MIET2510

Mechanical Design

Week 3 – Dynamic Fundamentals – Part 2

School of Science and Technology, RMIT Vietnam

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Outlines

1. Dynamic Force Analysis
2. Dynamic Force Analysis – Single Link in Pure Rotation
3. Dynamic Force Analysis – Three-bar linkage
4. Dynamic Force Analysis – Four-bar linkage

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1. Dynamic Force Analysis

- The following section deals with force analysis in mechanisms with significant accelerations.
- We will concentrate on solving for the forces and torques that result from, and are required to drive, our kinematic system in such a way as to provide the designed accelerations.

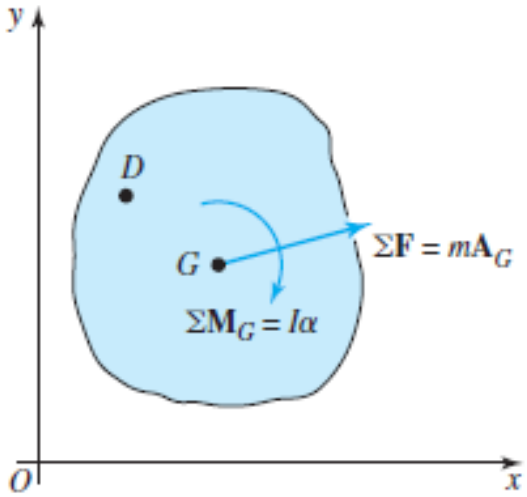
1. Dynamic Force Analysis

Apply Newton's second law,

- For linear motion: $\sum \mathbf{F} = m\mathbf{a}$

Ag acceleration of the link's centre of gravity.

- For angular motion: $\sum \mathbf{T} = I_G \alpha$



Suppose we wish to take moments about arbitrary point D, the sum of the moments about point D can be written as:

$$\sum \mathbf{M}_D = I_G \boldsymbol{\alpha} + \mathbf{R}_{GD} \times m\mathbf{A}_G.$$

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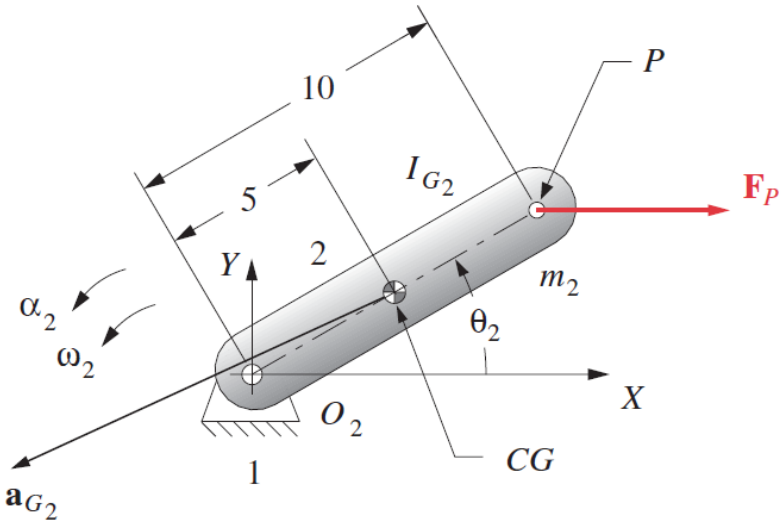
1. Dynamic Force Analysis – Solution Methods

- **Superposition:** The method of superposition attacks the problem by solving for parts of the solution and then adding (superposing) the partial results together to get the complete result.
- **Linear simultaneous equation solution:** All the relevant equations for the entire system are written as a set of linear simultaneous equations. These equations can then be solved simultaneously to obtain the results. This can be thought of as analogous to a “parallel processing” approach.

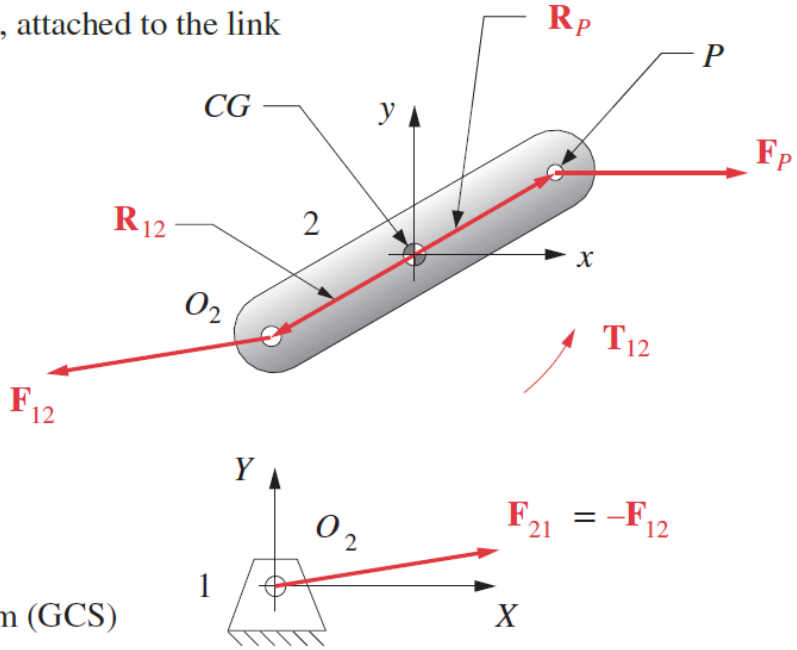
2. Dynamic $|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$ tion

Note: x, y is a local, nonrotating coordinate system (LNCS), attached to the link



Note: X,Y is the fixed, global coordinate system (GCS)

(*a*) Kinematic diagram



(*b*) Force (free-body) diagrams

Reference: Design of Machinery by Robert L. Norton.

$$\sum \mathbf{F} = \mathbf{F}_P + \mathbf{F}_{12} = m_2 \mathbf{a}_G$$

$$\sum \mathbf{T} = \mathbf{T}_{12} + (\mathbf{R}_{12} \times \mathbf{F}_{12}) + (\mathbf{R}_P \times \mathbf{F}_P) = I_G \alpha$$

$$F_{P_x} + F_{12_x} = m_2 a_{G_x}$$

$$F_{P_y} + F_{12_y} = m_2 a_{G_y}$$

$$T_{12} + \left(R_{12_x} F_{12_y} - R_{12_y} F_{12_x}\right) + \left(R_{P_x} F_{P_y} - R_{P_y} F_{P_x}\right) = I_G \alpha$$

2. Dynamic Force Analysis – Single Link in Pure Rotation

$$F_{P_x} + F_{12_x} = m_2 a_{G_x}$$

$$F_{P_y} + F_{12_y} = m_2 a_{G_y}$$

$$T_{12} + \left(R_{12_x} F_{12_y} - R_{12_y} F_{12_x}\right) + \left(R_{P_x} F_{P_y} - R_{P_y} F_{P_x}\right) = I_G \alpha$$

$$[\mathbf{A}] \times [\mathbf{B}] = [\mathbf{C}]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -R_{12_y} & R_{12_x} & 1 \end{bmatrix} \times \begin{bmatrix} F_{12_x} \\ F_{12_y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_x} - F_{P_x} \\ m_2 a_{G_y} - F_{P_y} \\ I_G \alpha - \left(R_{P_x} F_{P_y} - R_{P_y} F_{P_x}\right) \end{bmatrix}$$

Reference: Design of Machinery by Robert L. Norton.

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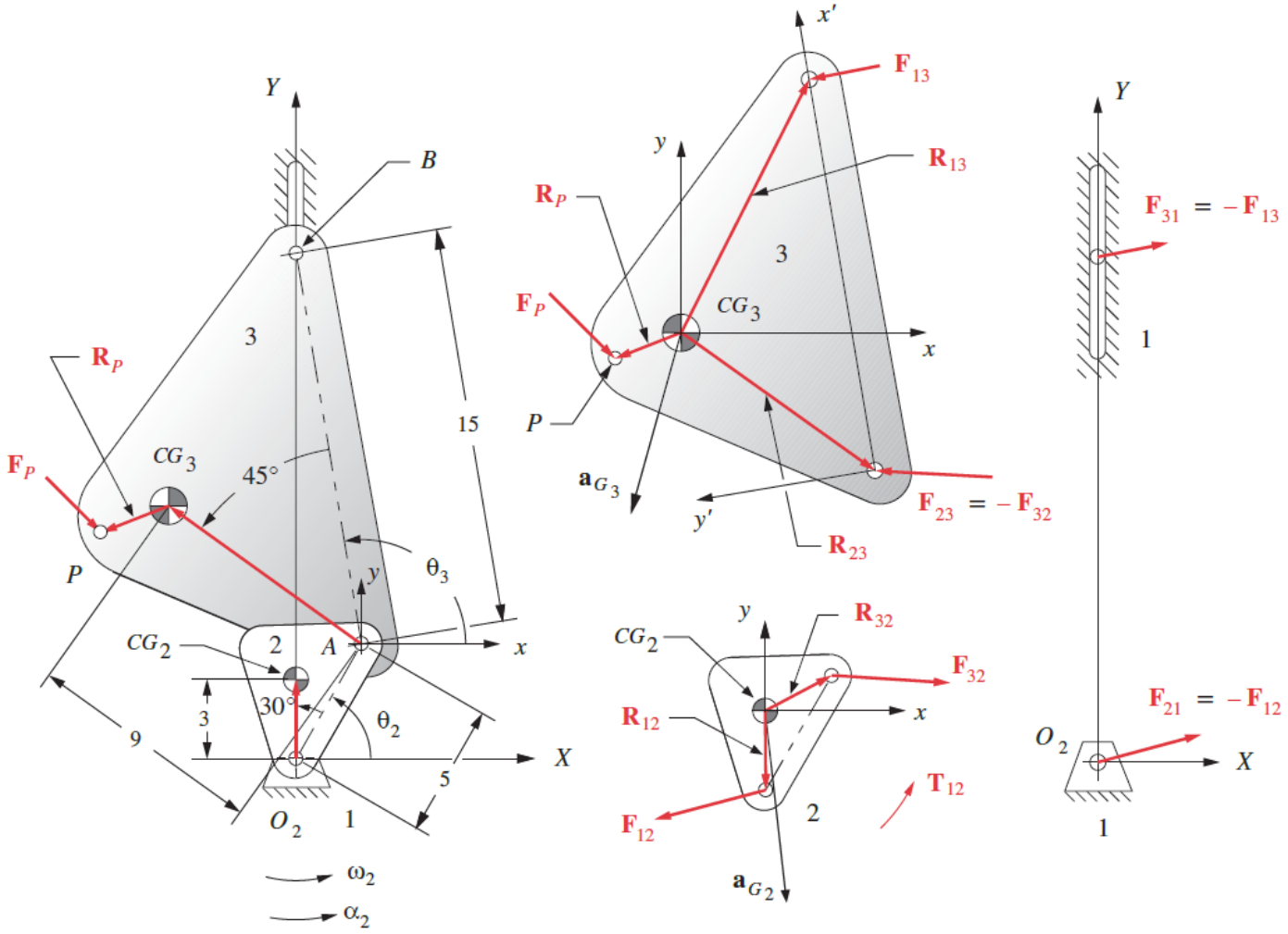
3. Dynamic Force Analysis – Three-bar Crank-slide Linkage



Reference: Design of Machinery by Robert L. Norton.

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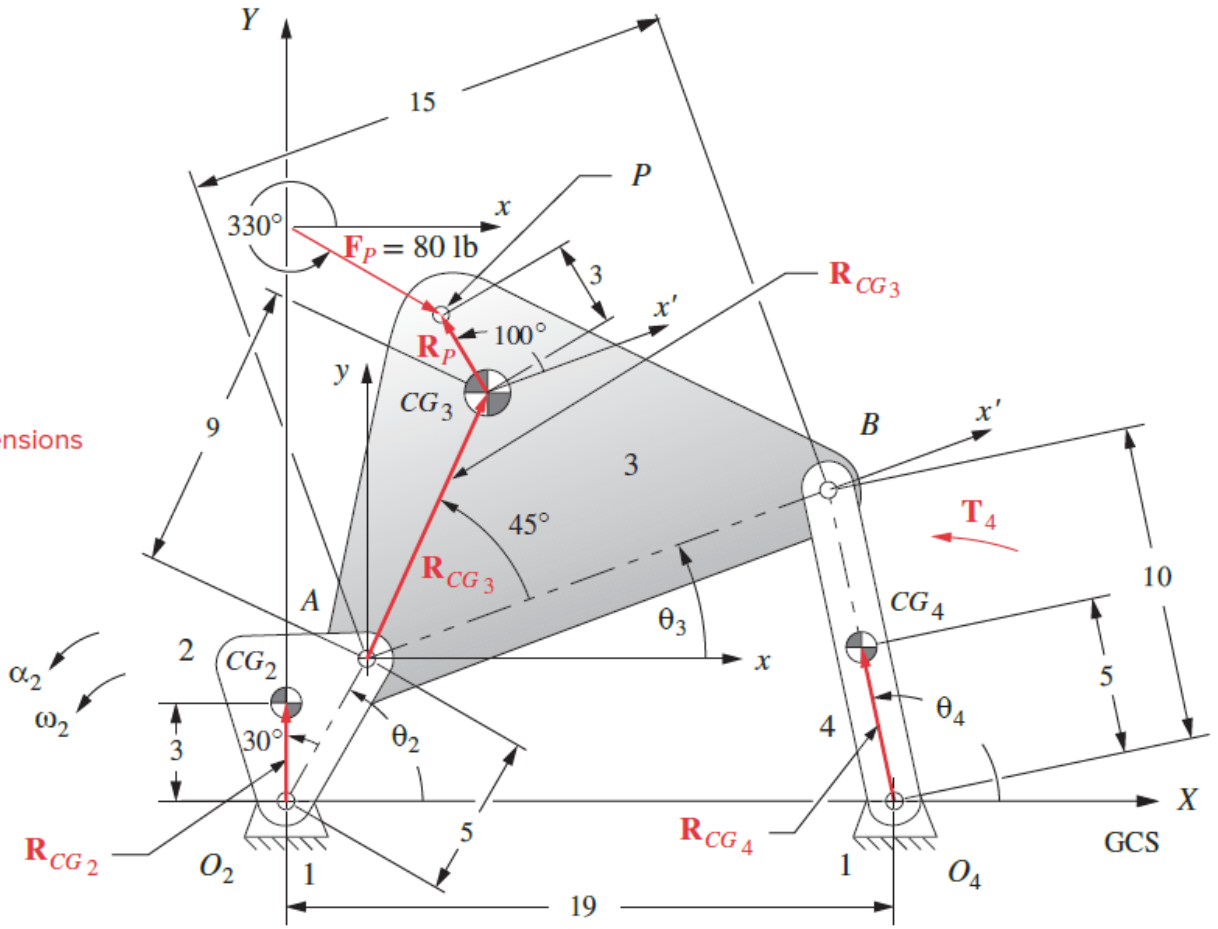
3. Dynamic Force Analysis – Four-bar Linkage



(a) The linkage and dimensions

Reference: Design of Machinery by Robert L. Norton.

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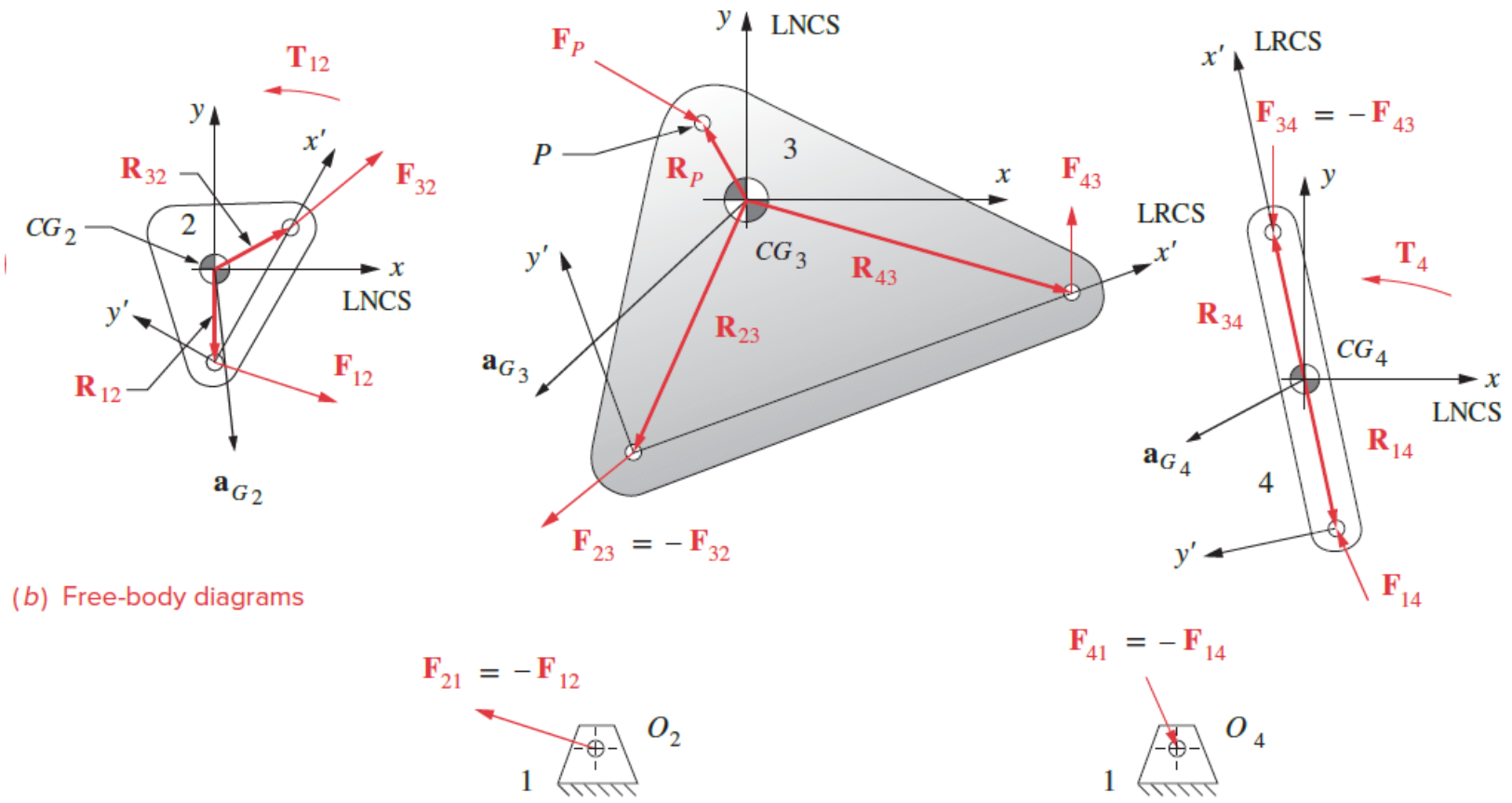
3. Dynamic Force Analysis – Four-bar Linkage



(b) Free-body diagrams

Reference: Design of Machinery by Robert L. Norton.

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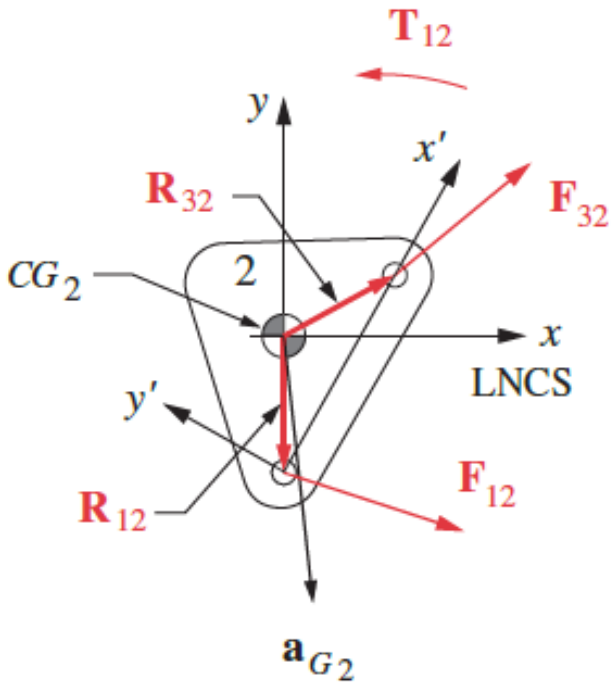

For link 2 the equations are,

$$F_{12x} + F_{32x} = m_2 a_{G_{2x}}$$
$$F_{12y} + F_{32y} = m_2 a_{G_{2y}}$$
$$T_{12} + (R_{12x}F_{12y} - R_{12y}F_{12x}) + (R_{32x}F_{32y} - R_{32y}F_{32x}) = I_{G2}\alpha_2$$

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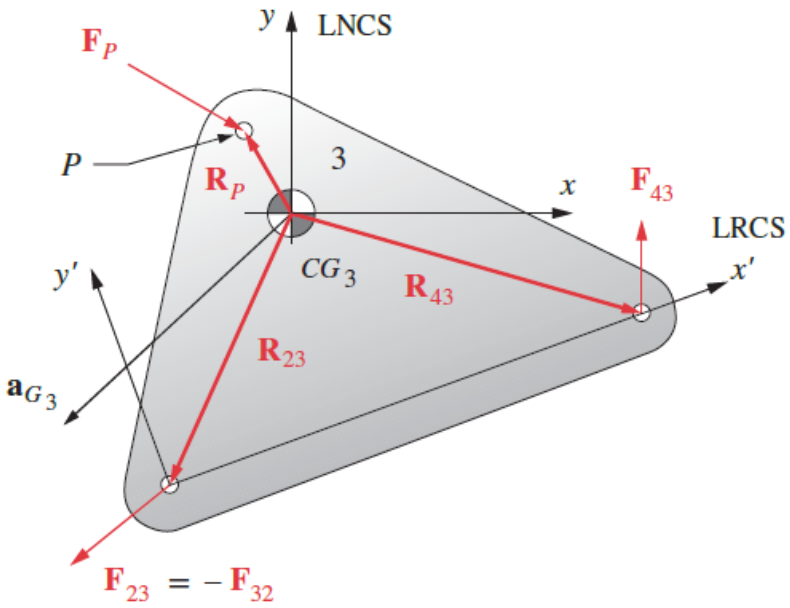

and for link 3,

$$F_{43x} - F_{32x} + F_{Px} = m_3 a_{G_{3x}}$$
$$F_{43y} - F_{32y} + F_{Py} = m_3 a_{G_{3y}}$$
$$(R_{43x}F_{43y} - R_{43y}F_{43x}) - (R_{23x}F_{32y} - R_{23y}F_{32x}) + (R_{Px}F_{Py} - R_{Py}F_{Px}) = I_{G3}\alpha_3$$

$\mathbf{R} \times \mathbf{F} = (R_x \cdot F_y) - (R_y \cdot F_x)$

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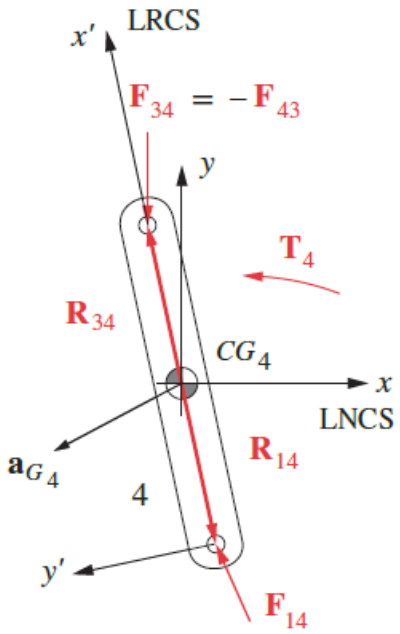

for link 4 we substitute the reaction force $-\mathbf{F_{43}}$ for $\mathbf{F_{34}}$,

$$F_{14x} - F_{43x} = m_4 a_{G_x}$$
$$F_{14y} - F_{43y} = m_4 a_{G_{4y}}$$
$$(R_{14x}F_{14y} - R_{14y}F_{14x}) - (R_{34x}F_{43y} - R_{34y}F_{43x}) + T_4 = I_{G_4}\alpha_4$$

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$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -R_{12y} & R_{12x} & -R_{32y} & R_{32x} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & R_{23y} & -R_{23x} & -R_{43y} & R_{43x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & R_{34y} & -R_{34x} & -R_{14y} & R_{14x} & 0 \end{bmatrix} \times \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{32x} \\ F_{32y} \\ F_{43x} \\ F_{43y} \\ F_{14x} \\ F_{14y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_{2x}} \\ m_2 a_{G_{2y}} \\ I_{G_2}\alpha_2 \\ m_3 a_{G_{3x}} - F_{Px} \\ m_3 a_{G_{3y}} - F_{Py} \\ I_{G_3}\alpha_3 - R_{Px}F_{Py} + R_{Py}F_{Px} \\ m_4 a_{G_{4x}} \\ m_4 a_{G_{4y}} \\ I_{G_4}\alpha_4 - T_4 \end{bmatrix}$$

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4. Further Topics to be investigated as Homework

- Force Analysis Of A Four-bar Crank-slider Linkage.
- Force Analysis Of The Inverted Crank-slider.
- Force Analysis - Linkages With More Than Four Bars.

Preparation for Class Test

1. Mobility analysis (week 1 tutorial)
2. Dynamic force analysis (week 3 tutorial)
    i. Free body diagram
    ii. Equilibrium equations
    iii. System equations
3. Velocity Analysis (week 2 tutorial)

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Thank you for your attendance :D

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Reference

- *Design of Machinery by Robert L. Norton.*

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