MIET2510

Mechanical Design

Week 3 – Dynamic Fundamentals – Part 1

School of Science and Technology, RMIT Vietnam

RMIT UNIVERSITY

Outlines

1. Dynamic Fundamentals
2. Static Force Analysis
3. Further information about Centre of Gravity and Mass Moment of Inertia.

1. Dynamic Fundamentals – Force Analysis

- We have focused on kinematic analysis up until now.
  - Position
  - Velocity
  - Acceleration
- We are now ready for a study of the dynamics of machines and systems, since the general function of any machine is to transmit motion and forces from an actuator to the components that perform the desired task.

RMIT UNIVERSITY

1. Dynamic Fundamentals - Forces

- Energy, work, power?
- A force, **F**, is a vector quantity that represents a pushing or pulling action on a part. Force has a magnitude and a direction.
- Two or more forces that are applied to a part can be combined to determine the resulting effect of the forces.
- One force can be broken into two components along orthogonal axes.

RMIT UNIVERSITY

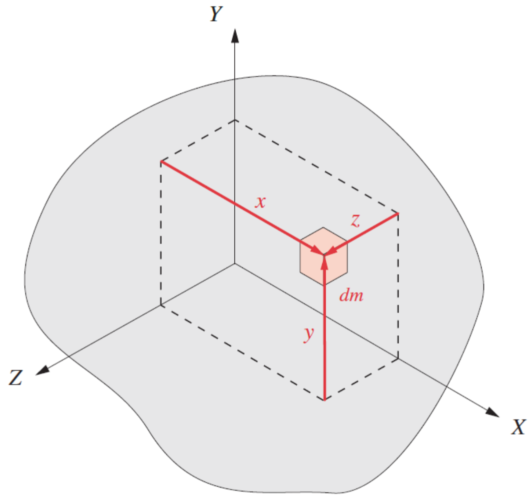
1. Mass Moment and Centre of Gravity

When the mass of an object is distributed over some dimensions, it will possess a moment with respect to any axis of choice. **The mass moment** of the differential element (dm) is equal to the product of its mass and its distance from the axis of interest.

$$dM_x = x\,dm \qquad M_x = \int x\,dm$$

$$dM_y = y\,dm \qquad M_y = \int y\,dm$$

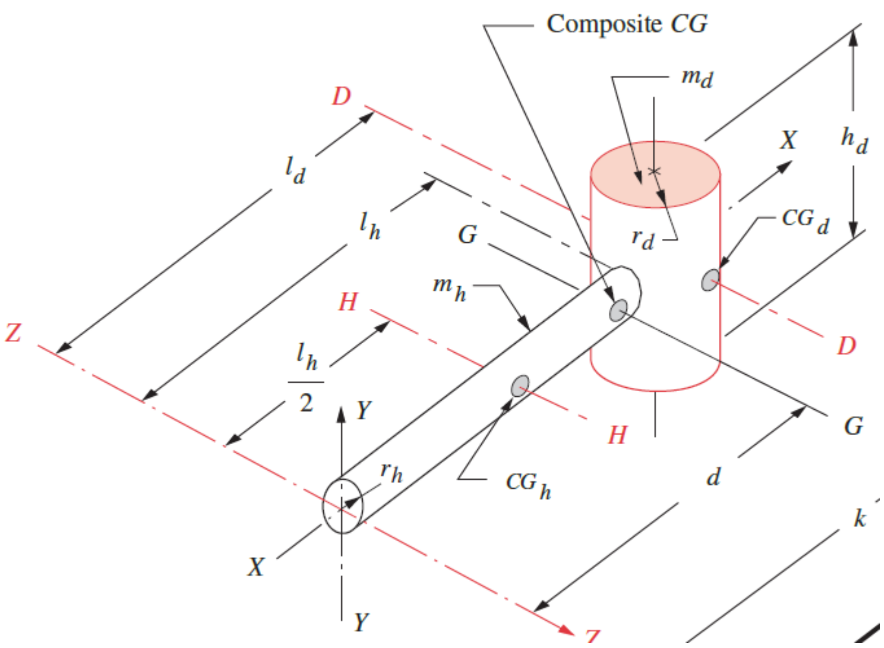
$$dM_z = z\,dm \qquad M_z = \int z\,dm$$



RMIT UNIVERSITY

1. Mass Moment and Centre of Gravity

If the mass moment with respect to a particular axis is numerically zero, then that axis passes through the centre of mass (CM) of the object, which for earthbound systems is coincident with its center of gravity (CG). By definition the summation of first moments about all axes through the center of gravity is zero.

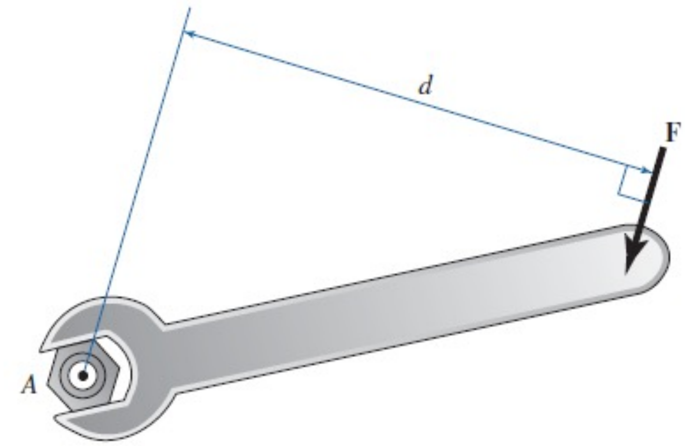


Reference: Design of Machinery by Robert L. Norton.

Find the centre of mass of some objects in the room.

1. Moments and Torques

A moment, or torque, is the twisting action produced by a force. Pushing on the handle of a wrench produces an action that tends to rotate a nut on a bolt. Thus, the force causes a twisting action around the center of a bolt. This resulting action is termed a moment or torque.



$$M_A = (F)(d)$$

1. Dynamic Fundamentals – Newton’s Laws of Motion

Isaac Newton developed three laws of motion that serve as the basis of all analysis of forces acting on machines. These laws are stated as follows:

- FIRST LAW: A body at rest tends to remain at rest and a body in motion at constant velocity will tend to maintain that velocity unless acted upon by an external force.
- SECOND LAW: The time rate of change of momentum of a body is equal to the magnitude of the applied force and acts in the direction of the force.
- THIRD LAW: For every action force there is an equal and opposite reaction force.

RMIT UNIVERSITY

Demonstrate!

2. Static Force Analysis

- Static Force Analysis deals with force analysis in mechanisms without accelerations, or where the accelerations can be neglected. This condition is termed static equilibrium.
- Static equilibrium is applicable in many machines where the changes in movement are gradual, or the mass of the components is negligible. These include clamps, latches, support linkages, and many hand operated tools, such as pliers and cutters.

RMIT UNIVERSITY

2. Static Force Analysis – Static Equilibrium

For an object to be in static equilibrium, the following two necessary and sufficient conditions must be met:

- The combination, or resultant, of all external forces acting on the object is equivalent to zero and does not cause it to translate.
- The moment due to any external force is cancelled by the moments of the other forces acting on the object and do not cause it to rotate about any point.

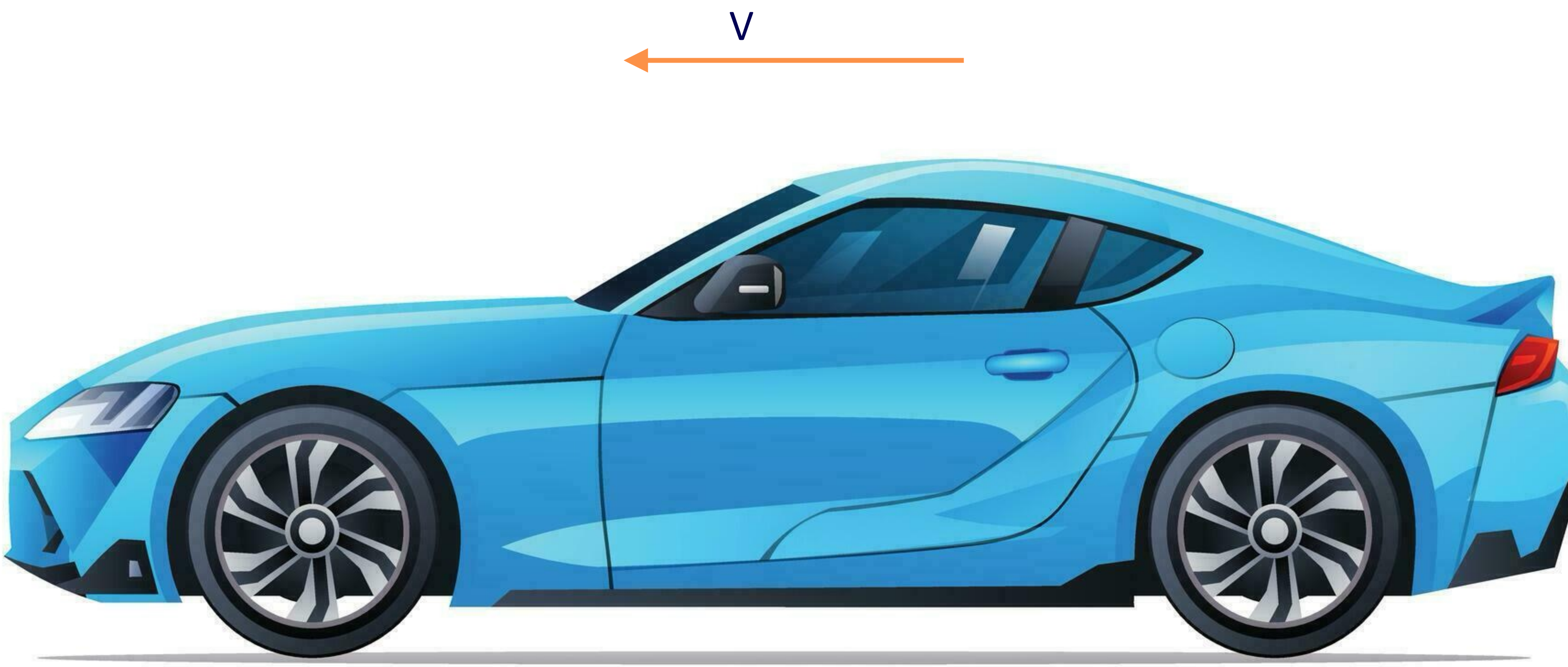
$$\sum \mathrm{F} = 0 \qquad \sum M_A = 0$$

RMIT UNIVERSITY

2. Static Force Analysis – Friction Force

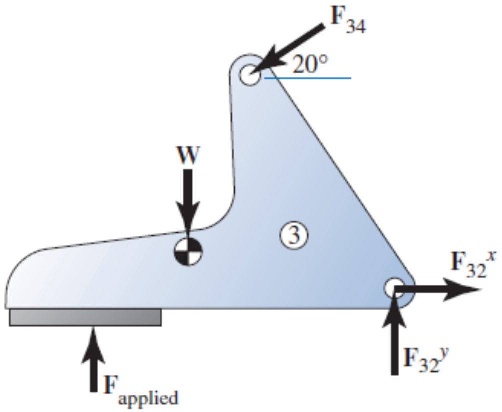
- A contact force, as a result of a sliding joint, always acts perpendicular to the surface in contact. This contact force is commonly referred to as a normal force because it acts perpendicular to the surfaces in contact.
- When friction cannot be neglected in machine analysis, an additional force, friction force, is observed. Friction always acts to impede motion and in opposite to the direction of motion. There are two types of friction, static friction and dynamic friction.

$$F_f = \mu N$$


V

2. Static Force Analysis – Free-Body Diagrams

The best way to track forces acting on the links of mechanism is to construct a **free-body diagram**.



A free-body diagram is a picture of the isolated part, as if it were floating freely. The part appears to be floating because all the supports and contacts with other parts have been removed. All these supports and contacts are then replaced with forces that represent the action of the support.

2. Static Force Analysis – Free-Body Diagrams

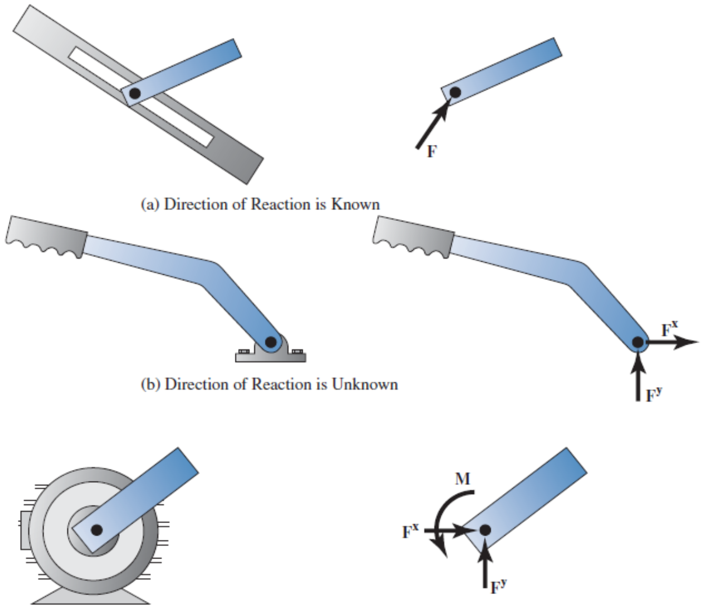
The following steps can assist in systematically drawing a free-body diagram:

a) Isolate the components to be studied.

b) Draw the component as if it were floating freely in space by removing all visible supports and physical contact that it has with other objects.

c) Replace the supports, or physical contacts, with the appropriate force and/or moments, which have the same effect as the supports.

It is essential that all forces are shown on the free-body diagram.

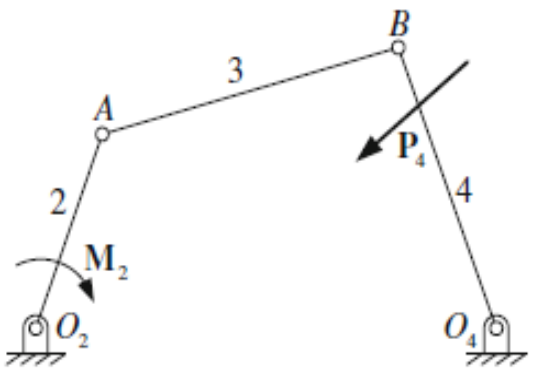


(a) Direction of Reaction is Known

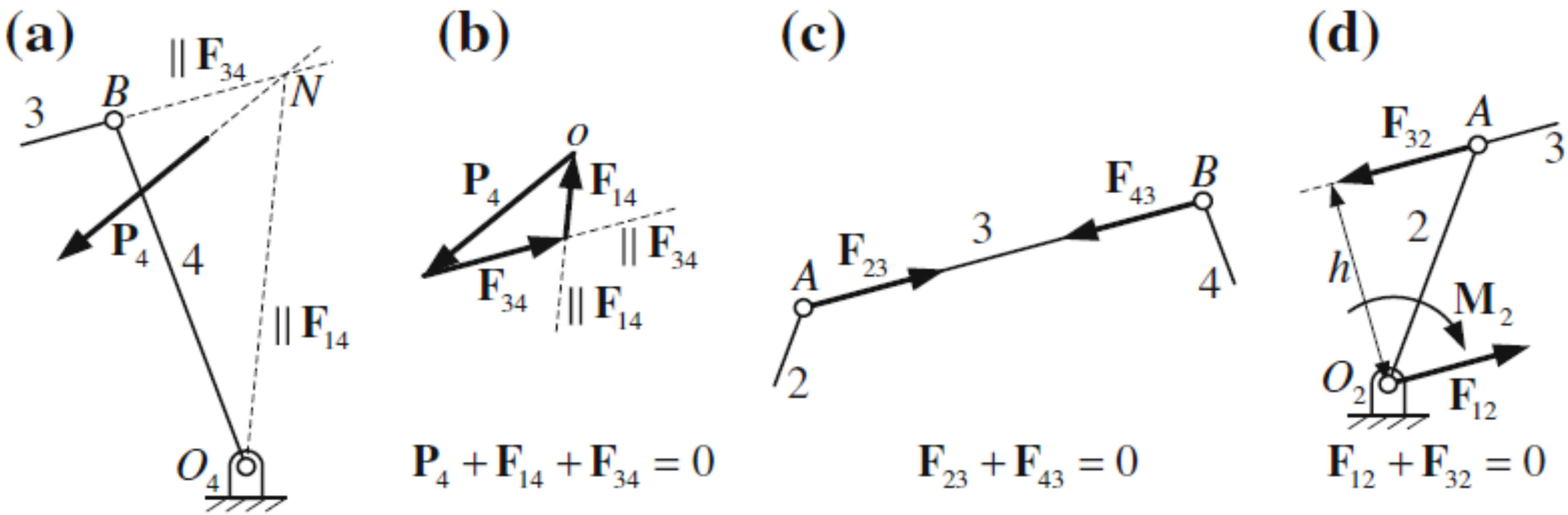
(b) Direction of Reaction is Unknown

(c) Reaction Prohibits Translation and Rotation

2. Static Force Analysis – Free-Body Diagrams



Understand the notation i.e. F23 is the force of link to acting on link 3



$\mathbf{P}_4 + \mathbf{F}_{14} + \mathbf{F}_{34} = 0$

$\mathbf{F}_{23} + \mathbf{F}_{43} = 0$

$\mathbf{F}_{12} + \mathbf{F}_{32} = 0$

2. Static Force Analysis – Analysis of a two-force member

A special case of equilibrium, which is of considerable interest, is that of a member that is subjected to only two forces. This type of machine component is termed a two-force member. Many mechanism links, particularly couplers and connecting rods, are two-force members.

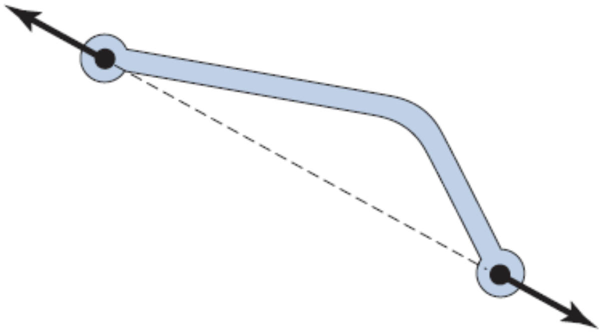


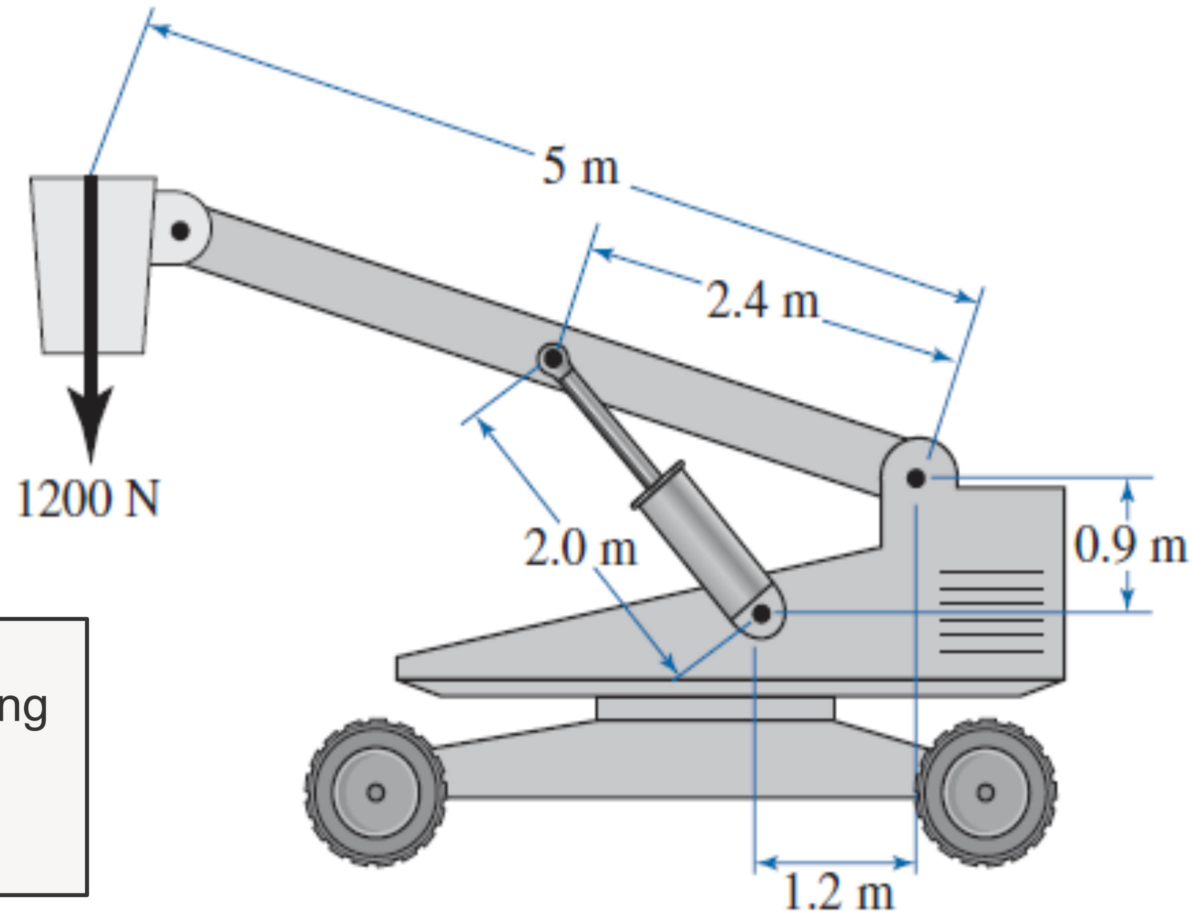
FIGURE 13.9 Two-force member.

In order for a two-force member to be in equilibrium the two forces must:

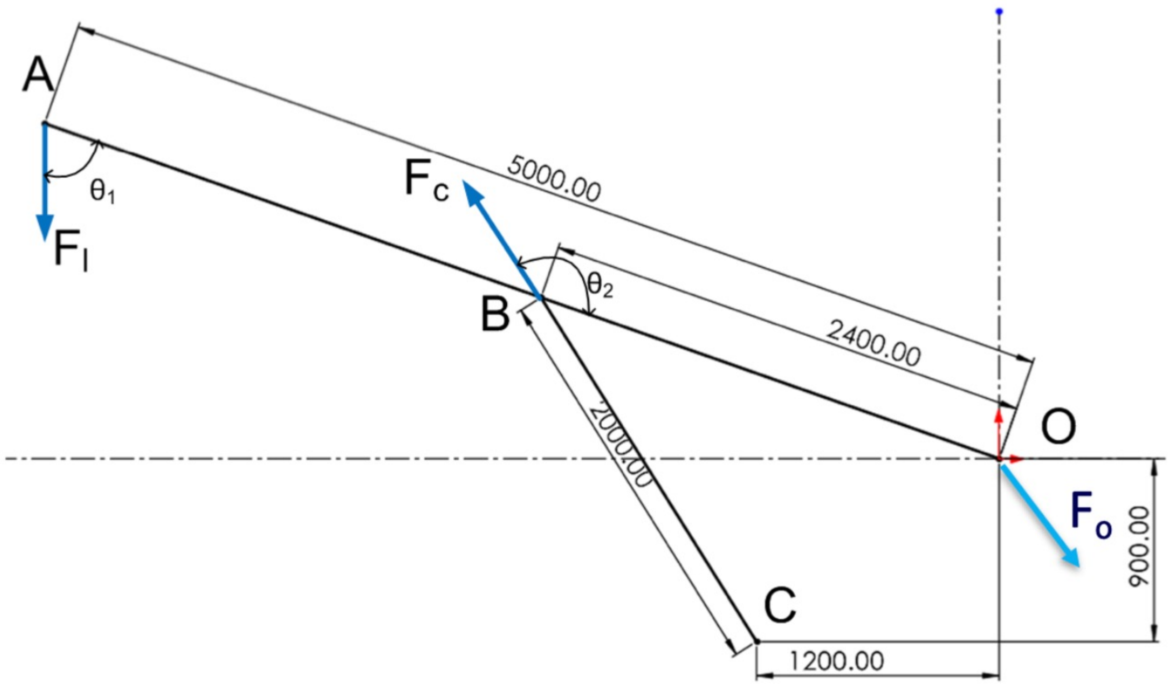
1. Have the same magnitude,
2. Act along the same line, and
3. Be opposite in sense.

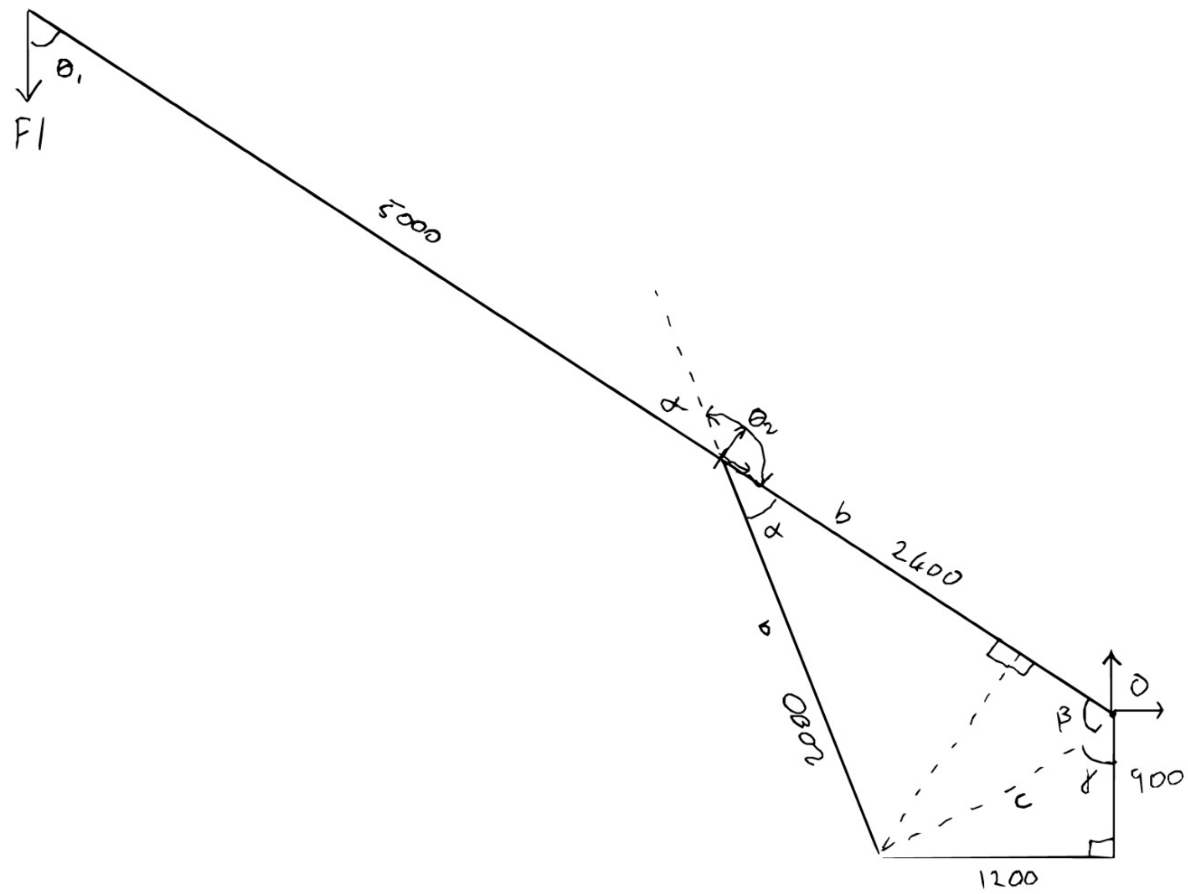
RMIT UNIVERSITY

2. Static Force Analysis



Consider the forces acting on the machine


A
B
C
O
F_c
F_l
F_o
θ_1
θ_2
5000.00
2400.00
2000.00
1200.00
900.00



θ1
F1
5000
α
θ2
α
b
2400
a
2000
β
O
γ
900
c
1200

3. Further details on Centre of Gravity

- The center of gravity, CG, of an object is the balance point of that object. Locating the center of gravity becomes important in force analysis because this is the location of the force of gravity, or weight.
- In dynamic force analysis, any inertia effects due to the acceleration of the part will also act at this point.

3. Further details on Centre of Gravity

For parts made of homogeneous material, the CG is the three-dimensional, geometric center of the object. For complex parts, a common method of determining the center of gravity is to divide the complex part into simple shapes, where the center of gravity of each is apparent.

RMIT UNIVERSITY

3. Further details on Centre of Gravity

- The composite centre of gravity can be determined from a weighted average of the coordinates of the individual CG. Because the acceleration due to gravity will be the same for the entire body, weight can be substituted for mass in equation:

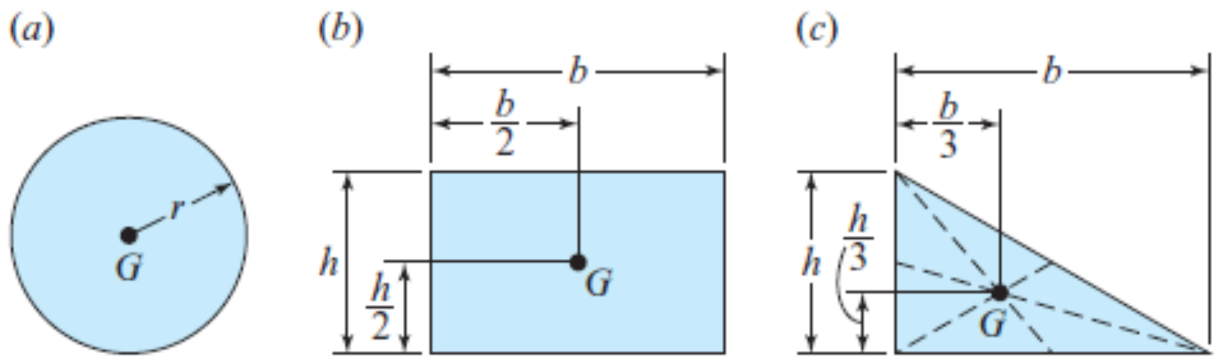
$$x_{cg\ \mathrm{total}} = \frac{m_1 x_{cg\ 1} + m_2 x_{cg\ 2} + m_3 x_{cg\ 3} + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

- Similar equations can be written for the y- and z-coordinates.

RMIT UNIVERSITY

3. Further details on Centre of Gravity

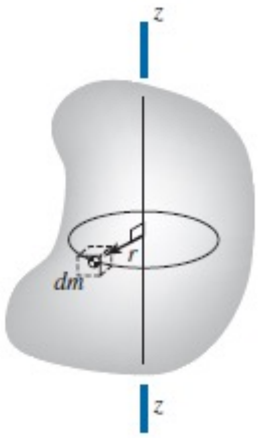
When mass is uniformly distributed along a line, over a plane, or volume, the center of mass can often be found by symmetry. Figure below illustrates the locations of the centers of mass for a circular solid, a rectangular solid, and a triangular solid. Each is assumed to have constant thickness and uniform density distribution.



3. Further information about Mass moment of Inertia

The **mass moment of inertia**, I, of a part is a measure of the resistance of that part to rotational acceleration. Mass moment of inertia, or simply moment of inertia, is dependent on the mass of the object along with the shape and size of that object. In addition, inertia is a property that is stated relative to a reference point (or axis when three dimensions are considered). This reference point is commonly the *centre of gravity of the part*.

The mass moment of inertia of this small element is determined by multiplying its mass, *dm*, by the square of the distance, *r*, to a reference axis, *z*. This distance is the perpendicular distance from the axis to the arbitrary element *dm*.



$$I_z = \int r^2 \, dm$$

1. Dynamic Fundamentals - Moments and Torques

The **mass moment of inertia** of the differential element is equal to the product of its mass and the square of its distance from the axis of interest.

$$dI_x = r_x^2 dm = (y^2 + z^2) dm$$
$$dI_y = r_y^2 dm = (x^2 + z^2) dm$$
$$dI_z = r_z^2 dm = (x^2 + y^2) dm$$

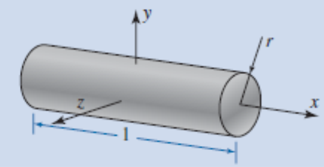
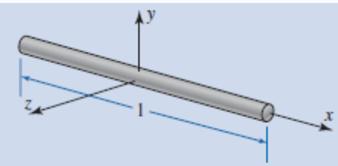
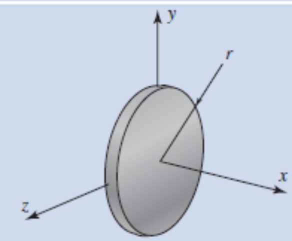
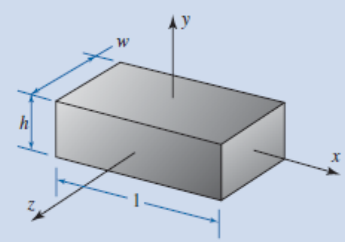
$$I_x = \int (y^2 + z^2) dm$$
$$I_y = \int (x^2 + z^2) dm$$
$$I_z = \int (x^2 + y^2) dm$$

$$\mathbf{T} = I\alpha$$

RMIT UNIVERSITY

3. Further information about Mass moment of Inertia

In practice, parts cannot always be simply approximated by the basic shapes shown in Table. However, for more complex parts, the determination of the moment of inertia can be done by dividing the complex parts into several basic shapes from the Table. The mass moment of inertia of each basic shape is computed relative to an axis through the center of the entire part. Finally, the total mass moment of inertia is determined by combining the values from the individual shapes.

Shape Name	Rendering	Mass Moment of Inertia
Cylinder		$I_x = \frac{1}{2}[mr^2]$ $I_y = \frac{1}{12}[m(3r^2 + l^2)]$ $I_z = \frac{1}{12}[m(3r^2 + l^2)]$
Slender rod		$I_x = 0$ $I_y = \frac{1}{12}[ml^2]$ $I_z = \frac{1}{12}[ml^2]$
Thin disk		$I_x = \frac{1}{2}[mr^2]$ $I_y = \frac{1}{4}[mr^2]$ $I_z = \frac{1}{4}[mr^2]$
Rectangular block		$I_x = \frac{1}{12}[m(w^2 + h^2)]$ $I_y = \frac{1}{12}[m(w^2 + l^2)]$ $I_z = \frac{1}{12}[m(h^2 + l^2)]$

RMIT UNIVERSITY

Next Class

- Dynamic Force Analysis
- Examples

Thank you for your attendance :D

RMIT
UNIVERSITY

Copyright Claim

RMIT UNIVERSITY

Reference

- *Design of Machinery by Robert L. Norton.*