MIET2510

Mechanical Design

Week 2 – Kinematic Analysis of Mechanism – Part 2

School of Science and Technology, RMIT Vietnam

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Outlines

1. Velocity Analysis
2. Graphical Velocity Analysis (HW)
3. Analytical Solution for Velocities Analysis
4. Homework and Further Topics

Approaches to Analysis

- Just as with position analysis there are several approaches to analysing linkages/mechanisms
  - Graphical approach
  - Analytic approach
  - Complex-algebraic approach
  - Vector-algebraic approach
  - Numeric approach

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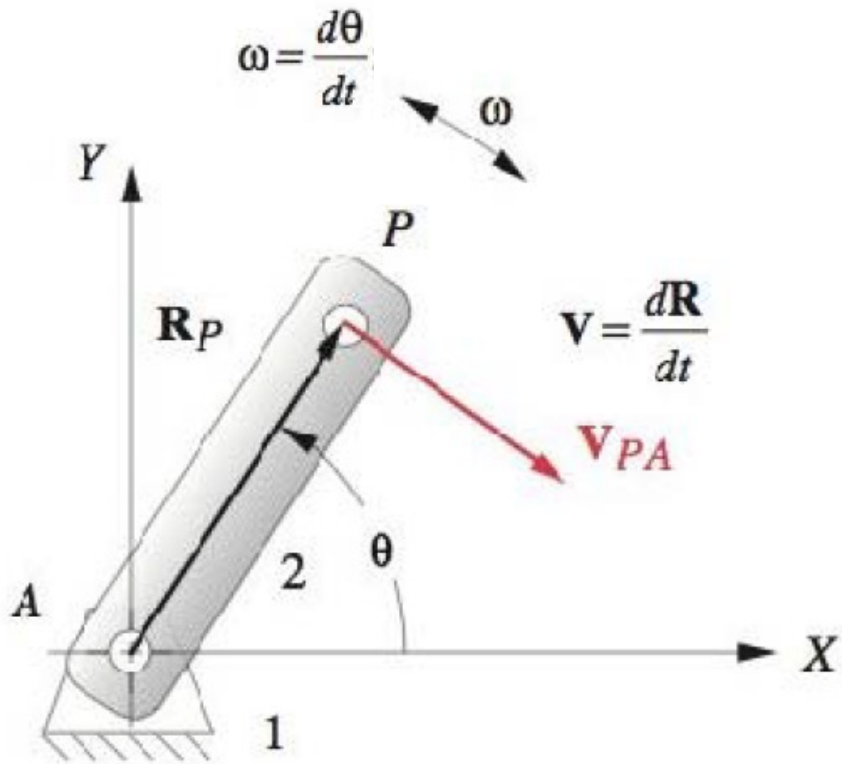
1. Velocity Analysis

- Once a position analysis is done, the next step is to determine the velocities of all links and points of interest in the mechanism.
- The velocity analysis is required to determine the link's acceleration which are needed for dynamic force calculations.
- Many methods and approaches exist to find velocities in mechanisms, and we will focus on the most general approach using analytical solution.

1. Velocity Analysis



$$\mathbf{R}_{PA} = pe^{j\theta}$$

$$\mathbf{V}_{PA} = \frac{d\mathbf{R}_{PA}}{dt} = p\,je^{j\theta}\frac{d\theta}{dt} = p\omega\,je^{j\theta}$$

$$\mathbf{V}_{PA} = p\omega j(\cos\theta + j\sin\theta) = p\omega(-\sin\theta + j\cos\theta)$$

What do we need to know to calculate $\boldsymbol{V}_{PA}$?

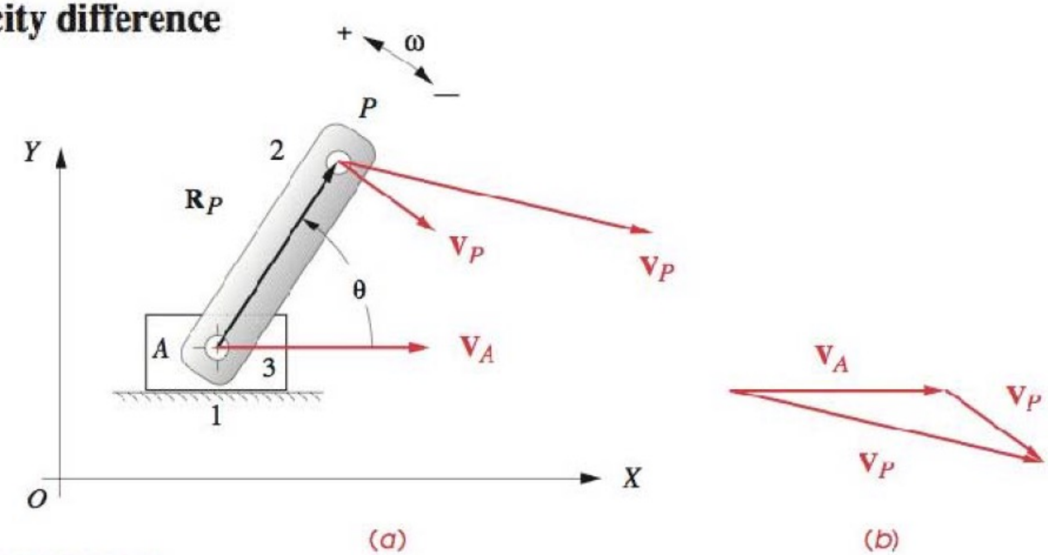
Remember Euler's equation: $e^{j\theta} = (\cos\theta + j\sin\theta)$

Reference: *Design of Machinery by Robert L. Norton*

1. Velocity Analysis

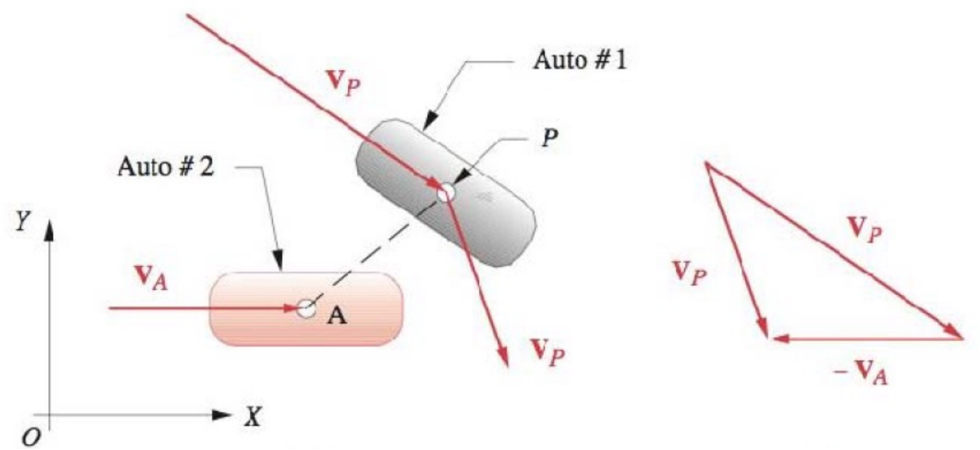
CASE 1: *Two points in the same body* => **velocity difference**

$$\mathbf{V}_P = \mathbf{V}_A + \mathbf{V}_{PA}$$



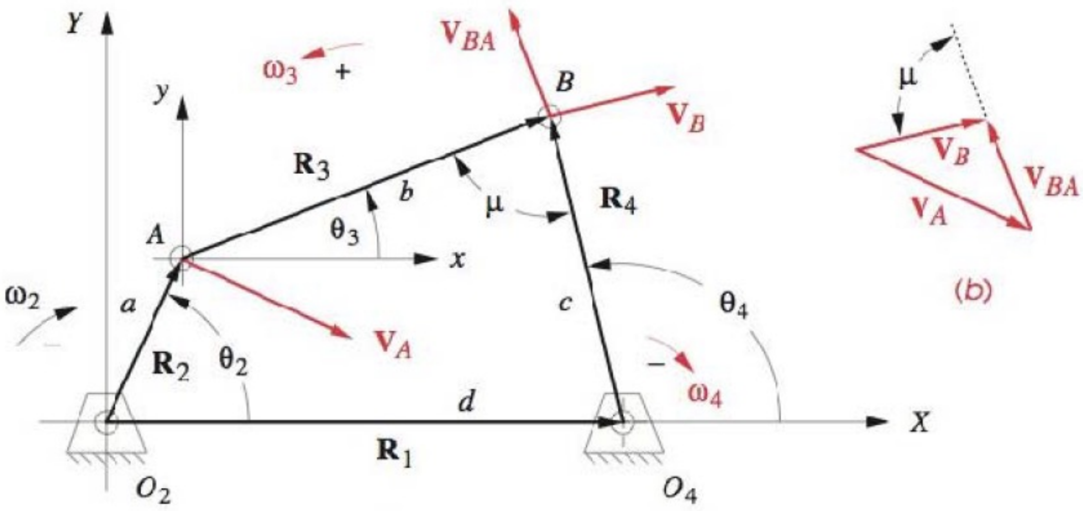
CASE 2: *Two points in different bodies* => **relative velocity**

Reference: Design of Machinery by Robert L. Norton



3. Analytical Solution for Velocity Analysis

- Write the vector loop equation for the following fourbar linkage



$$\boldsymbol{R}_A + \boldsymbol{R}_{BA} - \boldsymbol{R}_{BO_4} - \boldsymbol{R}_{O_4} = 0$$

Reference: *Design of Machinery by Robert L. Norton*

3. Analytical Solution for Velocity Analysis

- Use the complex form of the vector loop equation, noting that *a, b, c* and *d* represent the scalar lengths of the links

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0$$

$$\boldsymbol{R}_A = ae^{j\theta_2}$$

etc

we can differentiate wrt t to get

$$jae^{j\theta_2}\frac{d\theta_2}{dt} + jbe^{j\theta_3}\frac{d\theta_3}{dt} - jce^{j\theta_4}\frac{d\theta_4}{dt} \boxed{- jde^{j\theta_1}\frac{d\theta_1}{dt}} = 0$$

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3. Analytical Solution for Velocities Analysis

- Noting that $\frac{d\theta}{dt} = \omega$

$$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

- Using Euler's equation $e^{j\theta} = \cos\theta + j\sin\theta$

$$ja\omega_2(cos\theta_2 + jsin\theta_2) + jb\omega_3(cos\theta_3 + jsin\theta_3) - jc\omega_4(cos\theta_4 + jsin\theta_4) = 0$$

- Multiplying out the above remembering $j^2 = -1$ we get an expression that can be separated into real and imaginary components

$$-a\omega_2 sin\theta_2 - b\omega_3 sin\theta_3 + c\omega_4 sin\theta_4 = 0$$

$$a\omega_2 cos\theta_2 + b\omega_3 cos\theta_3 - c\omega_4 cos\theta_4 = 0$$

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3. Analytical Solution for Velocities Analysis

- We can solve these two equations simultaneously to find ω_3 and ω_4

$$\omega_3 = \omega_2 \frac{a}{b} \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)}$$

$$\omega_4 = \omega_2 \frac{a}{c} \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)}$$

- We can also find

$$\boldsymbol{V}_A = ja\omega_2(\cos\theta_2 + j\sin\theta_2) = a\omega_2(-\sin\theta_2 + j\cos\theta_2)$$

$$\boldsymbol{V}_{\boldsymbol{BA}} = jb\omega_3(\cos\theta_3 + j\sin\theta_3) = b\omega_3(-\sin\theta_3 + j\cos\theta_3)$$

$$\boldsymbol{V}_B = jc\omega_4(\cos\theta_4 + j\sin\theta_4) = c\omega_4(-\sin\theta_4 + j\cos\theta_4)$$

Make a note for each of the above which variables are required for the calculation

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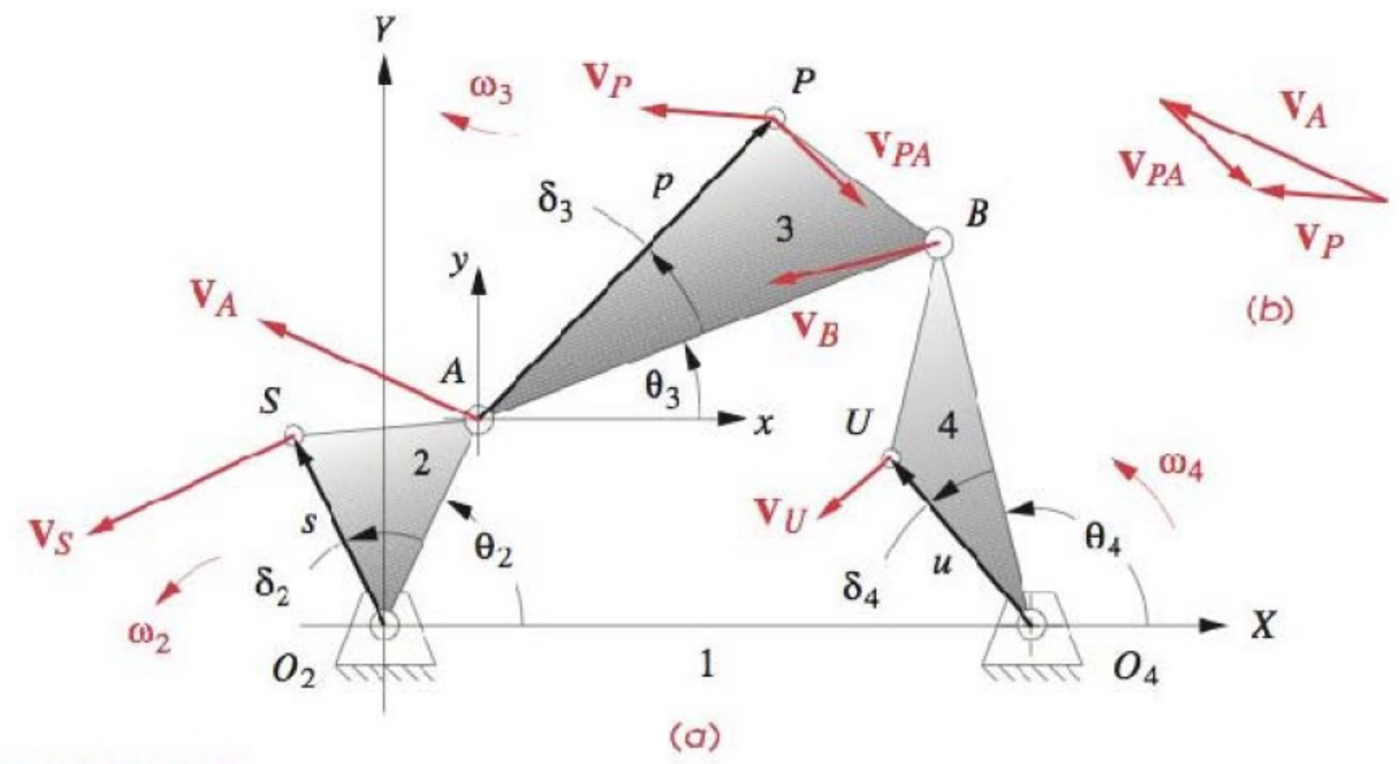
4. Homework

Make sure you understand the derivation in the notes for the fourbar linkage position analysis: https://tinyurl.com/miet2510-notes

Sample Problems in the Reading Notes about Velocity Analysis applied to the following mechanisms: Conventional Pin-Jointed Four-bar linkage, Four-bar crank slider, Four-bar slider crank, Four-bar inverted crank-slider.

4. Further Topic to be Investigated

Velocity of Any Point on a Linkage



Reference: Design of Machinery by Robert L. Norton

Thank you for your attendance :D

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