MIET2510

Mechanical Design

Week 5 – Shaft Design, Key, and Seal – Part 1

School of Science and Technology, RMIT Vietnam

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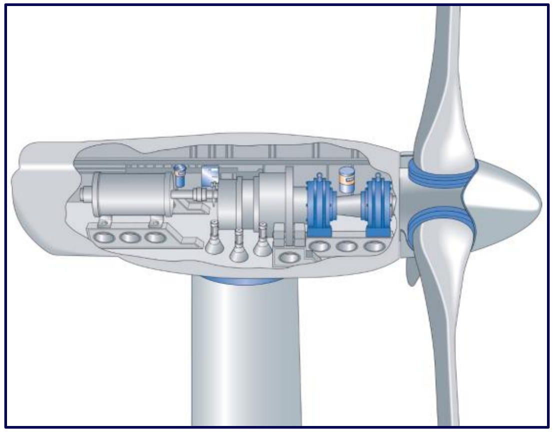
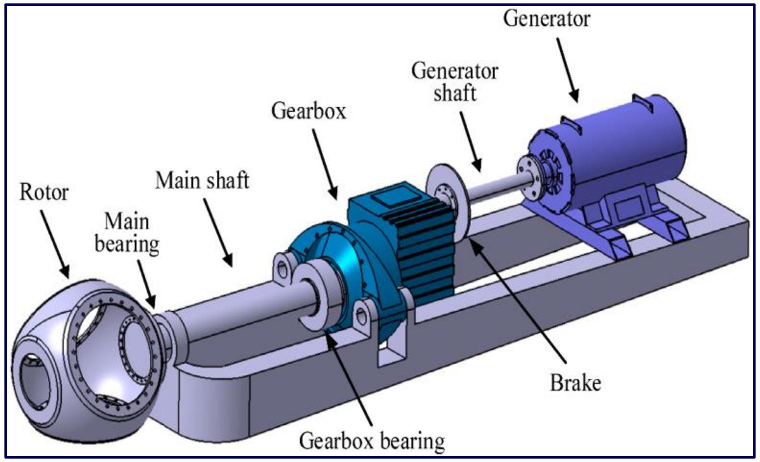
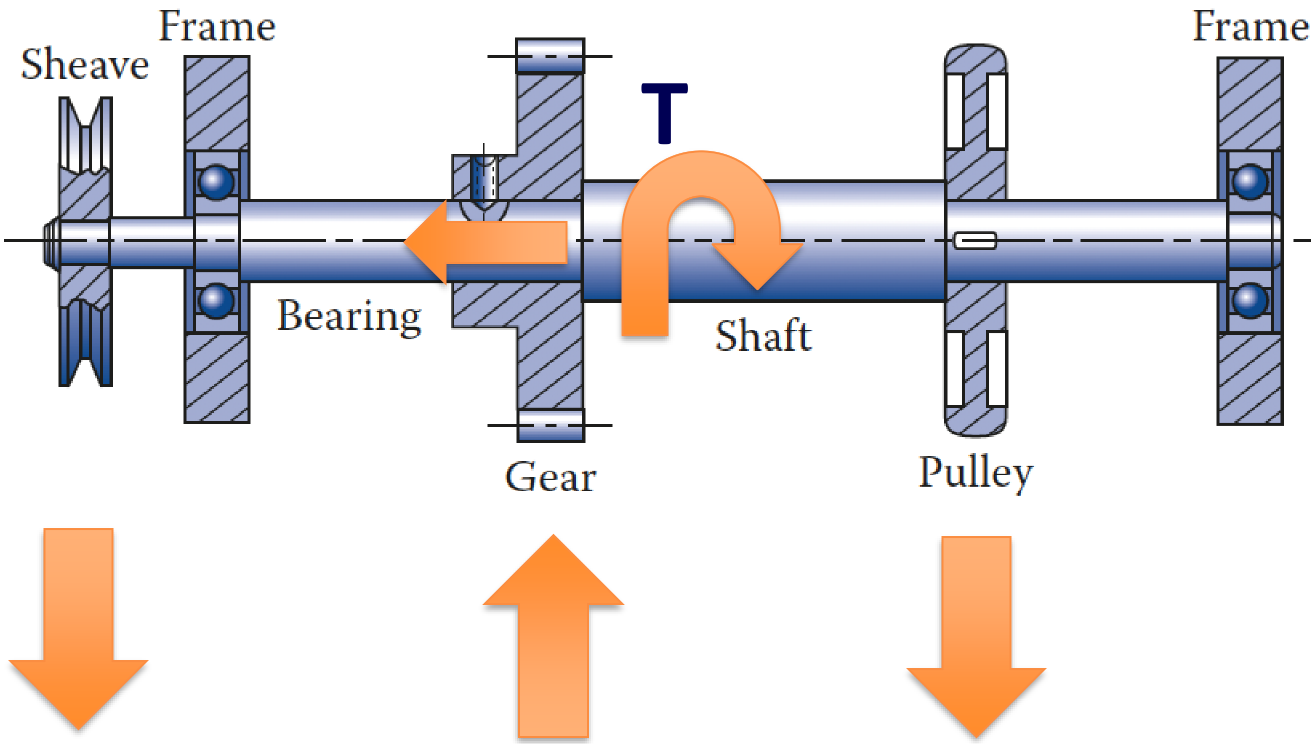
Shaft Introduction

- SHAFT is a relevant and important machine element.
- The term "shaft" applies to rotating machine members used for transmitting power or torque.
- Transmission shafts transmit torque from one location to another and have a circular cross-section much smaller in diameter than in length.



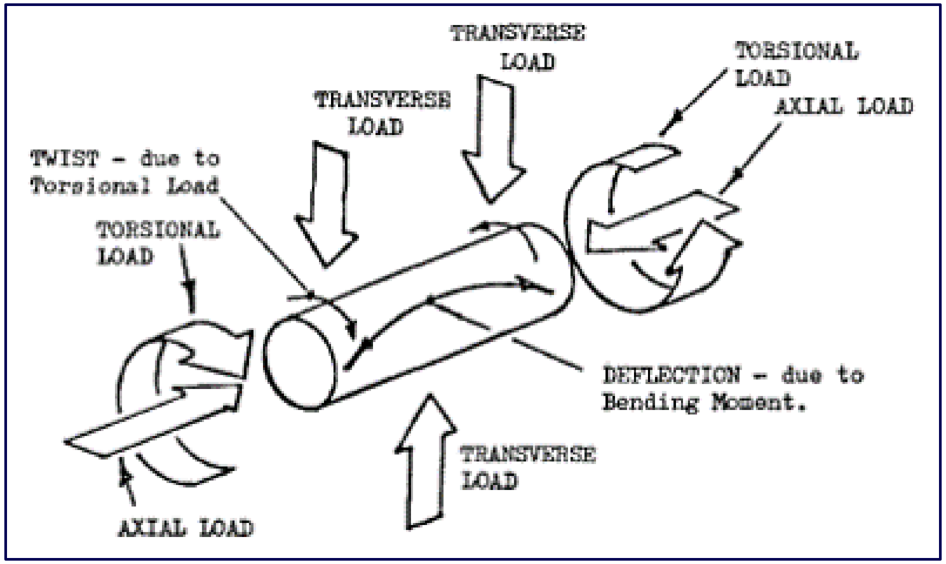


Shaft Introduction



Loads on the Shaft

- Axial loads result from transmission elements (e.g., gears).
- Torsional loads result from the torque being transmitted.
- Transverse loads result from elements like belts, gears.



Shaft Design

It is normal practice to buy items such as bearings, gears, pulleys and many other items as standard components, "off the shelf". In other words, a manufacturer makes large quantities of industry-standard items which are sold in many different locations for many different purposes.

Shafts generally do not fall into this category. A shaft is usually designed to perform a specific task in a specific machine.

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Shaft Design Problems

1. Given required power to be transmitted and shaft geometry

- Calculate torque, forces, and stresses.
- Select material.

2. Given required power to be transmitted and shaft material

- Calculate torque, and forces.
- Determine shaft diameter.

Shaft Design Problems

- Power – 100 kW
- Speed – 2000 rpm

$$P = T\omega$$

$$T = \frac{100\ kW}{2000\ rpm \cdot \frac{2\pi}{60}}$$

$$T = 477.46\ Nm$$

Shaft Design Problems (cont.)

Some of the main considerations in designing a shaft are **strength**, using **yield or fatigue** (or both) as a criterion; or the **dynamics** established by the **critical speeds**.

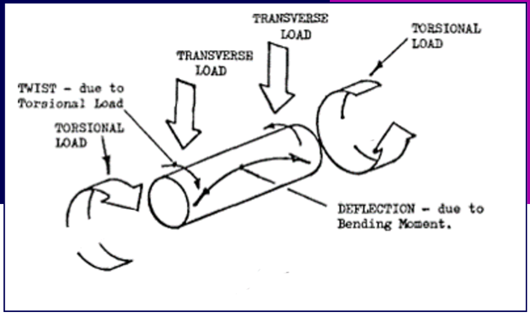
- **Yield -** the static loading (constant load) must be considered
- **Fatigue -** the cyclic loading (varying load) needs to be considered.
- **Critical speeds -** the speed at which the rotating shaft becomes dynamically unstable and large deflections due to vibration. This varies with given material and geometry.

Shaft Design Problems - Failure

- The **Distortion Energy Theory** predicts yielding in shafts when the distortion **energy** exceeds a critical value, using the equivalent von Mises stress.

- The **Maximum Shear Stress Theory** predicts yielding in shafts when the maximum **shear stress** reaches a critical value, considering the principal stress differences.

Design of Shafts for Static Loading



1. Shaft under Bending moment and Torsion

The smallest diameter where failure will occur is predicted as:

- Applying the Distortion – Energy Theory (DET):

$$d = \left(\frac{32n_s}{\pi S_y} \sqrt{M^2 + \frac{3}{4}T^2} \right)^{1/3}$$

- Applying the Maximum-Shear-Stress Theory (MSST):

$$d = \left(\frac{32n_s}{\pi S_y} \sqrt{M^2 + T^2} \right)^{1/3}$$

d = diameter
S_y= yield strength
M= bending moment
T= torque
n_s= safety factor

where d is shaft minimum diameter, M is the moment (Nm), T is torque (Nm), n_S is the safety factor, S_Y is yield strength (Pa).

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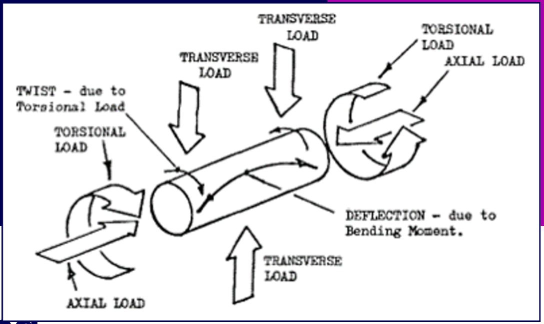
Design of Shafts for Static Loading



2. Shaft under Bending moment, Torsion, and Axial Loading

The smallest diameter where failure will occur is predicted as:

- Applying the Distortion – Energy Theory (DET):

$$\frac{4}{\pi d^3}\sqrt{(8M + Pd)^2 + 48T^2} = \frac{S_y}{n_s}.$$

- Applying the Maximum-Shear-Stress Theory (MSST):

$$\frac{4}{\pi d^3}\sqrt{(8M + Pd)^2 + 64T^2} = \frac{S_y}{n_s}.$$

where P is the normal force (N).

d = diameter
S_y= yield strength
M= bending moment
T= torque
n_s= safety factor
P= axial force

Fatigue Design of Shafts - Ductile Materials

The smallest diameter where failure will occur is predicted as:

- Applying the Distortion – Energy Theory (DET):

$$d^3 = \frac{32 n_s}{\pi S_y} \sqrt{\left(M_m + \frac{S_y}{S_e} K_f M_a\right)^2 + \frac{3}{4}\left(T_m + \frac{S_y}{S_e} K_{fs} T_a\right)^2}$$

- Applying the Maximum-Shear-Stress Theory (MSST):

$$d = \left[\frac{32 n_s}{\pi S_y} \sqrt{\left(M_m + \frac{S_y}{S_e} K_f M_a\right)^2 + \left(T_m + \frac{S_y}{S_e} K_{fs} T_a\right)^2}\right]^{1/3}$$

d = diameter
S_y = yield strength
S_e = endurance limit
K_f = concentration factor
M_m = mean moment
M_a = alternating moment
T_m = mean torque
T_a = alternating torque
n_s = safety factor

where d is shaft minimum diameter, S_e is modified endurance limit (Pa), K_f is fatigue stress concentration factor, Subscript “a” designates alternating, “m” designates mean or steady stress, “s” designates shear loading.

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Fatigue Design of Shafts - Brittle Materials

The smallest diameter where failure will occur is predicted as:

$$d = \left\{ \frac{16n_s}{\pi S_u} \left[K_c \Psi + \sqrt{K_c^2 \Psi^2 + K_{cs}^2 \left(T_m + \frac{S_u}{S_e} T_a \right)^2} \right] \right\}^{1/3}$$

$$\Psi = M_m + \frac{S_u}{S_e} M_a.$$

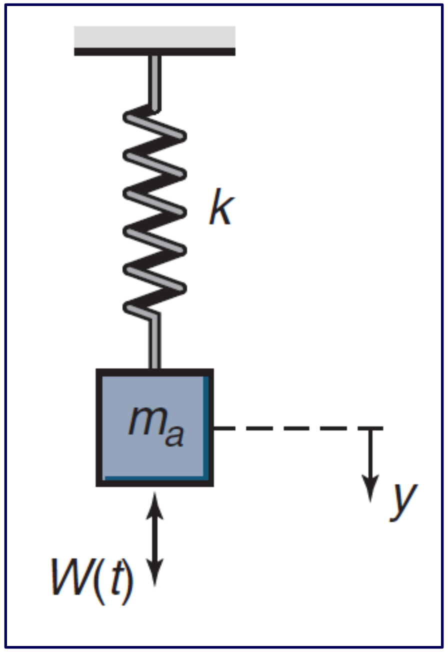
d = diameter
S_u= ultimate strength
S_e= endurance limit
K_c= fracture toughness
M_m= mean moment
M_a= alternating moment
T_m= mean torque
T_a= alternating torque
n_s= safety factor

Critical Speed of Rotating Shaft

- For any shaft there are an infinite number of critical speeds, but only the lowest (first) and occasionally the second are generally of interest to designers. The others are usually so high as to be well out of the operating range of shaft speed.
- Two approximate methods of finding the first critical speed (or lowest natural frequency) of a system are given, one attributed to **Rayleigh** and the other to **Dunkerley.**

Critical Speed of Rotating Shaft

- **Single-Mass System**



$$\omega = \sqrt{\frac{k}{m_a}} = \sqrt{\frac{W/y}{W/g}} = \sqrt{\frac{g}{\delta}}.$$

Critical Speed of Rotating Shaft

- **Multiple-Mass System**

Applying Rayleigh equation:

$$\omega_{\mathrm{cr}} = \sqrt{\frac{g \sum_{i=1,\ldots,n} W_i \delta_{i,\max}}{\sum_{i=1,\ldots,n} W_i \delta_{i,\max}^2}}.$$

δ = deflection, m

where W_i is the ith weight placed on the shaft and g is gravitation acceleration, 9.807 $\mathrm{m/s^2}$.

Applying Dunkerley equation:

$$\frac{1}{\omega_{\mathrm{cr}}^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \cdots + \frac{1}{\omega_n^2},$$

where

ω_1 = critical speed if only mass 1 exists
ω_2 = critical speed if only mass 2 exists
ω_n = critical speed if only the nth mass exists

$$\omega_i = \sqrt{g/\delta_i}.$$

Thank you for your attendance :D

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Reference

- *Mechanical Design of Machine Components (2nd) by Ansel C.Ugural.*
- *Mechanical Engineering Design (10th) by Richard G.Budynas and J. Keith Nisbett.*
- *Theory of Machines and Mechanisms (5th) by John J.Uicker, Gordon R.Pennock, Joseph E. Singley.*

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