MIET2510

Mechanical Design

Week 2 – Kinematic Analysis of Mechanism – Part 1

School of Science and Technology, RMIT Vietnam

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Outlines

1. Position Analysis
2. Graphical Position Analysis of Linkages
3. Algebraic Position Analysis of Linkages
4. Vector Loop Method for Analysis of Linkages

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1. Position Analysis

- For mechanisms, it is important to predict its position in space to analyse its motion since velocity and acceleration are require a position analysis.
- Dynamic forces are linked with accelerations via Newton's 2nd law. Given dynamic forces and static forces, stresses in the components are calculated and proper materials are selected.

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Approaches to Analysis

- There are several approaches to analysing linkages/mechanisms
  - Graphical approach
  - Analytic approach
  - Complex-algebraic approach
  - Vector-algebraic approach
  - Numeric approach

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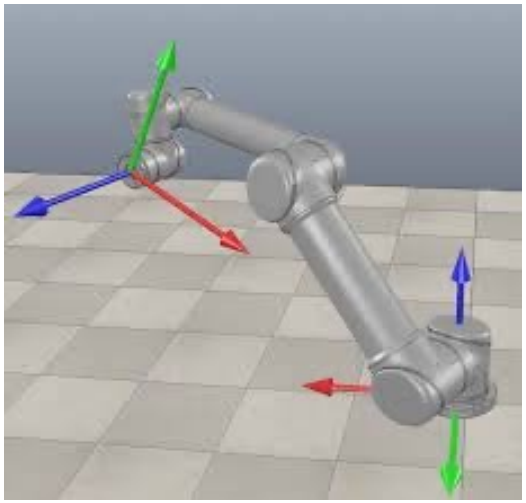
1. Position Analysis - Representation

All position parameters are taken with respect to the selected reference frame. In most mechanism, reference point is Ground. ⏚



The choice of reference axes depends on what analysis is being undertaken but is usually chosen to make calculations as simple as possible.

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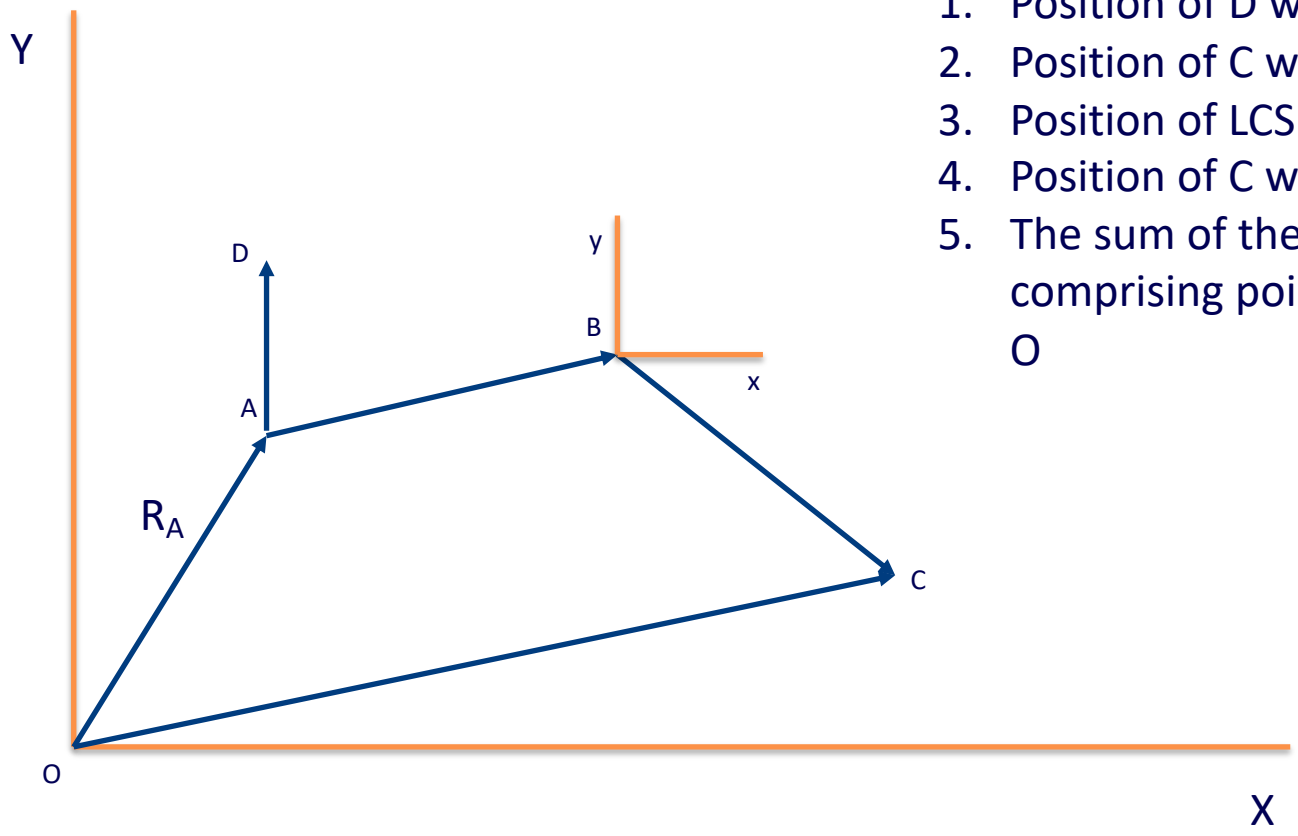
1. Position Analysis – Reference Frame



Use vector notation to describe

1. Position of D wrt O
2. Position of C wrt A
3. Position of LCS wrt GCS
4. Position of C wrt LCS
5. The sum of the vector loop comprising points O, A, B, C, O

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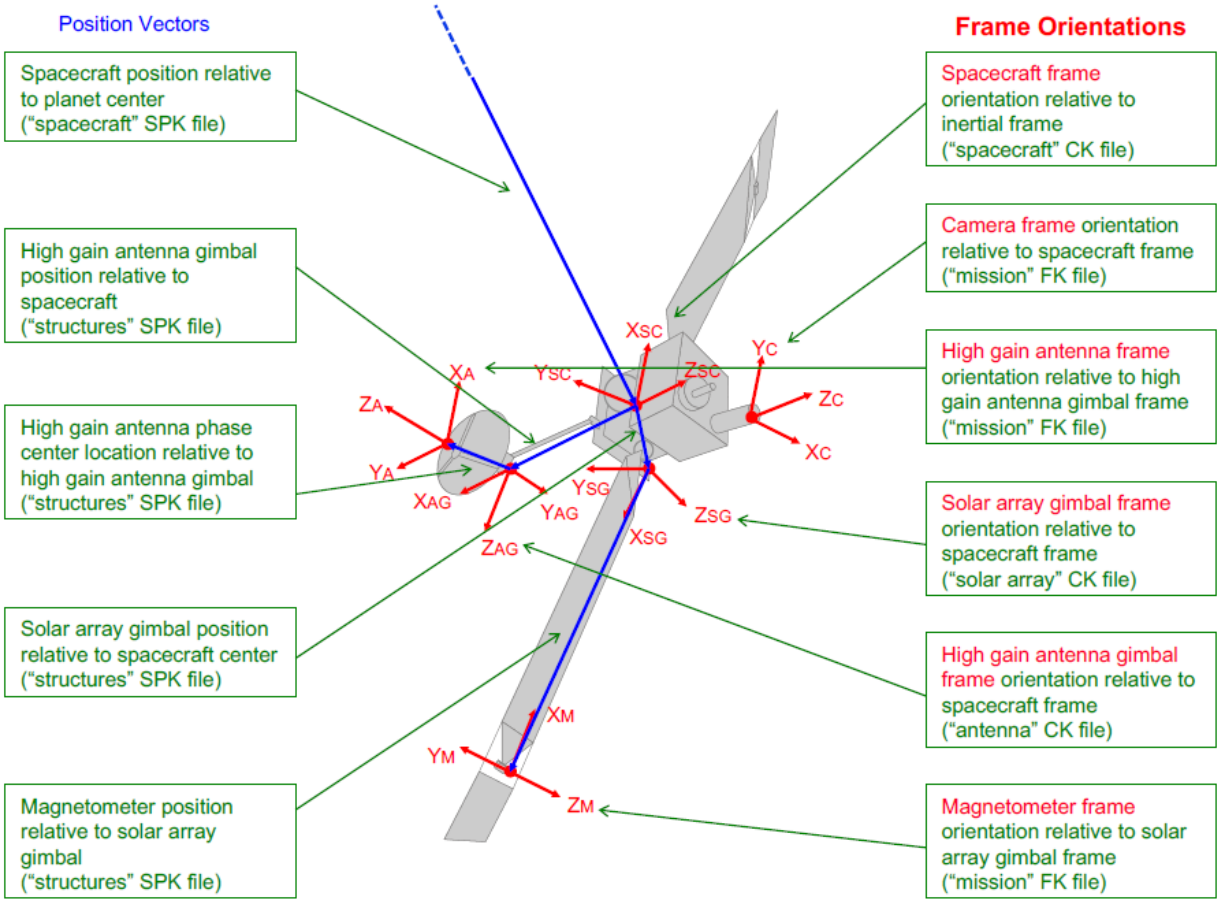
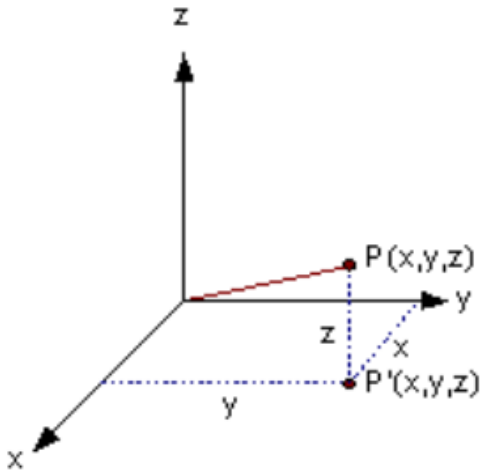
1. Position Analysis – Reference Frame

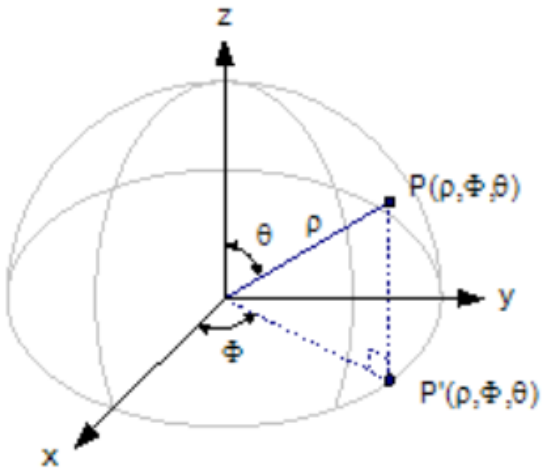


Reference: An Overview of Reference Frames and Coordinate Systems in the SPICE Context.

1. Position Analysis – Coordinate System



Rectangular or Cartesian coordinates:
X, Y, Z



Spherical coordinates:
ϕ, θ, *ρ*

Why would we choose one coordinate system over the other?

Reference: An Overview of Reference Frames and Coordinate Systems in the SPICE Context.

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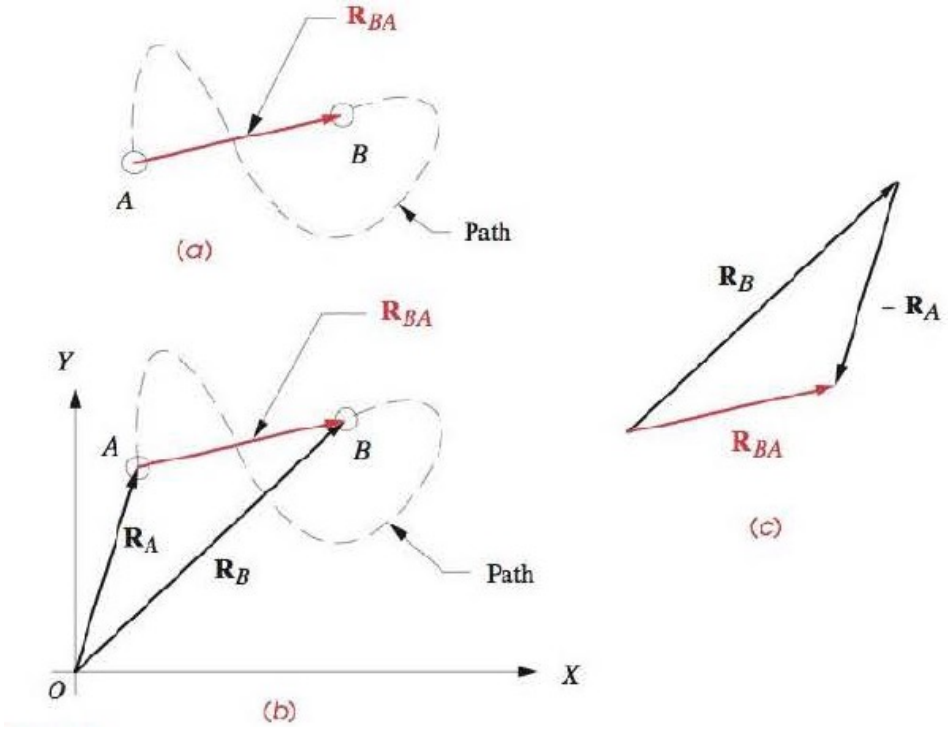
1. Position Analysis - Displacement

Displacement is a straight-line distance between initial and final position in the reference frame.



What would the difference be if R_{BA} describes the relative position of two points?

Reference: Design of Machinery by Robert L. Norton

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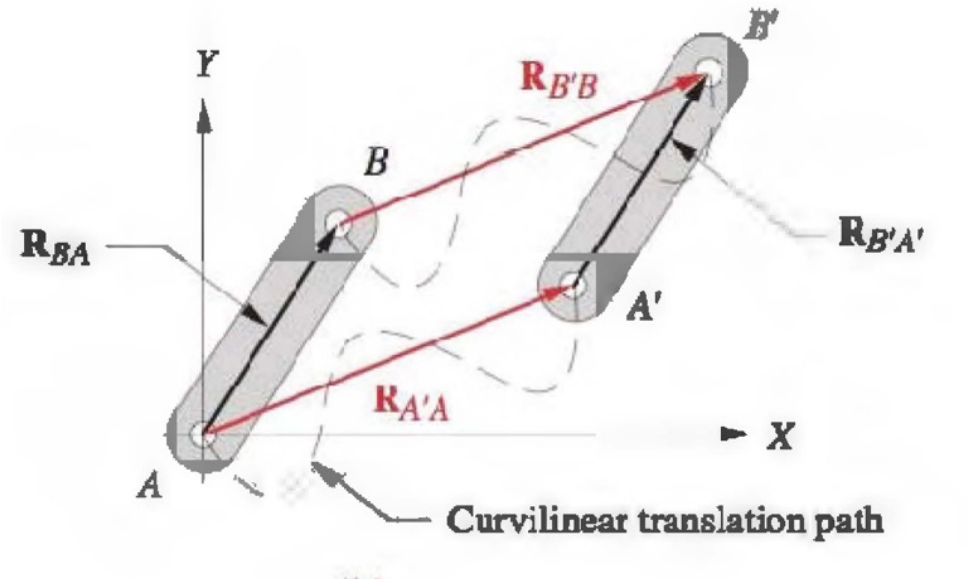
1. Position Analysis – Types of Motion

- **Translation:** all points of the body experience the same displacement

$$\boldsymbol{R}_{A'A} = \boldsymbol{R}_{B'B}$$

What is the difference between $R_{A'A}$ and $R_{AA'}$?



Reference: Design of Machinery by Robert L. Norton

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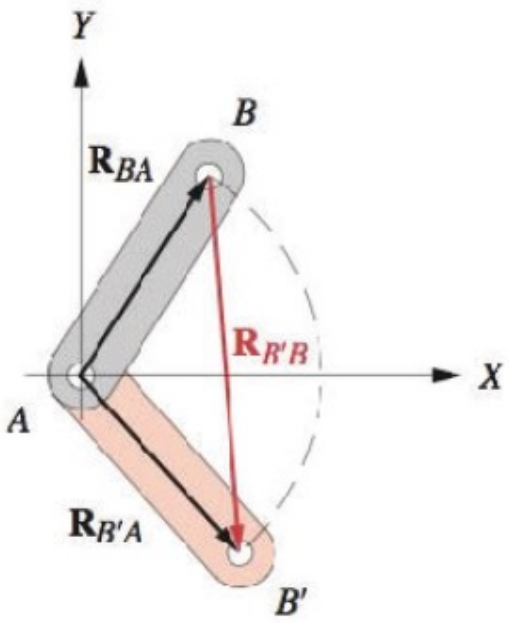
1. Position Analysis – Types of Motion

- **Rotation:** all points undergo different displacements and there is a displacement difference between any two points.

$$\boldsymbol{R}_{B'B} = \boldsymbol{R}_{B'A} - \boldsymbol{R}_{BA}$$



Reference: Design of Machinery by Robert L. Norton

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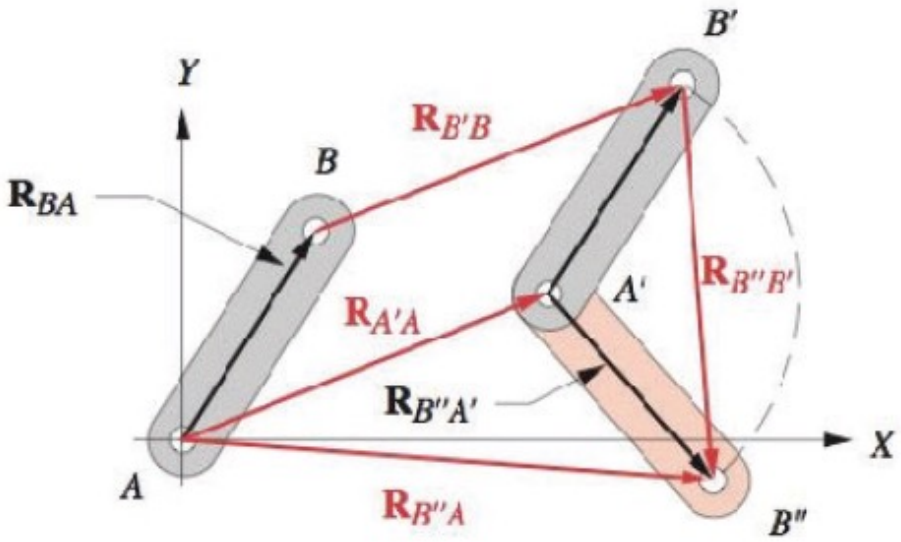
1. Position Analysis – Types of Motion

- **Complex motion** is sum of translation and rotation

$$\boldsymbol{R}_{B''A} = \boldsymbol{R}_{A'A} + \boldsymbol{R}_{B''A'}$$

Is there a difference between translation-then-rotation and rotation-then-translation?

What is the total displacement of B?



Reference: Design of Machinery by Robert L. Norton

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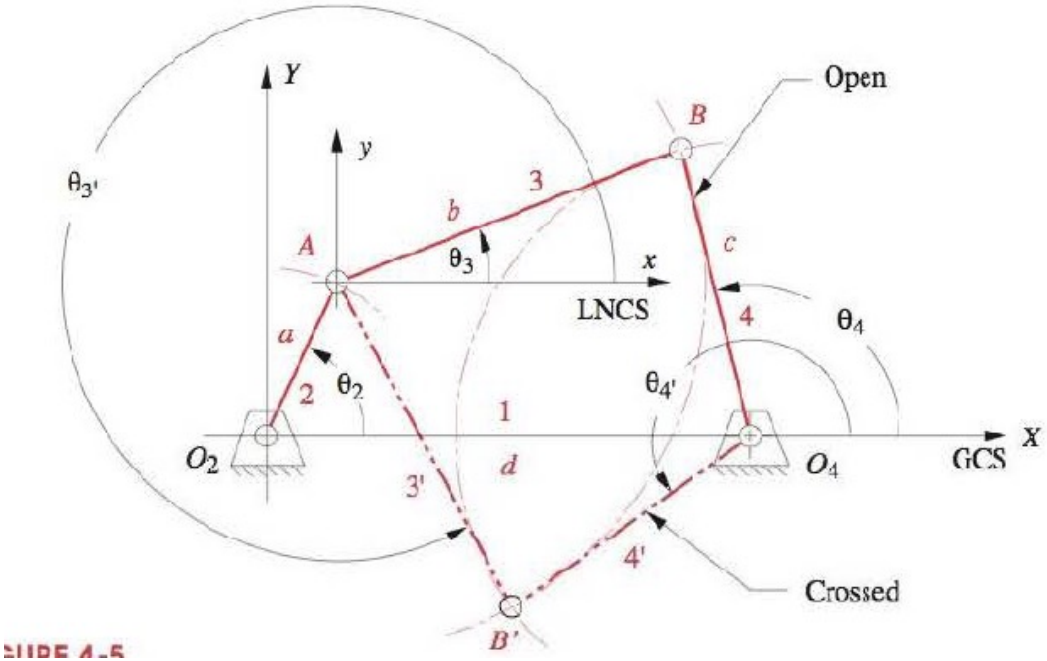
2. Graphical Position Analysis of Linkages

Position Analysis for Four-bar Linkage



Read the notes https://tinyurl.com/miet2510-notes relevant to this section

Reference: Design of Machinery by Robert L. Norton

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3. Algebraic Position Analysis of Linkages

The coordinates (reference X,Y) of point A are found from:

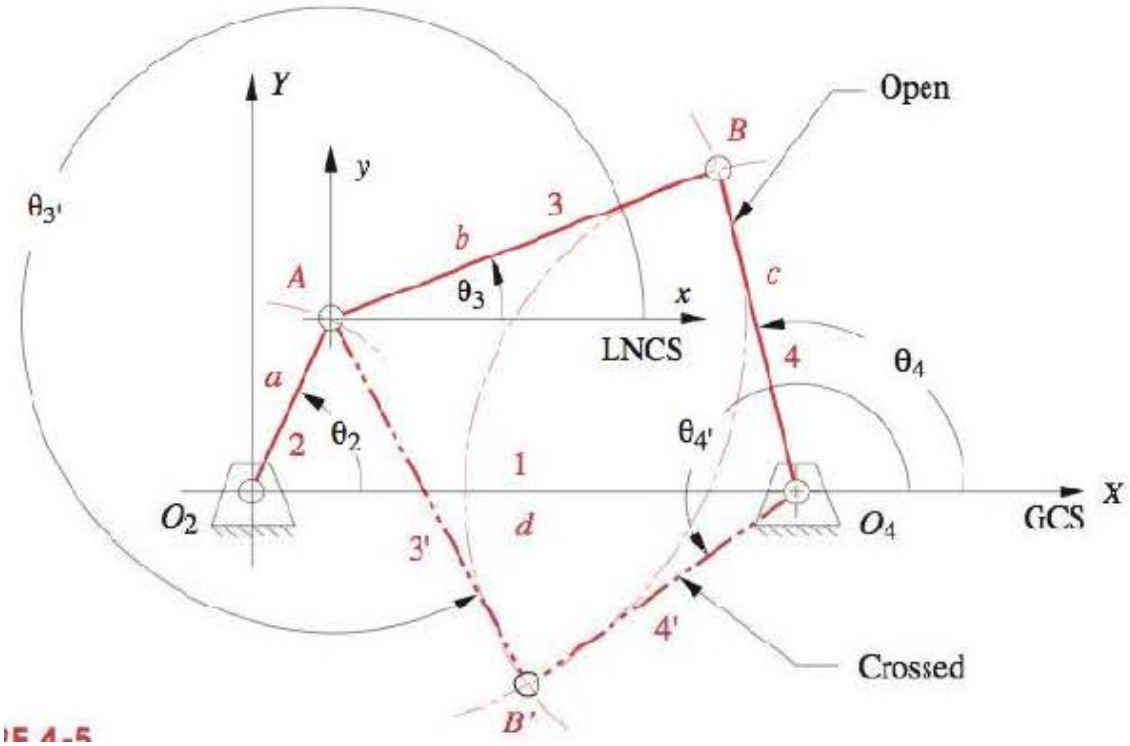
$A_X = a\cos\theta_2$ and $A_Y = a\sin\theta_2$

The coordinates of point B are found from:

$$b^2 = (B_X - A_X)^2 + (B_Y - A_Y)^2$$

$$c^2 = (d - B_X)^2 + B_Y^2$$

Using above equations, the solution for B_X and B_Y are obtained.



Derive the above equations using the local reference x,y

Reference: Design of Machinery by Robert L. Norton

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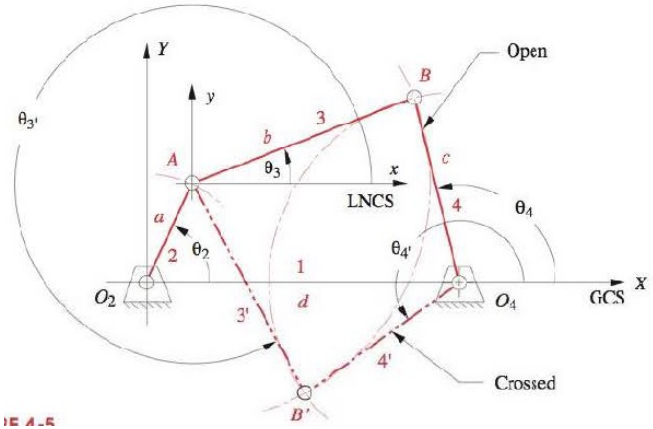
3. Algebraic Position Analysis of Linkages

- If the solution for B_X and B_Y are imaginary, it indicates that the links cannot connect at the given input angle or at all.
- If the obtained values are real, the link angles are found from:

$$\theta_3 = \tan^{-1}\left(\frac{B_Y - A_Y}{B_X - A_X}\right)$$

$$\theta_4 = \tan^{-1}\left(\frac{B_Y}{B_X - d}\right)$$



See here for more details of this derivation; https://bamason2.github.io/miet2510-module/notes/fourbar_crank_slider_%20pos_analysis_algebraic.html

Reference: Design of Machinery by Robert L. Norton

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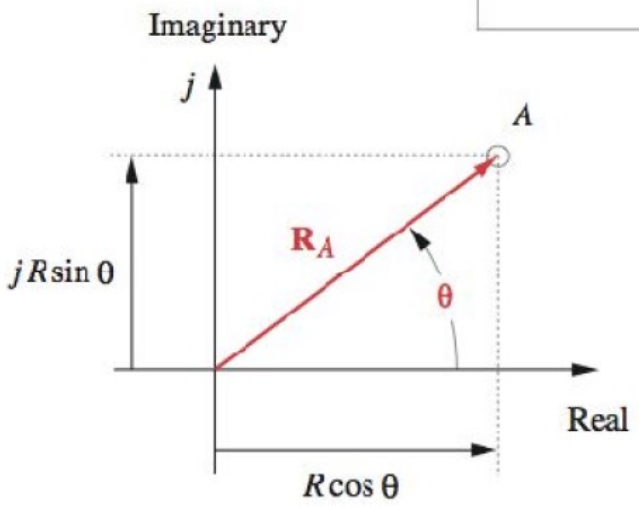
3. Algebraic Position Analysis of Linkages

Solution Procedure Outline

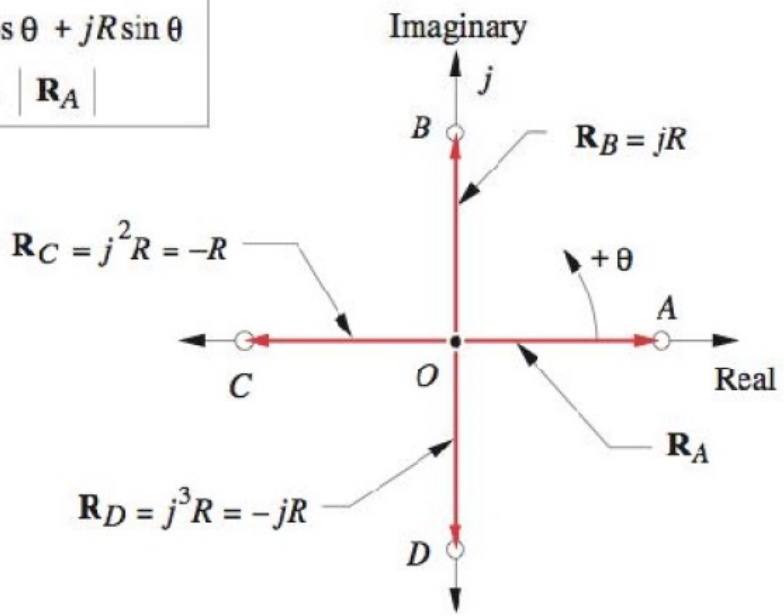
- 1. …

4. Vector Loop Method for Analysis of Linkages

Polar form:	$R\,e^{j\theta}$
Cartesian form:	$R\cos\theta + jR\sin\theta$
	$R = \lvert \mathbf{R}_A \rvert$



(a) Complex number representation of a position vector



(b) Vector rotations in the complex plane

Remember Euler's equation: $e^{j\theta} = (\cos\theta + j\sin\theta)$

Reference: Design of Machinery by Robert L. Norton

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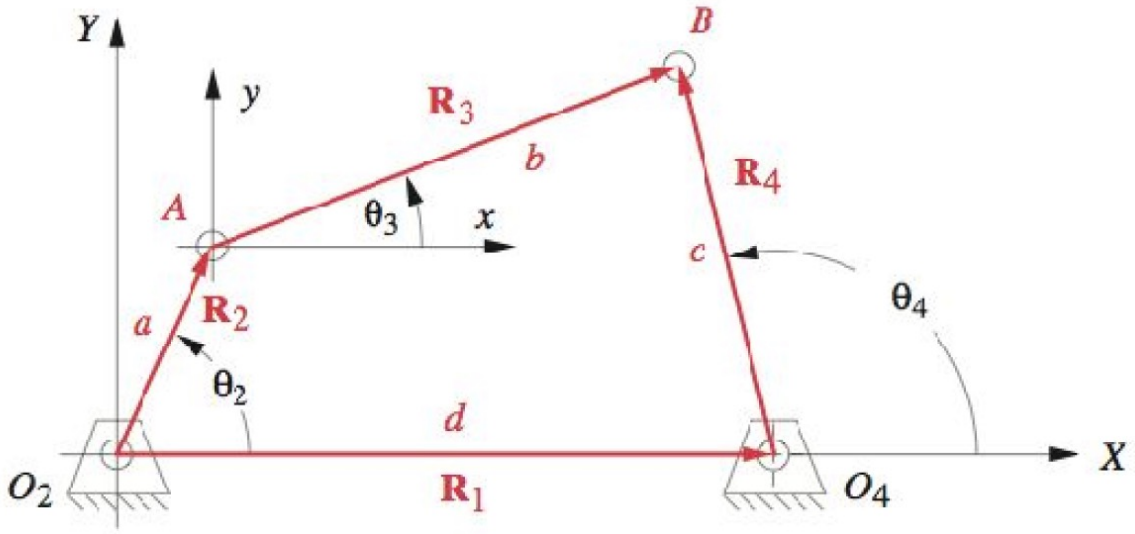
4. Vector Loop Method for Analysis of Linkages

- Write the vector loop equation for the following fourbar linkage



$$\boldsymbol{R}_A + \boldsymbol{R}_{BA} - \boldsymbol{R}_{BO_4} - \boldsymbol{R}_{O_4} = 0$$

Reference: Design of Machinery by Robert L. Norton

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4. Vector Loop Method for Analysis of Linkages

- Replacing vectors with complex numbers, the following equation can be derived:

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0$$

$\boldsymbol{R}_A = \boldsymbol{a}e^{j\theta_2}$
etc

- Replacing the exponential form with complex form gives the following.

$$a(\cos\theta_2 + j\sin\theta_2) + b(\cos\theta_3 + j\sin\theta_3) - c(\cos\theta_4 + j\sin\theta_4) - d(\cos\theta_1 + j\sin\theta_1) = 0$$

Reference: Design of Machinery by Robert L. Norton

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4. Vector Loop Method for Analysis of Linkages

Solving the Vector Loop Equation

1. Separate the final equation on the previous slide into real and imaginary components.
2. After some algebraic manipulation and the use of identities we obtain an equation in quadratic form

$$\theta_{4_{1,2}} = 2\tan^{-1}\left(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right)$$

3. And following a similar process

$$\theta_{3_{1,2}} = 2\tan^{-1}\left(\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right)$$

4. From the above all angles are now known. The above indicates that there are two possible solutions for each angle.
5. If the solution is imaginary then it is an **infeasible** mechanism.

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Homework

Read the notes and understand the derivation for the vector position analysis of the four-bar linkage: https://tinyurl.com/miet2510-notes

Using the Vector Loop Method, conduct the position analysis for the following mechanisms which are included in the Reading Note for Week 2:

- Example 4.1: Four-bar Linkage
- Example 4.2: Four-bar crank-slider Linkage
- Example 4.3: Four-bar slider-crank Linkage
- Example 4.4: Inverted crank-slider Linkage

Reference: Design of Machinery by Robert L. Norton

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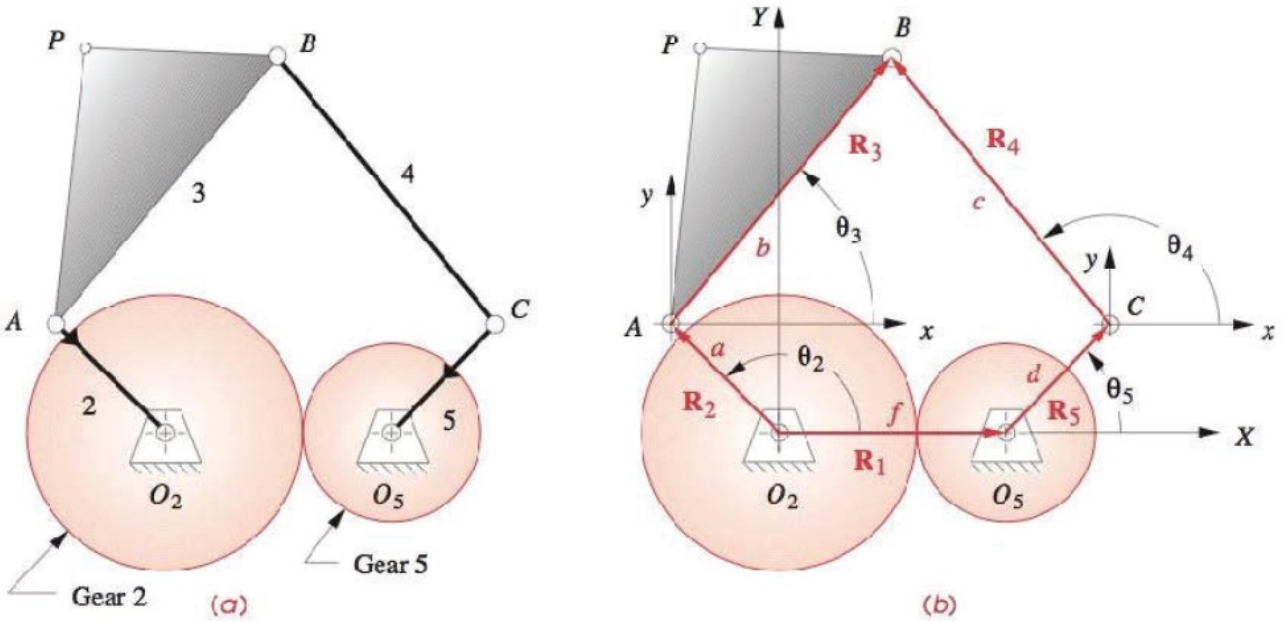
Further topics to be investigated

1. Position Analysis for Linkages of more than four bars



Reference: Design of Machinery by Robert L. Norton

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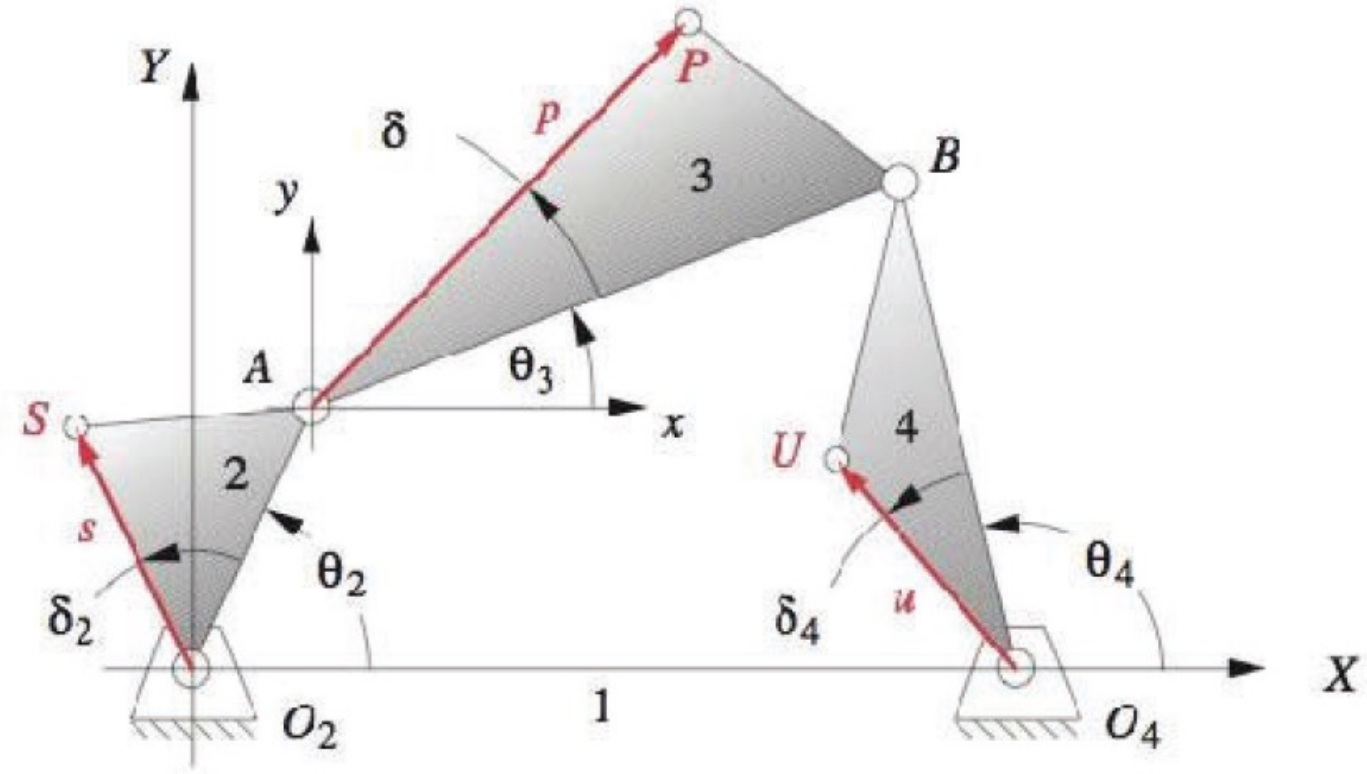
Further topics to be investigated

2. Position Analysis of any Point on a Linkage



Reference: Design of Machinery by Robert L. Norton

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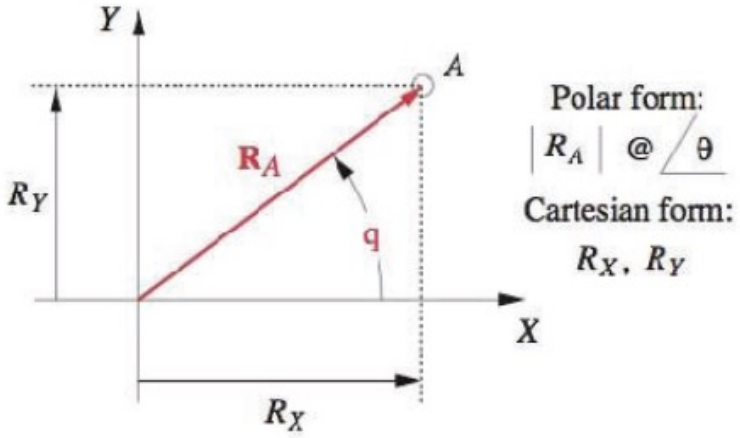
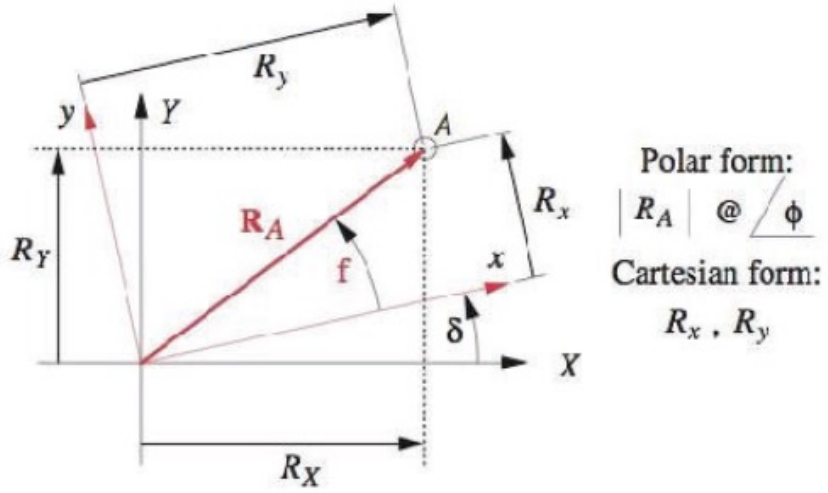


(a) Global coordinate system XY



(b) Local coordinate system xy

Reference: Design of Machinery by Robert L. Norton

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Thank you for your attendance :D

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Reference

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