MIET2510

Mechanical Design

Week 3 – Kinematic Analysis of Mechanism – Part 3

School of Science and Technology, RMIT Vietnam

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Outlines

1. Acceleration Analysis
2. Analytical Solution for Acceleration Analysis
3. Homework and Further Discussion Topics

1. Acceleration Analysis

- After velocity analysis the next step is to determine the acceleration of all links and points of interest in the mechanism or machine. We need to know the accelerations to calculate the dynamic force.
- The dynamic force will contribute to the stresses in the links and other components.
- We will focus on the analytical solution for acceleration analysis.

Reference: Design of Machinery by Robert L. Norton.

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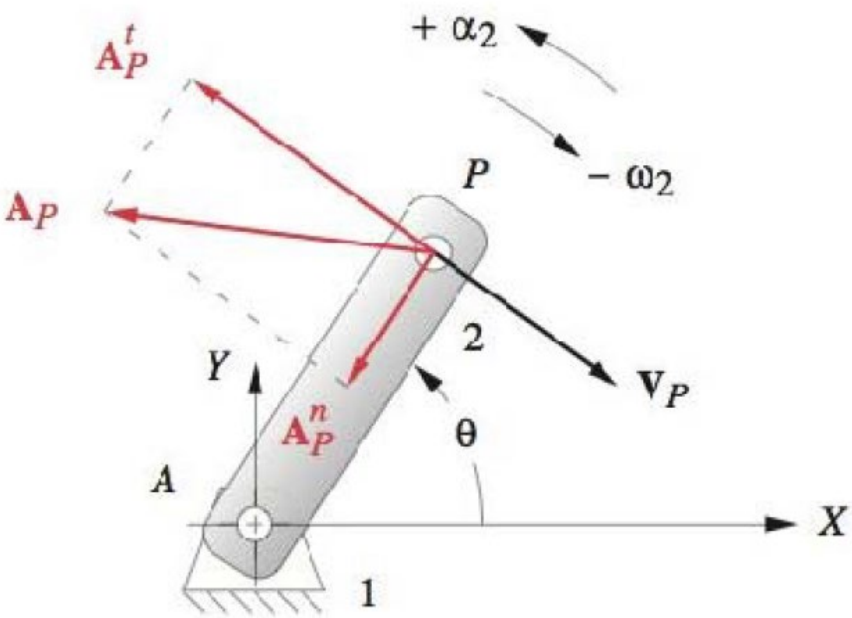
1. Acceleration Analysis

- Angular Acceleration will be defined as **α** and linear acceleration as **A**.



$$\mathbf{R}_{PA} = pe^{j\theta}$$

$$\mathbf{V}_{PA} = \frac{d\mathbf{R}_{PA}}{dt} = p\,je^{j\theta}\frac{d\theta}{dt} = p\omega\,je^{j\theta}$$

$$\mathbf{A}_{PA} = \frac{d\mathbf{V}_{PA}}{dt} = \frac{d\left(p\omega je^{j\theta}\right)}{dt}$$

$$\mathbf{A}_{PA} = jp\left(e^{j\theta}\frac{d\omega}{dt} + \omega je^{j\theta}\frac{d\theta}{dt}\right)$$

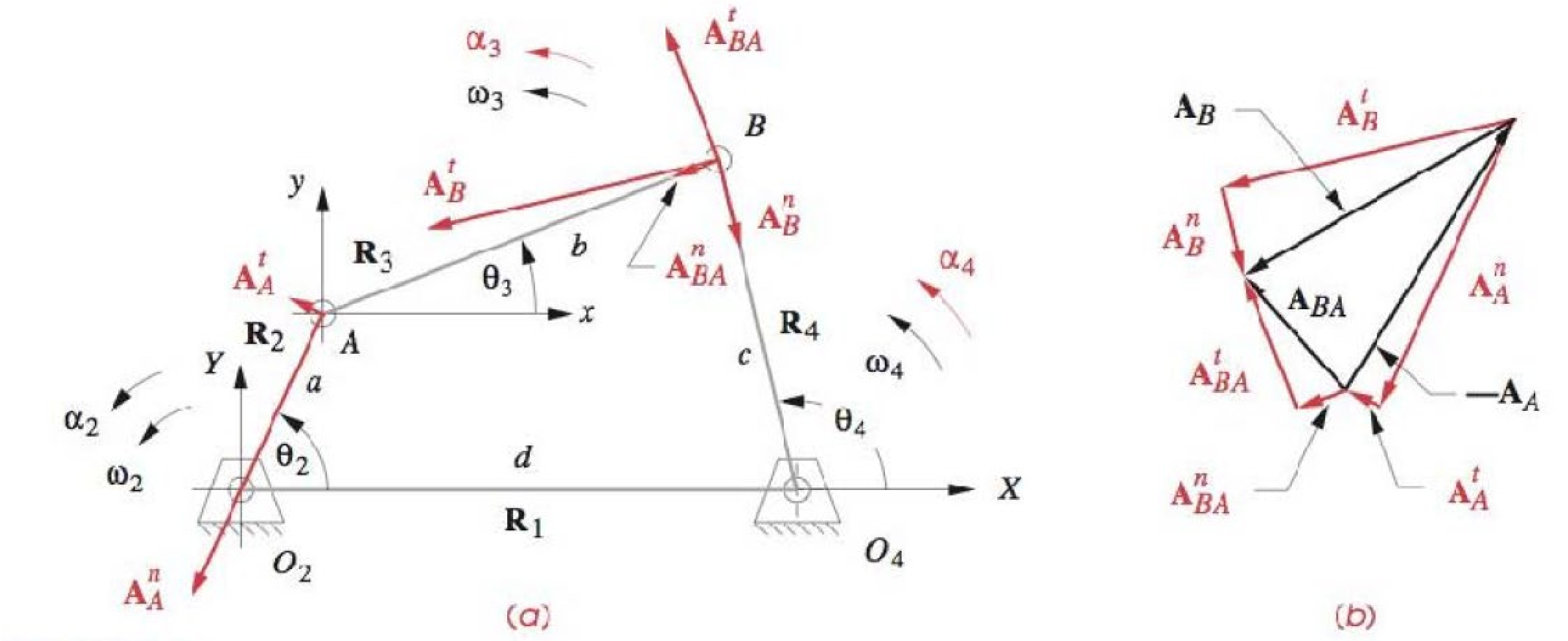
$$\mathbf{A}_{PA} = p\alpha je^{j\theta} - p\omega^2 e^{j\theta}$$

$$\mathbf{A}_{PA} = \mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n$$

$$\mathbf{A}_{PA} = p\alpha(-\sin\theta + j\cos\theta) - p\omega^2(\cos\theta + j\sin\theta)$$

Reference: Design of Machinery by Robert L. Norton.

2. Analytical Solution for Acceleration Analysis



Conventional Pin-Jointed Four-bar linkage

Reference: Design of Machinery by Robert L. Norton.

2. Analytical Solution for Acceleration Analysis

- From previous part (velocity analysis), we have:

$$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

- If we take derivative from velocity equation for the loop in 4-link mechanism, then:

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\mathrm{j}a\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4}\right) = 0$$

$$\left(j^2 a\omega_2^2 e^{j\theta_2} + ja\alpha_2 e^{j\theta_2}\right) + \left(j^2 b\omega_3^2 e^{j\theta_3} + jb\alpha_3 e^{j\theta_3}\right) - \left(j^2 c\omega_4^2 e^{j\theta_4} + jc\alpha_4 e^{j\theta_4}\right) = 0$$

Reference: Design of Machinery by Robert L. Norton.

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2. Analytical Solution for Acceleration Analysis

- Simplifying the grouping terms:

$$\left(a\alpha_2 je^{j\theta_2} - a\omega_2^2 e^{j\theta_2}\right) + \left(b\alpha_3 je^{j\theta_3} - b\omega_3^2 e^{j\theta_3}\right) - \left(c\alpha_4 je^{j\theta_4} - c\omega_4^2 e^{j\theta_4}\right) = 0$$

- Above equation is in fact:

$$\mathbf{A}_A + \mathbf{A}_{BA} - \mathbf{A}_B = 0$$

$$\mathbf{A}_A = \left(\mathbf{A}_A^t + \mathbf{A}_A^n\right) = \left(a\alpha_2 je^{j\theta_2} - a\omega_2^2 e^{j\theta_2}\right)$$

$$\mathbf{A}_{BA} = \left(\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n\right) = \left(b\alpha_3 je^{j\theta_3} - b\omega_3^2 e^{j\theta_3}\right)$$

$$\mathbf{A}_B = \left(\mathbf{A}_B^t + \mathbf{A}_B^n\right) = \left(c\alpha_4 je^{j\theta_4} - c\omega_4^2 e^{j\theta_4}\right)$$

Reference: Design of Machinery by Robert L. Norton.

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2. Analytical Solution for Acceleration Analysis

- Our problem is to find angular acceleration α_3 and α_4 knowing the angular input acceleration α_2, the link length, all link angles, and angular velocities.

$$\alpha_3 = f(a, b, c, d, \theta_2, \theta_3, \theta_4, \omega_2, \omega_3, \omega_4, \alpha_2)$$
$$\alpha_4 = g(a, b, c, d, \theta_2, \theta_3, \theta_4, \omega_2, \omega_3, \omega_4, \alpha_2)$$

Reference: Design of Machinery by Robert L. Norton.

2. Analytical Solution for Acceleration Analysis

- The strategy of solution will be the same as was done for position and velocity analysis.

$$\left[a\alpha_2\, j(\cos\theta_2 + j\sin\theta_2) - a\omega_2^2(\cos\theta_2 + j\sin\theta_2)\right]$$
$$+\left[b\alpha_3\, j(\cos\theta_3 + j\sin\theta_3) - b\omega_3^2(\cos\theta_3 + j\sin\theta_3)\right]$$
$$-\left[c\alpha_4\, j(\cos\theta_4 + j\sin\theta_4) - c\omega_4^2(\cos\theta_4 + j\sin\theta_4)\right] = 0$$

Reference: Design of Machinery by Robert L. Norton.

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2. Analytical Solution for Acceleration Analysis

- The real component is

$$-a\alpha_2 \sin\theta_2 - a\omega_2^2 \cos\theta_2 - b\alpha_3 \sin\theta_3 - b\omega_3^2 \cos\theta_3 + c\alpha_4 \sin\theta_4 + c\omega_4^2 \cos\theta_4 = 0 \qquad [1]$$

- The imaginary component is:

$$a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 + b\alpha_3 \cos\theta_3 - b\omega_3^2 \sin\theta_3 - c\alpha_4 \cos\theta_4 + c\omega_4^2 \sin\theta_4 = 0 \qquad [2]$$

Reference: Design of Machinery by Robert L. Norton.

2. Analytical Solution for Acceleration Analysis

- Solving [1] and [2] simultaneously we get

$$\alpha_3 = \frac{(a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 - b\omega_3^2\sin\theta_3 + c\omega_4^2\sin\theta_4)(c\cos\theta_4)}{(c\sin\theta_4)(b\cos\theta_3) - (b\sin\theta_3)(c\cos\theta_4)}$$
$$- \frac{(c\sin\theta_4)(a\alpha_2\sin\theta_2 + a\omega_2^2\cos\theta_2 + b\omega_3^2\cos\theta_3 - c\omega_4^2\cos\theta_4)}{(c\sin\theta_4)(b\cos\theta_3) - (b\sin\theta_3)(c\cos\theta_4)}$$

$$\alpha_4 = \frac{(a\alpha_2\sin\theta_2 + a\omega_2^2\cos\theta_2 + b\omega_3^2\cos\theta_3 - c\omega_4^2\cos\theta_4)(b\cos\theta_3)}{(c\sin\theta_4)(b\cos\theta_3) - (b\sin\theta_3)(c\cos\theta_4)}$$
$$- \frac{(b\sin\theta_3)(a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 - b\omega_3^2\sin\theta_3 + c\omega_4^2\sin\theta_4)}{(c\sin\theta_4)(b\cos\theta_3) - (b\sin\theta_3)(c\cos\theta_4)}$$

2. Analytical Solution for Acceleration Analysis

- Having obtained α_3 and α_4 we can obtain the accelerations.

$$\mathbf{A}_A = a\alpha_2\left(-\sin\theta_2 + j\cos\theta_2\right) - a\omega_2^2\left(\cos\theta_2 + j\sin\theta_2\right)$$
$$\mathbf{A}_{BA} = b\alpha_3\left(-\sin\theta_3 + j\cos\theta_3\right) - b\omega_3^2\left(\cos\theta_3 + j\sin\theta_3\right)$$
$$\mathbf{A}_B = c\alpha_4\left(-\sin\theta_4 + j\cos\theta_4\right) - c\omega_4^2\left(\cos\theta_4 + j\sin\theta_4\right)$$

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3. Homework and Further Topics

- Graphical Approach in Acceleration Analysis.
- Sample Problems in the Reading Notes.
- Acceleration of Any Point in the Linkage.

Reference: Design of Machinery by Robert L. Norton.

Thank you for your attendance :D

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Reference

- *Design of Machinery by Robert L. Norton.*