similarly, $R_{xd} = -137.5$ N.

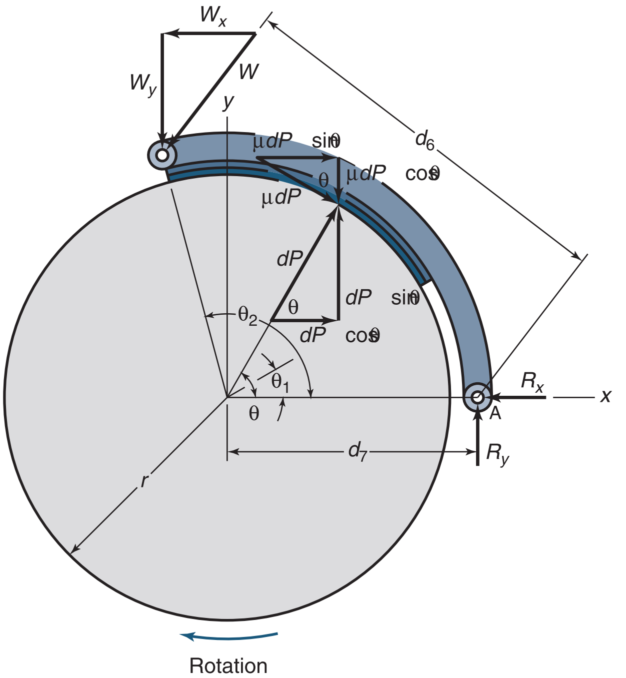

Figure 18.14: Forces and dimensions of long-shoe, external, contracting rim brake.

Deenergizing shoes: From Eq. (18.52),

$$R_{yd} = \left[\frac{(0.6194)(1\times10^6)(0.075)(0.2)}{4\sin 75^\circ}\right] \times\left[2(75^\circ-10^\circ)\frac{\pi}{180^\circ}-\sin 150^\circ+\sin 20^\circ -2(0.24)(\sin^2 75^\circ-\sin^2 10^\circ)\right] = 4034\text{ N}.$$

The resultant forces of the reactions in the hinge pin in the horizontal and vertical directions are

$$R_H = -654.6 - 137.5 = -792.1\text{ N},$$

$$R_V = 9878 - 4034 = 5844\text{ N}.$$

The resultant force at the hinge pin is

$$R = \sqrt{R_H^2 + R_V^2} = \sqrt{(-792.1)^2+(5844)^2} = 5897\text{ N}.$$

18.7 Long-Shoe, External, Contracting Rim Brakes

Figure 18.14 shows the forces and dimensions of a long-shoe, external, contracting rim brake. In Fig. 18.12, the brake is internal to the drum, whereas in Fig. 18.14 the brake is external to the drum. The symbols used in these figures are similar. The equations developed in Section 18.6 for internal shoe brakes are exactly the same as those for external shoe brakes as long as one properly identifies whether the brake is self-energizing or deenergizing.

The *internal* brake shoe in Fig. 18.12 was found to be *self-energizing* for clockwise rotation, since in the moment summation of Eq. (18.42), the actuating and friction moments have the same sign. The *external* brake shoe in Fig. 18.14 is *deenergizing* for clockwise rotation. Summing the moments and equating the sum to zero

$$Wd_6 - M_F - M_P = 0. \qquad (18.53)$$

The actuating and friction moments have opposite signs and thus the external brake shoe shown in Fig. 18.14 is deenergizing.

If, in Figs. 18.12 and 18.14, the direction of rotation were changed from clockwise to counterclockwise, the friction moments in Eqs. (18.42) and (18.53) would have opposite signs. Therefore, the internal brake shoe would be deenergizing and the external brake shoe would be self-energizing.

Example 18.5: External Long-Shoe Brake

Given: An external, long-shoe rim brake is to be cost optimized. Three lining geometries are being considered, covering the entire 90° of the shoe, covering only 45° of the central portion of the shoe, or covering only 22.5° of the central portion of the shoe. The braking torque must be the same for all three geometries, and the cost of changing any of the brake linings is half of the cost of a 22.5° lining. The cost of the lining material is proportional to the wrap angle. The wear rate is proportional to the pressure. The input parameters for the 90° lining are $d_7 = 100$ mm, $r = 80$ mm, $b = 25$ mm, $\theta_1 = 0^\circ$, $\theta_2 = 90^\circ$, $\mu = 0.27$, and $T = 125$ N-m.

Find: Which of the wrap angles (90°, 45°, or 22.5°) would be the most economical?

Solution: The braking torque is given by Eq. (18.44) and is the same for all three geometries. For the 90° wrap angle ($\theta_1 = 0^\circ$, $\theta_2 = 90^\circ$, and $\theta_a = 90^\circ$):

$$(p_{\max})_{90^\circ} = \frac{T\sin\theta_a}{\mu b r^2(\cos\theta_1-\cos\theta_2)} = \frac{(125)\sin 90^\circ}{(0.27)(0.025)(0.08)^2(\cos 0^\circ - \cos 90^\circ)} = 2.894\times10^6\text{ Pa} = 2.894\text{ MPa}.$$

For the 45° wrap angle ($\theta_1 = 22.5^\circ$, $\theta_2 = 67.5^\circ$, and $\theta_a = 67.5^\circ$):

$$(p_{\max})_{45^\circ} = \frac{(125)\sin 67.5^\circ}{(0.27)(0.025)(0.08)^2(\cos 22.5^\circ-\cos 67.5^\circ)} = 4.940\times10^6\text{ Pa} = 4.940\text{ MPa}.$$

For the 22.5° wrap angle ($\theta_1 = 33.75^\circ$, $\theta_2 = 56.25^\circ$, and $\theta_a = 56.25^\circ$):

$$(p_{\max})_{22.5^\circ} = \frac{(125)\sin 56.25^\circ}{(0.27)(0.025)(0.08)^2(\cos 33.7^\circ-\cos 56.2^\circ)}$$

which is solved as 8.720 MPa.

The cost of changing the lining is C. The lining costs are $2C$ for a 22.5° lining, $4C$ for a 45° lining, and $8C$ for a 90° lining. The wear rate is proportional to the pressure, or the time it takes for the shoe to wear out is inversely proportional to the pressure. Thus, the times it takes for the shoe to wear out for the three geometries are

$$t_{90^\circ} = \frac{A}{(p_{\max})_{90^\circ}} = \frac{A}{(2.894 \times 10^6)} = 3.455 \times 10^{-7} A,$$

where A is a constant independent of geometry. Similarly,

$$t_{45^\circ} = \frac{A}{(p_{\max})_{45^\circ}} = \frac{A}{(4.940 \times 10^6)} = 2.024 \times 10^{-7} A,$$

$$t_{22.65^\circ} = \frac{A}{(p_{\max})_{22.5^\circ}} = \frac{A}{(8.720 \times 10^6)} = 1.147 \times 10^{-7} A.$$

The costs per unit time for the three geometries are:

90° wrap angle:

$$\frac{8C + C}{(3.455 \times 10^{-7})\, A} = \left(26.05 \times 10^6\right) \frac{C}{A}.$$

45° wrap angle:

$$\frac{4C + C}{(2.024 \times 10^{-7})\, A} = \left(24.70 \times 10^6\right) \frac{C}{A}.$$

22.5° wrap angle:

$$\frac{2C + C}{(1.147 \times 10^{-7})\, A} = \left(26.16 \times 10^6\right) \frac{C}{A}.$$

The 45°-wrap-angle shoe gives the lowest cost, 5.6% lower than the 22.5°-wrap-angle shoe and 5.2% lower than the 90°-wrap-angle shoe.

18.8 Symmetrically Loaded Pivot-Shoe Brakes

Figure 18.15 shows a symmetrically loaded **pivot-shoe brake**. The major difference between the internal and external rim brakes considered previously and the symmetrically loaded pivot-shoe brake shown in Fig. 18.15 is the pressure distribution around the shoe. Recall from Eq. (18.37) for the internal rim brake that the maximum pressure was at $\theta = 90^\circ$ and the pressure distribution from the heel to the top of the brake was sinusoidal. For the symmetrically loaded pivot-shoe brake (Fig. 18.15), the maximum pressure is at $\theta = 0^\circ$, which suggests the pressure variation is

$$p = \frac{p_{\max} \cos\theta}{\cos\theta_a} = p_{\max}\cos\theta. \tag{18.54}$$

For any angular position from the pivot, θ, a differential normal force dP acts with a magnitude of

$$dP = pbr\, d\theta = p_{\max} br\, \cos\theta\, d\theta. \tag{18.55}$$

The design of a symmetrically loaded pivot-shoe brake is such that the distance d_7, which is measured from the center of the drum to the pivot, is chosen so that the resulting friction moment acting on the brake shoe is zero. From Fig. 18.15,

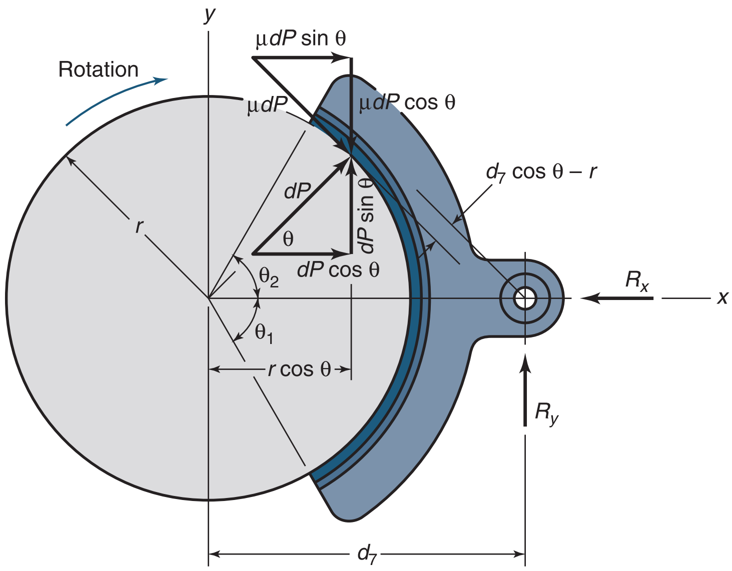

Figure 18.15: Symmetrically loaded pivot-shoe brake.

the friction moment, when set equal to zero, is

$$M_F = 2\int_0^{\theta_2} \mu\, dP\, (d_7\cos\theta - r) = 0. \tag{18.56}$$

Substituting Eq. (18.55) into Eq. (18.56) gives

$$2\mu p_{\max} br \int_0^{\theta_2} \left(d_7\cos^2\theta - r\cos\theta\right) = 0.$$

This reduces to

$$d_7 = \frac{4r\sin\theta_2}{2\theta_2\left(\frac{\pi}{180^\circ}\right) + \sin 2\theta_2}. \tag{18.57}$$

This value of d_7 produces a friction moment equal to zero ($M_F = 0$). The braking torque is

$$\begin{aligned} T &= 2\int_0^{\theta_2} r\mu\, dP \\ &= 2\mu r^2 b p_{\max}\int_0^{\theta_2}\cos\theta\, d\theta \\ &= 2\mu r^2 b p_{\max}\sin\theta_2. \end{aligned} \tag{18.58}$$

Note from Fig. 18.15 that, for any x, the horizontal friction force components of the upper half of the shoe are equal and opposite in direction to the horizontal friction force components of the lower half of the shoe. For a fixed x, the horizontal *normal* components of both halves of the shoe are equal and in the same direction, so that the horizontal reaction force is

$$R_x = 2\int_0^{\theta_2} dP\cos\theta = \frac{p_{\max} br}{2}\left[2\theta_2\left(\frac{\pi}{180^\circ}\right) + \sin 2\theta_2\right]. \tag{18.59}$$

Making use of Eq. (18.57) gives

$$R_x = \frac{2br^2 p_{\max}\sin\theta_2}{d_7}. \tag{18.60}$$

For a fixed y, the vertical *friction force* components of the upper half of the shoe are equal and in the same direction as the vertical friction force components of the lower half of the shoe. For a fixed y, the vertical *normal* components of both halves of the shoe are equal and opposite in direction, so that the vertical reaction force is

$$R_y = 2\int_0^{\theta_2} \mu\, dP \cos\theta = \frac{\mu p_{\max} br}{2}\left[2\theta_2\left(\frac{\pi}{180^\circ}\right) + \sin 2\theta_2\right]. \qquad (18.61)$$

Making use of Eq. (18.59) gives

$$R_y = \frac{2\mu b r^2 p_{\max}\sin\theta_2}{d_7} = \mu R_x.$$

Example 18.6: Pivot-Shoe Brake

Given: A symmetrically loaded, pivot-shoe brake has the distance d_7 shown in Fig 18.11 optimized for a 180° wrap angle. When the brake lining is worn out, it is replaced with a 90°-wrap-angle lining symmetrically positioned in the shoe. The actuating force is 11,000 N, the coefficient of friction is 0.31, the brake drum radius is 100 mm, and the brake width is 45 mm.

Find: Calculate the pressure distribution in the brake shoe and the braking torque.

Solution: The distance d_7 can be expressed from Eq. (18.57) for the 180° wrap angle as

$$\begin{aligned}(d_7)_{180^\circ} &= \frac{4r\sin\theta_2}{2\theta_2\left(\frac{\pi}{180^\circ}\right) + \sin 2\theta_2}\\ &= \frac{4(0.1)\sin 90^\circ}{2(90)\left(\frac{\pi}{180^\circ}\right) + \sin 180^\circ}\\ &= 0.1273 \text{ m}.\end{aligned}$$

For the 90°-wrap-angle, symmetrically loaded, pivot-shoe brake, the pressure distribution will be unsymmetrical. The maximum pressure, which will occur at θ_2, needs to be determined from shoe equilibrium, or

$$\int_{-\pi/4}^{\pi/4} pr\, d\theta(b)(d_7\sin\theta) - \int_{-\pi/4}^{\pi/4} \mu pr\, d\theta(b)(d_7\cos\theta - r) = 0. \qquad (a)$$

If the wear rate is proportional to the pressure and the maximum pressure is at $\theta = \theta_o$, the pressure distribution is

$$p = p_{\max}\cos(\theta - \theta_o). \qquad (b)$$

Substituting Eq. (b) into Eq. (a) gives

$$\begin{aligned}0 &= \int_{-\pi/4}^{\pi/4} d_7\cos(\theta-\theta_o)\sin\theta\, d\theta\\ &\quad - \int_{-\pi/4}^{\pi/4} \mu\cos(\theta-\theta_o)(d_7\cos\theta - r)\, d\theta.\end{aligned}$$

But

$$\cos(\theta-\theta_o) = \cos\theta\cos\theta_o + \sin\theta\sin\theta_o.$$

Therefore,

$$\int_{-\pi/4}^{\pi/4} d_7\left(\cos\theta_o\cos\theta\sin\theta + \sin\theta_o\sin^2\theta\right) d\theta$$

$$= \int_{-\pi/4}^{\pi/4} \mu(\cos\theta_o\cos\theta + \sin\theta_o\sin\theta)(d_7\cos\theta - r)d\theta.$$

Integrating gives

$$\frac{d_7\cos\theta_o}{2}\left[\sin^2\frac{\pi}{4} - \sin^2\left(-\frac{\pi}{4}\right)\right] + d_7\sin\theta_o\left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$\begin{aligned}&= \mu d_7\cos\theta_o\left(\frac{\pi}{4}+\frac{1}{2}\right)\\ &\quad + \mu d_7\sin\theta_o\left(\frac{\sin^2\frac{\pi}{4}}{2} - \frac{\sin^2\left(-\frac{\pi}{4}\right)}{2}\right)\\ &\quad - \mu r\cos\theta_o\sqrt{2} + \mu r\sin\theta_o\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right).\end{aligned}$$

This reduces to

$$d_7\sin\theta_o\left(\frac{\pi}{4} - \frac{1}{2}\right) = \mu\cos\theta_o\left[d_7\left(\frac{\pi}{4}+\frac{1}{2}\right) - r\sqrt{2}\right],$$

or

$$\begin{aligned}\tan\theta_o &= \frac{\mu\left[d_7\left(\frac{\pi}{4}+\frac{1}{2}\right) - r\sqrt{2}\right]}{d_7\left(\frac{\pi}{4}-\frac{1}{2}\right)}\\ &= \frac{0.31\left[0.1273\left(\frac{\pi}{4}+\frac{1}{2}\right) - (0.100)\sqrt{2}\right]}{0.1273\left(\frac{\pi}{4}-\frac{1}{2}\right)}\\ &= 0.1895,\end{aligned}$$

or $\theta_o = 10.73^\circ$. The actuating force is

$$W = \int_{-\pi/4}^{\pi/4} pr\, d\theta b\cos\theta + \int_{-\pi/4}^{\pi/4} \mu pr\, d\theta b\sin\theta. \qquad (c)$$

Substituting Eq. (b) into Eq. (c) gives

$$\begin{aligned}W &= rbp_{\max}\int_{-\pi/4}^{\pi/4}\cos(\theta-\theta_o)\cos\theta\, d\theta\\ &\quad + rbp_{\max}\int_{-\pi/4}^{\pi/4}\mu\cos(\theta-\theta_o)\sin\theta\, d\theta. \qquad (d)\end{aligned}$$

But

$$\int_{-\pi/4}^{\pi/4}\cos(\theta-\theta_o)\cos\theta\, d\theta = \left(\frac{\pi}{4}+\frac{1}{2}\right)\cos\theta_o, \qquad (e)$$

and

$$\int_{-\pi/4}^{\pi/4}\cos(\theta-\theta_o)\sin\theta\, d\theta = \left(\frac{\pi}{4}-\frac{1}{2}\right)\sin\theta_o. \qquad (f)$$

Substituting Eqs. (e) and (f) into Eq. (d) while solving for the maximum pressure gives

$$\begin{aligned}p_{\max} &= \frac{W}{rb\left[\left(\frac{\pi}{4}+\frac{1}{2}\right)\cos\theta_o + \mu\left(\frac{\pi}{4}-\frac{1}{2}\right)\sin\theta_o\right]}\\ &= \frac{11,000}{(0.0045)\left[(1.285)\cos 10.7^\circ + (0.08835)\sin 10.7^\circ\right]}\\ &= 1.911\times 10^6 \text{ Pa} = 1.911 \text{ MPa}.\end{aligned}$$

From Eq. (*b*) the pressure distribution can be expressed as

$$p = \left(1.911 \times 10^6\right)\cos(\theta - 10.73^\circ).$$

The braking torque is

$$\begin{aligned} T &= \mu p_{\max} b r^2 \int_{-\pi/4}^{\pi/4} \cos(\theta - \theta_o)\, d\theta \\ &= \mu p_{\max} b r^2 \left[\sin\left(\frac{\pi}{4} - \theta_o\right) + \sin\left(\frac{\pi}{4} + \theta_o\right)\right] \\ &= (0.31)\left(1.911 \times 10^6\right)(0.045)(0.1)^2 \\ &\quad \times [\sin 34.27^\circ + \sin 55.73^\circ] \\ &= 370.4 \text{ N-m}. \end{aligned}$$

18.9 Band Brakes

Figure 18.16 shows the components of a **band brake**, which consists of a band wrapped partly around a drum. The brake is actuated by pulling the band tighter against the drum, as shown in Fig. 18.17a. The band is assumed to be in contact with the drum over the entire wrap angle, ϕ in Fig. 18.17a. The pin reaction force is given as F_1 and the actuating force as F_2. In Fig. 18.17a, the heel of the brake is near F_1 and the toe is near F_2. Since some friction will exist between the band and the drum, the actuating force will be less than the pin reaction force, or $F_2 < F_1$.

Figure 18.17b shows the forces acting on an element of the band. The forces are the normal force, P, and the friction force, F. Summing the forces in the radial direction while using Fig. 18.17b gives

$$(F + dF)\sin\left(\frac{d\theta}{2}\right) + F\sin\left(\frac{d\theta}{2}\right) - dP = 0;$$

$$dP = 2F\sin\left(\frac{d\theta}{2}\right) + dF\sin\left(\frac{d\theta}{2}\right).$$

Since $dF \ll F$,

$$dP = 2F\sin\left(\frac{d\theta}{2}\right).$$

Since $d\theta/2$ is small, then $\sin d\theta/2 \approx d\theta/2$. Therefore,

$$dP = F d\theta. \tag{18.62}$$

Figure 18.16: A typical band brake. *Source:* Courtesy of Northern Tool & Equipment Company, Inc.

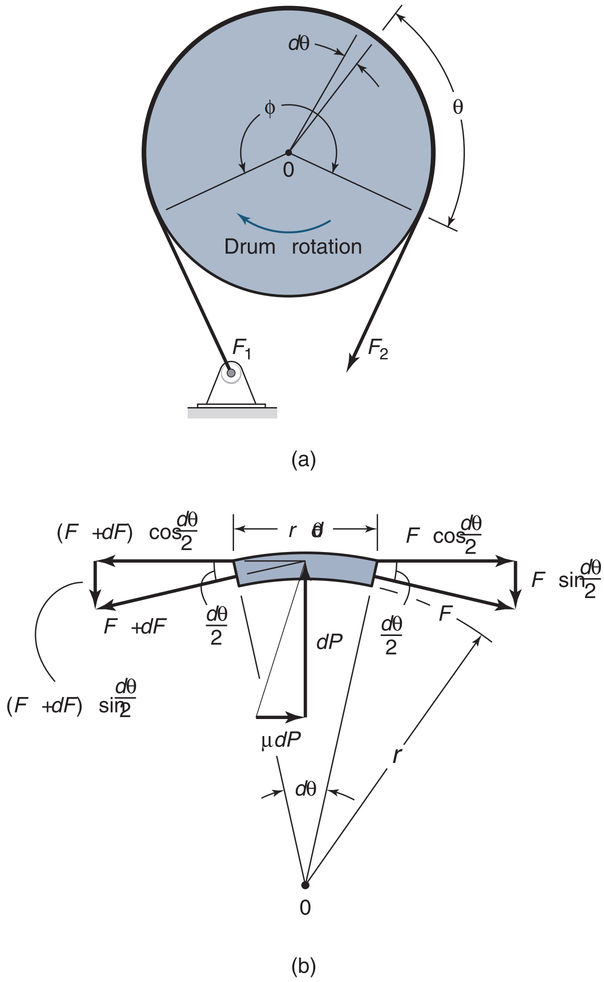

Figure 18.17: Band brake. (a) Forces acting on band; (b) forces acting on element.

Summing the forces in the *horizontal* (tangential) direction while using Fig. 18.17b gives

$$(F + dF)\cos\left(\frac{d\theta}{2}\right) - F\cos\left(\frac{d\theta}{2}\right) - \mu dP = 0;$$

$$dF\cos\left(\frac{d\theta}{2}\right) - \mu dP = 0.$$

Since $d\theta/2$ is small, then $\cos(d\theta/2) \approx 1$. Therefore,

$$dF - \mu dP = 0. \tag{18.63}$$

Substituting Eq. (18.62) into Eq. (18.63) gives

$$dF - \mu F d\theta = 0 \quad \text{or} \quad \int_{F_2}^{F_1} \frac{dF}{F} = \mu \int_0^{\phi} d\theta.$$

Integrating gives

$$\ln\left(\frac{F_1}{F_2}\right) = \frac{\mu\phi\pi}{180^\circ},$$

or

$$\frac{F_1}{F_2} = e^{\mu\phi\pi/180^\circ}, \tag{18.64}$$

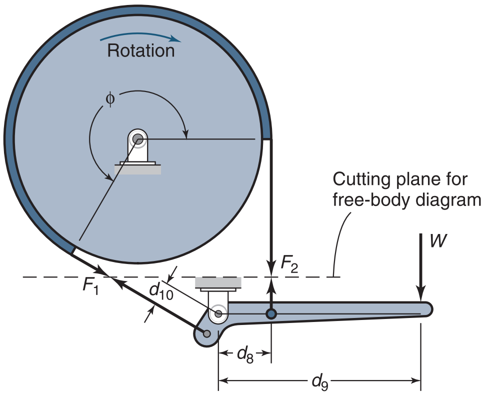

Figure 18.18: Band brake used in Example 18.7.

where ϕ is the wrap angle. The torque applied to the drum is

$$T = r(F_1 - F_2). \tag{18.65}$$

The differential normal force dP acting on the element in Fig. 18.17b, with width b (coming out of the paper) and length $r\,d\theta$, is

$$dP = pbr\,d\theta, \tag{18.66}$$

where p is the contact pressure. Substituting Eq. (18.66) into Eq. (18.62) gives

$$p = \frac{F}{br}. \tag{18.67}$$

The pressure is proportional to the tension in the band. The maximum pressure occurs at the heel, or near the pin reaction force, and has the value

$$p_{\max} = \frac{F_1}{br}. \tag{18.68}$$

Example 18.7: Band Brake

Given: The band brake shown in Fig. 18.18 has $r = 100$ mm, $b = 25$ mm, $d_9 = 225$ mm, $d_8 = 50$ mm, $d_{10} = 12$ mm, $\phi = 270°$, $\mu = 0.2$, and $p_{\max} = 500$ kPa.

Find: Determine the braking torque, actuating force, and value of d_{10} when the brake force locks.

Solution: From Eq. (18.68), the pin reaction force is

$$F_1 = p_{\max}br = \left(500 \times 10^3\right)(0.025)(0.100) = 1250 \text{ N}.$$

From Eq. (18.64), the actuating force is

$$F_2 = F_1 e^{-\mu\phi\pi/180°} = 1250e^{-0.2(270)\pi/180} = 487 \text{ N}.$$

From Eq. (18.65), the braking torque is

$$T = r(F_1 - F_2) = (0.100)(1250 - 487) = 76.3 \text{ Nm}.$$

Summing the moments about the hinge pin and setting the sum equal to zero give

$$-d_9W + d_8F_2 - d_{10}F_1 = 0.$$

Solving for the actuating force W,

$$W = \frac{d_8F_2 - d_{10}F_1}{d_9} = \frac{(0.050)(487) - (0.012)(1250)}{0.225} = 41.56 \text{ N}.$$

If $W = 0$, the brake will self-lock, therefore:

$$d_8F_2 - d_{10}F_1 = 0,$$

$$d_{10} = \frac{d_8F_2}{F_1} = \frac{(0.050)(487)}{1250} = 0.0195 \text{ m}.$$

The brake will self-lock if $d_{10} \geq 19.5$ mm.

18.10 Slip Clutches

A clutch will often be used as a torque-limiting device, usually to prevent machinery damage in a malfunction or undesired event, although they can also be used to control peak torques during startup. A **slip clutch**, in a simple manifestation, consists of two surfaces held together by a constant force so that they slip when a preset level of torque is applied to them. Slip clutches come in a wide variety of sizes but are very compact. They are designed to be actuated only rarely and thus the friction elements do not need to be sized for wear. Also, slip clutches are almost always contacting disks, mainly because it is imperative to prevent the possibility of a self-energizing shoe (which would compromise the torque-limiting control).

Slip clutches are mainly used to protect machinery elements and are not relied upon to prevent personal injury. After slip clutches spend long time periods in contact, the friction surfaces can stick or weld together, requiring a larger torque to initiate slippage. This torque is usually not large enough to break a gear, for example, but can be enough of an increase to result in a serious injury. Regardless, a slip clutch is an effective torque limiter and can be used instead of shear pins or keyways with the advantage that no maintenance is required after the excessive torque condition has been corrected.

Case Study 18.1: Roller Coaster Braking System

This case study discusses some of the design decisions that are made for the braking system of a roller coaster such as shown in Fig. 18.19. Figure 18.20 is a schematic of a typical roller coaster brake. The brake systems for automobiles or other vehicles, that is, drum and disk brakes associated with wheels, will not work for a roller coaster. The friction between the steel wheel and the steel rail is extremely low, so that the cars can slide on the rails without losing much energy. This is important for roller coaster operation, as the cars are pulled up a main hill (usually by a chain drive — see Section 19.6), and then coast for the remainder of the ride.

Roller coasters have braking stations, where the cars must be brought to a stop so that passengers can get on or off. A typical brake system uses pads that bear against a flange, or fin, on the roller coaster cars. The roller coaster operator controls a pneumatic actuator that applies force through the linkage to aggressively decelerate the cars. The maximum braking force should not expose the passengers to greater than 1/2 g of deceleration, and preferably not more than 2.5 m/s^2 or so.

Typically, several brakes are used at the point of access to a roller coaster. This case study will show a four-brake system, which is not unusual. Note that the thermal effects

Figure 18.19: A typical roller coaster. (Shutterstock)

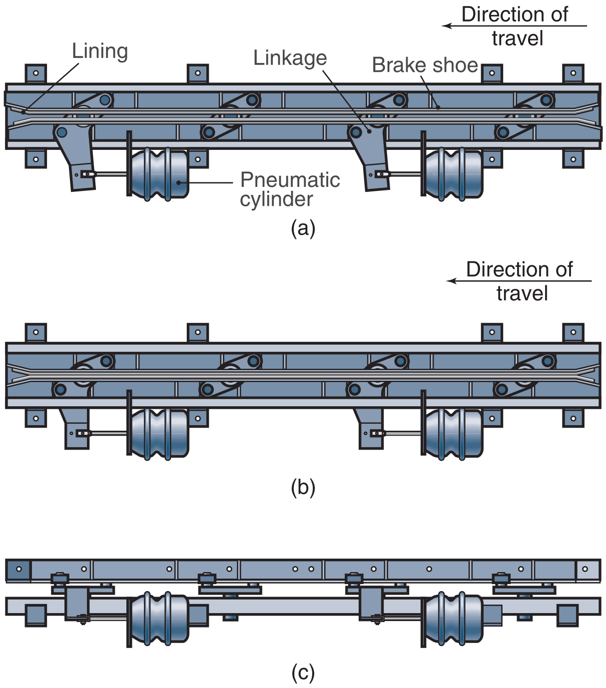

Figure 18.20: Schematic illustration of a roller coaster brake system. (a) Components of the roller coaster and shown when the brake is not engaged, as seen by the gap between the liner pads; (b) top view of an engaged brake; (c) side view of engaged brake.

that are so prominent in other brake designs are not considered here because roller coaster brakes are actuated only intermittently, so that they avoid the thermal checking and temperature problems typical of automotive disk or drum brakes.

Brake Pad–Fin Interaction

The first part of the design problem is to correlate brake system variables (actuating force, pad area, etc.) with performance (braking force developed). The coefficient of friction can be reasonably taken as at least 0.25 for sintered brass on steel (it is usually higher for these materials). Friction-reducing contaminants on the roller coaster are not a serious concern. This is mainly because the braking system is pneumatic, not hydraulic, so no leaking hydraulic fluids can contaminate the brake pads.

Figure 18.21 illustrates a three-car chain of roller coaster cars, but the manufacturer allows as many as seven cars provided that the front and rear cars are configured as shown. Many details, especially case-specific decorations, have been omitted for clarity. Each car can weigh 1900 lb and carry as many as four adults. (It is more common that two adults and two children will ride in a roller coaster car.) The 95th percentile adult male weighs 216 lb, and weight of the car and contents is calculated accordingly. The maximum incoming velocity is approximately 30 mph (44 ft/s) on the basis of the energy attainable from the first roller coaster hill.

A braking force is then determined to achieve the deceleration of 0.5 g, which is then used to obtain the actuating force. However, the brake pad arrangement shown in Fig. 18.20 develops frictional force on both sides of the fin. The normal force obtained from such a calculation represents the worst-case design load in the cars. No increase is necessary from a safety standpoint for the following reasons:

1. For most cases this friction force is more than adequate.
2. Typical installations will have a series of at least three brakes spaced in alternate bays. If deceleration can be maintained throughout the entire contact length between the cars and the brakes, extra braking capacity is inherently placed in the system.
3. Even in the event of insufficient braking (perhaps due to excessive wear of the braking elements), emergency brakes in the operator's station can stop the cars. The alternative is to generate larger braking forces with the subject braking system, leading to excessive deceleration and possible injuries to riders from jarring and jerking. The target deceleration has been chosen to provide good braking potential while minimizing the risk of such injuries. A stronger brake would actually lead to a less safe system.

Brake Actuation System

Figure 18.22 shows a detail of the brake-actuating cylinder and associated linkage. Recall from Fig. 18.20 that two cylinders are used on each brake and two unpowered stabilizing elements are used to distribute the contact forces over the whole shoe. The brake is based on a pneumatic cylinder/spring system so that the normal cylinder position is with the piston totally extended, corresponding to the brake generating maximum brake force. The reason for such a system is that many difficulties can arise with pneumatic systems: air hoses can leak, pneumatic couplings can decouple, power can fail, etc. The system is fail safe if the brakes are engaged in such a reasonably foreseeable event.

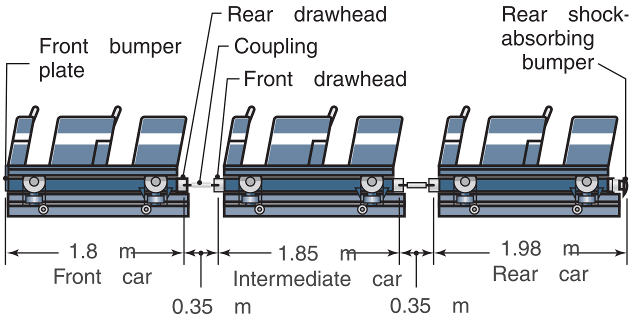

Figure 18.21: Schematic illustration of typical roller coaster cars.

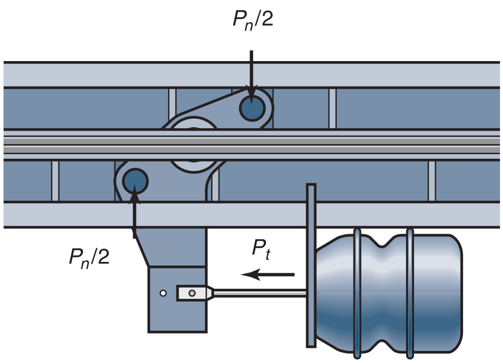

Figure 18.22: Detail of brake-actuating cylinder with forces shown.

Pneumatic brake cylinders are available in a range of capacities from a brake manufacturer, each of which has a characteristic spring force curve. The nonlinear nature of this curve is typical of pneumatic springs (as discussed in Section 17.7), but this cylinder uses a combination of air pressure and helical springs for actuation as shown in the breakaway view in Fig. 18.23.

A subtlety in sizing such brake cylinders must be explained. During assembly the piston is totally extended and may be difficult to affix to the hinge mechanism. Therefore, a piston extension is threaded for a distance on its length. The cylinder can be assembled and then the collar on the piston extension adjusted for brake actuation.

Summary

Short-shoe brakes have a long history of safe operation on roller coasters. As discussed in this case study, the brake system has a number of redundancies to make sure that failure of any one braking station does not result in catastrophic injury of the passengers.

Also, it should be noted that roller coasters have a stringent maintenance schedule, and the ride is inspected before the ride is placed in service every day. If a lining is worn, there will be an audible indication from the exposed rivets on the brake shoes “grinding” against the fin, and this is a clear indication that the pads need replacement. Any reasonably attentive maintenance staff can diagnose and repair such conditions as needed.

Source: Courtesy of Brian King, Recreation Engineering LLC.

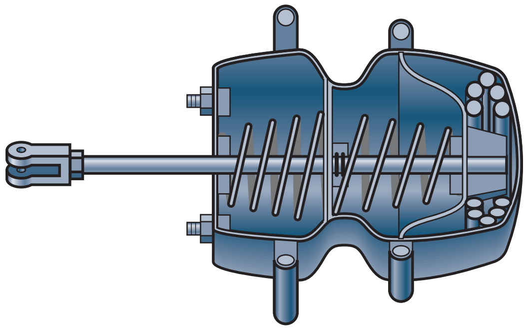

Figure 18.23: Cross-section of the actuating pneumatic cylinder, highlighting the helical springs incorporated into the design.

18.11 Summary

This chapter focused on two machine elements, clutches and brakes, that are associated with motion and have the common function of dissipating and/or transferring rotating energy. In analyzing the performance of clutches and brakes, the actuating force, the torque transmitted, and the reaction force at the hinge pin were the major focuses of this chapter. The torque transmitted is related to the actuating force, the coefficient of friction, and the geometry of the clutch or brake. This is a problem in statics, where different geometries were studied separately.

For long-shoe clutches and brakes, two theories were studied: the uniform pressure model and the uniform wear model. It was found that for the same dimensionless torque the uniform wear model requires a larger radius ratio than does the uniform pressure model for the same maximum pressure. This larger radius ratio implies that a larger area is needed for the uniform wear model. Thus, the uniform wear model was viewed as a safer approach.

Key Words

band brake brake that uses contact pressure of flexible band against outer surface of drum

brake device used to bring moving system to rest through dissipation of energy to heat by friction

clutches power transfer devices that allow coupling and decoupling of shafts

cone disk brake or clutch that uses shoes pressed against convergent surface of cone

deenergizing brake or clutch shoe where frictional moment hinders engagement

rim type brake or clutch that uses internal shoes which expand onto inner surface of drum

self-energizing brake or clutch shoe where frictional moment assists engagement

slip clutch clutch where maximum transferred torque is limited

thrust disk brake or clutch that uses flat shoes pushed against rotating disk

Summary of Equations

Heat Transfer

First Law of Thermodynamics: $Q_f = Q_c + Q_h + Q_s$

Lining temperature rise: $\Delta t_m = \dfrac{Q_f}{C_p m_a}$

Thrust (Disk) Brakes:

Uniform pressure model:

Actuating force: $P_p = \pi p_o \left(r_o^2 - r_i^2\right)$

Torque: $T_p = \frac{2\pi\mu p_o}{3}\left(r_o^3 - r_i^3\right) = \frac{2\mu P_p\left(r_o^3-r_i^3\right)}{3\left(r_o^2-r_i^2\right)}$

Uniform wear model:

Actuating force: $P_w = 2\pi p_{\max} r_i \left(r_o - r_i\right)$

Torque: $T_w = \pi\mu r_i p_{\max}\left(r_o^2 - r_i^2\right) = \dfrac{\mu P_w \left(r_o + r_i\right)}{2}$

Cone Clutches and Brakes:

Uniform pressure model:

Actuating force: $W = \frac{\pi p_o}{4}\left(D^2 - d^2\right)$

Torque: $T = \dfrac{\mu W\left(D^3 - d^3\right)}{3\sin\alpha\left(D^2 - d^2\right)}$

Uniform wear model:

Actuating force: $W = \pi p r \left(D - d\right)$

Torque: $T = \dfrac{\mu W}{4\sin\alpha}\left(D + d\right)$

Block (Short-Shoe) Brakes:

Normal force: $P = \dfrac{W d_4}{d_3 + \mu d_2}$

Torque: $T = \dfrac{\mu d_4 r W}{d_3 - \mu d_1}$ (energizing)

$T = \dfrac{\mu d_4 r W}{d_3 + \mu d_2}$ (deenergizing)

Long-Shoe, Internal, Expanding Rim (Drum) Brakes:

Pressure distribution: $p = p_{\max}\left(\dfrac{\sin\theta}{\sin\theta_a}\right)$

Normal force moment:

$$M_P = \frac{b r d_7 p_{\max}}{4\sin\theta_1}\left[2\left(\theta_2 - \theta_1\right)\frac{\pi}{180^\circ} - \sin 2\theta_2 + \sin 2\theta_1\right]$$

Friction force moment:

$$M_F = \frac{\mu p_{\max} b r}{\sin\theta_a} \times \left[-r\left(\cos\theta_2 - \cos\theta_1\right) - \frac{d_7}{2}\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right]$$

Self-energizing shoe: $-W d_6 - M_F + M_P = 0$

Deenergizing shoe: $-W d_6 + M_F + M_P = 0$

Pivot-Shoe Brakes:

Pressure distribution: $p = p_{\max}\cos\theta$

Torque: $T = 2\mu r^2 b p_{\max}\sin\theta_2$

Band Brakes:

Forces: $\dfrac{F_1}{F_2} = e^{\mu\phi\pi/180^\circ}$

Maximum pressure: $p_{\max} = \dfrac{F_1}{br}$

Torque: $T = r(F_1 - F_2)$

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Questions

18.1 What is a clutch? How is it different from a brake?

18.2 What is the uniform pressure model? The uniform wear model?

18.3 When is a rectangular pad disk brake used?

18.4 What is a cone clutch?

18.5 What is a self-energizing shoe?

18.6 Can a short-shoe brake be self-energizing?

18.7 What is a long-shoe brake? What is the main difference between a long-shoe and a short-shoe brake?

18.8 What is the difference between an external long-shoe brake and a pivot-shoe brake?

18.9 Describe the operating mechanisms of a band brake.

18.10 What is a heat check? How are they avoided?

Qualitative Problems

18.11 List the material properties that are (*a*) necessary, and (*b*) useful for a brake or clutch lining.

18.12 List the advantages and disadvantages of cone clutches compared to thrust disc clutches.

18.13 Explain the conditions when the uniform pressure model is more appropriate than the uniform wear model. When is the uniform wear model more appropriate?

18.14 If a composite material is used as a brake or clutch lining, what preferred orientation of the reinforcement would you recommend, if any? Explain your answer.

18.15 Explain the difference between self-energizing and self-locking.

18.16 What are the similarities and differences between band and thrust disc brakes?

18.17 What are the main indications of an out-of-round brake drum?

18.18 What are the similarities and differences between heat checks and hard spots?