Chapter 19

Flexible Machine Elements

A rolling chain on a sprocket. *Source:* Shutterstock.

Scientists study the world as it is, engineers create the world that never has been.

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Rubber belts, roller chains, and wire ropes are three examples of power transmission devices that are widely used in many industries. These machine elements have the common characteristic of flexibility. Rubber belts are used around pulleys, and transmit a torque based on friction associated with the belt-pullet interface. Flat belts can be constructed from soft or hard rubber, depending on the power to be transmitted, but synchronous and V-belts are advanced composite materials. Reinforced rubber belts are economically attractive and reliable machine elements that have further advantages of reducing impact stresses, but they will slip if torque becomes too large. Synchronous (timing) belts have teeth that prevent slip, and have the further advantages of low preload and associated reduced bearing loads. Roller chains are longer-lived and more robust devices than belts, but are more expensive. Roller chains and their sprockets require effective lubrication to achieve long lives. There are a number of design variables that can be modified with roller chains, and conventional and silent chains are presented in this chapter. Wire rope consists of many strands of wire carefully wound around a core; such rope is widely used to hoist loads or provide a force through a winch or equivalent device. Smaller diameter ropes are often used as cables to transmit a force in order to open a clutch or engage a brake, for example. Wire ropes are especially useful when power needs to be transferred efficiently over long distances.

Machine elements in this chapter: Flat belts, V-belts, synchronous belts, roller and silent chains, and wire ropes.
Typical applications: Power transmission from electric motors or internal combustion engines to power other devices; bicycle power transmission; cranes, hoists, and derricks; elevators.
Competing machine elements: Shaft couplings (Ch. 11), gears (Ch. 14 and 15).

Symbols

A_m	cross-sectional area of metal strand in rope, m^2
a	link plate thickness, m
a_1	service factor
a_2	multiple-strand factor in rolling chain
c	distance from neutral axis to outer fiber, m
c_d	center distance, m
D	sheave or pulley diameter, m
d	diameter, m
d_w	wire diameter, m
E	modulus of elasticity, Pa
F	force, N
F_a	force due to acceleration, N
F_f	fatigue allowable force, N
F_h	static force, N
F_i	initial tensile force, N
F_r	rope weight, N
F_t	total friction force, N
F_w	deadweight, N
f_1	overload service factor
f_2	power correction factor
g	gravitational acceleration, 9.807 m/s^2
g_r	velocity ratio
HB	Brinell hardness, kg/mm^2
h_p	power transmitted, W
h_{pb}	rated power per belt, W
h_{pr}	rated power, W
h_t	V-belt height, m
I	area moment of inertia, m^4
L	belt length, m
M	bending moment, N-m
m	mass, kg
m'	mass per unit length, kg/m
N	number of teeth in sprocket or number of belts
N_a	angular speed, rpm
n_s	safety factor
p	bearing pressure, Pa
p_{all}	allowable bearing pressure, Pa
p_t	pitch or datum, m
r	sheave or pulley radius, m
Δr	chordal rise, m
r_c	chordal radius, m
S_u	ultimate strength, Pa
t	thickness, m
u	belt velocity, m/s
w_t	belt width, m
w_z	weight, N
w_z'	weight per unit length, N/m
α	angle used to describe loss in arc of contact, deg
θ_r	angle of rotation to give chordal rise, deg
μ	coefficient of friction
σ	normal stress, Pa
σ_b	bending stress, Pa
σ_t	tensile normal stress, Pa
ϕ	wrap angle; gantry line angle, deg
ω	angular velocity, rad/s

Subscripts

1	driver pulley or sheave
2	driven pulley or sheave

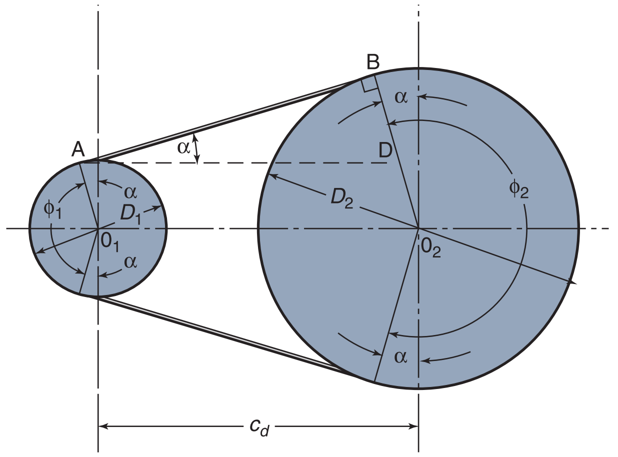

Figure 19.1: Dimensions, angles of contact, and center distance of open flat belt.

19.1 Introduction

The machine elements considered in this chapter, like the machine elements considered in Chapter 18, use friction as a useful agent to produce a high and uniform force and to transmit power. A belt, rope, or chain provides a convenient means for transferring power from one shaft to another, and are especially useful for transfer of power over long distances. Table 19.1 compares belt, chain, and gear power transmission approaches.

19.2 Flat Belts

Flat belts find considerable use in applications requiring small pulley diameters, high belt surface speeds, low noise levels, low weight, and/or low inertia. They cannot be used where absolute synchronization between pulleys must be maintained because they rely on friction for their proper functioning, so some slip is inevitable. Flat belts must be kept under tension to function, and therefore they require tensioning devices.

There are many applications where belts are used to transfer very low power levels. This book emphasizes machinery applications, where the power that is transferred is generally larger than 100 Nm/s (75 ft-lb/s). Smaller belts are readily available, and usually can be selected based solely on kinematic constraints.

19.2.1 Belt Length

Figure 19.1 shows the dimensions, angles of contact, and center distance of an open flat belt. The term "open" is used to distinguish it from the geometry of a crossed belt, where the belt forms a figure eight when viewed on end. Note that distance 0_2D is equal to $r_1 = D_1/2$, distance BD is equal to $r_2 - r_1 = (D_2 - D_1)/2$, and distance AD is the center distance c_d. Also, triangle ABD is a right triangle so that

$$\text{AB}^2 + \text{BD}^2 = \text{AD}^2,$$

so that

$$\text{AB}^2 + (r_2 - r_1)^2 = c_d^2;$$

$$\text{AB} = \sqrt{c_d^2 - (r_2 - r_1)^2}\,. \tag{19.1}$$

Table 19.1: Comparison of selected power transmission devices.

Constraint	Power transmission device: Flat belt	V-belt	Synchronous belt	Roller chain	Silent chain	Spur gear
Synchronization	1	1	1	4	4	4
Efficiency	1	1	2	4	4	4
Anti-shock	4	4	3	2	3	1
Noise/vibration	4	4	3	2	3	1
Compactness						
High speed/low load	2	3	3	1	4	3
Low speed/high load	1	1	2	4	3	2
Lubrication	None	None	None	Required	Required	Required
Bearing loads	2	1	2	4	3	4
Longevity	1	2	2	3	3	4

1, Poor; 2, Fair; 3, Good; 4, Excellent.

The length of the open flat belt can be expressed as

$$L = 2\text{AB} + r_1\phi_1\frac{\pi}{180} + r_2\phi_2\frac{\pi}{180}, \qquad (19.2)$$

where ϕ is the wrap angle. The wrap angles can be expressed as

$$\phi_1 = 180^\circ - 2\alpha \qquad \text{and} \qquad \phi_2 = 180^\circ + 2\alpha. \qquad (19.3)$$

Also, from right triangle ABD in Fig. 19.1, the angle used to describe the loss in arc of contact is

$$\sin\alpha = \frac{D_2 - D_1}{2c_d} \qquad \text{or} \qquad \alpha = \sin^{-1}\left(\frac{D_2 - D_1}{2c_d}\right). \qquad (19.4)$$

The angle α is in degrees in order to be consistent with Eq. (19.3), and is equal to zero only if the pulleys have a 1-to-1 ratio. By substituting Eqs. (19.3) and (19.4) into Eq. (19.2), the belt length can be expressed as

$$\begin{aligned} L &= \sqrt{(2c_d)^2 - (D_2 - D_1)^2} \\ &\quad + \frac{D_1\pi}{360}\left[180^\circ - 2\sin^{-1}\left(\frac{D_2 - D_1}{2c_d}\right)\right] \\ &\quad + \frac{D_2\pi}{360}\left[180^\circ + 2\sin^{-1}\left(\frac{D_2 - D_1}{2c_d}\right)\right], \end{aligned}$$

or

$$\begin{aligned} L &= \sqrt{(2c_d)^2 - (D_2 - D_1)^2} + \frac{\pi}{2}(D_1 + D_2) \\ &\quad + \frac{\pi(D_2 - D_1)}{180}\sin^{-1}\left(\frac{D_2 - D_1}{2c_d}\right). \end{aligned} \qquad (19.5)$$

19.2.2 Belt Forces

The basic equations developed for band brakes are also applicable here. From Section 18.9, the following torque and friction equations can be written:

$$\frac{F_1}{F_2} = e^{\mu\phi\pi/180^\circ}; \qquad (18.64)$$

$$T = \frac{(F_1 - F_2)D_1}{2}, \qquad (18.65)$$

where
- ϕ = wrap angle, deg
- μ = coefficient of friction
- F_1 = tight-side or driver force, N
- F_2 = slack-side or driven force, N

In obtaining the preceding equations, it was assumed that the coefficient of friction on the belt is uniform over the entire angle of wrap and that centrifugal forces on the belt can be neglected.

In transmitting power from one shaft to another by means of a flat belt and pulleys, the belt must have an initial tensile force, F_i. The required initial belt tension (or tensile force) depends on the elastic characteristics of the belt and friction, but can be approximated by

$$F_i = \frac{F_1 + F_2}{2}. \qquad (19.6)$$

Also, from Eq. (18.65), note that when power is being transmitted, the tensile force, F_1, in the tight side exceeds the tension in the slack side, F_2.

The power transmitted is

$$h_p = (F_1 - F_2)u, \qquad (19.7)$$

where u is the belt velocity. The centrifugal force acting on the belt can be expressed as

$$F_c = m'u^2 = \frac{w_z'}{g}u^2, \qquad (19.8)$$

where
- m' = mass per unit length, kg/m
- u = belt velocity, m/s
- w_z' = weight per unit length, N/m

When the centrifugal force is considered, Eq. (18.64) becomes

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\phi\pi/180^\circ} \qquad (19.9)$$

Of course, Eq. (18.65) is unchanged when centrifugal forces are considered.

Example 19.1: Forces on a Flat Belt

Given: A flat belt is 150 mm wide and 8 mm thick and transmits 12 kW. The center distance is 2.5 m. The driving pulley is 150 mm in diameter and rotates at 2000 rpm such that the loose side of the belt is on top. The driven pulley has a diameter of 450 mm. The belt material weighs 970 kg/m^3.

Find: Determine the following:

(*a*) If μ = 0.30, determine F_1 and F_2.

(*b*) If μ is reduced to 0.20 because of oil getting on part of the pulley, what are F_1 and F_2? Would the belt slip?

(*c*) What is the belt length?

Solution:

(*a*) The belt angular velocity is $\omega = 2000$ rpm $= 209.4$ rad/s. Therefore,

$$u = r\omega = \left(\frac{0.150}{2}\right)(209.4) = 15.7 \text{ m/s}.$$

From Eq. (19.7),

$$F_1 - F_2 = \frac{h_p}{u} = \frac{12,000}{15.7} = 764 \text{ N}. \qquad (a)$$

The mass per volume was given, so that the weight per unit volume is $(970)(9.81) = 9515 \text{ N/m}^3$, therefore,

$$\begin{aligned}\frac{w_z}{L} &= 9515 w_t t = (9515)(0.150)(0.008)\\ &= 11.4 \text{ N/m}.\end{aligned}$$

The centrifugal force acting on the belt is, from Eq. (19.8),

$$F_c = \frac{w_z}{L}\frac{u^2}{g} = \frac{(11.4)(15.7)^2}{(9.81)} = 286.4 \text{ N}.$$

From Eq. (19.4),

$$\begin{aligned}\alpha &= \sin^{-1}\left(\frac{D_2 - D_1}{2c_d}\right)\\ &= \sin^{-1}\left(\frac{450-150}{(2)(2500)}\right)\\ &= 3.440^\circ.\end{aligned}$$

The wrap angle is given by Eq. (19.3) as

$$\phi = 180^\circ - 2(\alpha) = 180^\circ - 2(3.440^\circ) = 173.1^\circ. \qquad (b)$$

Making use of Eq. (19.9) gives

$$\frac{F_1 - 286.4}{F_2 - 286.4} = e^{(0.3)(173.1)\pi/180} = 2.475.$$

Therefore,

$$F_1 - 286.4 = 2.475 F_2 - 708.9,$$

so that

$$F_1 = 2.475 F_2 - 422.5. \qquad (c)$$

Substituting Eq. (*c*) into Eq. (*a*) gives

$$2.475 F_2 - F_2 = 422.5 + 764;$$

$$F_2 = \frac{1186}{1.475} = 804 \text{ N}.$$

Therefore, from Eq. (*a*),

$$F_1 = 764 + 804 = 1568 \text{ N}.$$

From Eq. (19.6), the initial belt tension is

$$F_i = \frac{F_1 + F_2}{2} = \frac{1568 + 804}{2} = 1186 \text{ N}.$$

(*b*) If $\mu = 0.20$ instead of 0.30,

$$\frac{F_1 - 286.4}{F_2 - 286.4} = e^{(0.2)(173.1)\pi/180} = 1.830. \qquad (d)$$

so that

$$F_1 = 1.830 F_2 - (1.83)(286.4) + 286.4 = 1.830 F_2 - 237.7, \qquad (e)$$

Since the initial belt tension is still $F_i = 1186$ N, substituting Eq. (*e*) into Eq. (19.6) gives

$$\frac{1.830 F_2 - 237.7 + F_2}{2} = 1186,$$

or $F_2 = 922$ N. Substituting this result into Eq. (*e*) produces

$$F_1 = 1.830(922) - 237.7 = 1450 \text{ N}.$$

From Eq. (19.7) the power transmitted is

$$\begin{aligned}h_p &= (F_1 - F_2)\,u\\ &= (1450 - 922)(15.7)\\ &= 8.29 \text{ kW}.\end{aligned}$$

Since this is less than the required power, the belt will slip.

(*c*) From Eq. (19.5) while making use of Eq. (*c*),

$$\begin{aligned}L &= 2\sqrt{(2)(2500)^2 - (450-150)^2} + \frac{\pi}{2}(450+150)\\ &\quad + \frac{\pi(450-150)}{180}\sin^{-1}\left(\frac{450-150}{2(2500)}\right)\\ &= 4483 \text{ mm} = 4.483 \text{ m}.\end{aligned}$$

19.2.3 Slip

Slip is detrimental for a number of reasons. It reduces belt efficiency and can cause thermal damage to the belt, leading to elongation (and resulting in lower belt tension and more slip) or chemical degradation. To eliminate slip, the initial belt tension needs to be retained. But as the belt stretches over time, some of the initial tension will be lost. One solution might be to have a very high initial tension, but this would put large loads on the pulley, shaft, and bearings, and also shorten the belt life. Some better approaches are the following:

1. Develop means of adjusting tension during operation.
2. Increase the wrap angle.
3. Change the belt material to increase the coefficient of friction.
4. Use a larger belt section.
5. Use an alternative design, such as a synchronous belt (see Section 19.3) or a chain as discussed in Section 19.6.

There are many tensioning devices involving spring-loaded pulleys or weights to apply loads to belts. Figure 19.2 illustrates one way of maintaining belt tension. The slack side of the belt is on the top, so that the sag of the belt acts to increase its wrap angle.

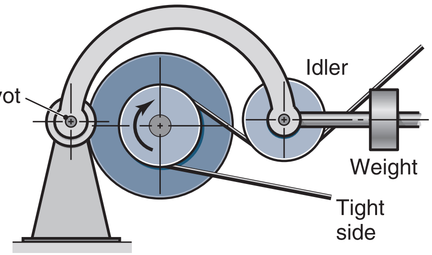

Figure 19.2: Weighted idler used to maintain desired belt tension.

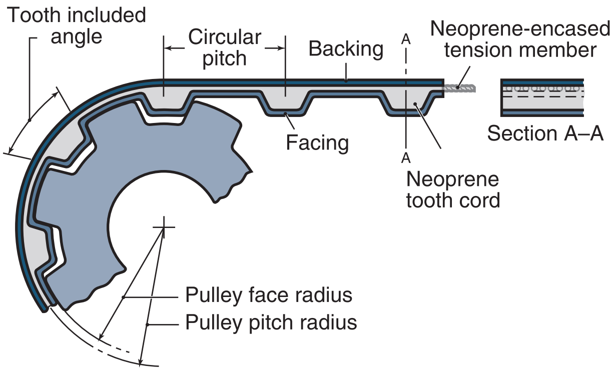

Figure 19.3: Synchronous, or timing, belt.

19.3 Synchronous Belts

Synchronous belts, or **timing belts**, are basically flat belts with a series of evenly spaced teeth on the inside circumference, thereby combining the advantages of flat belts with the excellent traction of gears and chains (see Section 19.6). A synchronous belt is shown in Fig. 19.3.

Unlike flat belts, synchronous belts do not slip or creep, and the required belt tension is low, resulting in very small bearing loads. Synchronous belts will not stretch and require no lubrication. Speed is transmitted uniformly because there is no chordal rise and fall of the pitch line as in rolling chains (Section 19.6). The equations developed for flat-belt length and torque in Section 19.2 are equally valid for synchronous belts.

Example 19.2: Optimum Pitch Diameter for a Timing Belt

Given: A timing belt is used to transfer power from a high-speed motor to a grinding wheel. The timing belt is 750 mm long and weighs 180 g. The maximum allowable force in the belt is 2000 N, and the speed of the turbine is the same as the speed of the grinding wheel, 5000 rpm.

Find: Calculate the optimum pulley pitch diameter for maximum power transfer.

Solution: If the total maximum force in the belt is F_1 and the centrifugal force is F_c, the maximum power transmitted is

$$h_p = u(F_1 - F_c).$$

Letting F_1 be equal to the maximum allowable force and making use of Eq. (19.8) for the centrifugal force gives

$$h_p = u\left(2000 - \frac{0.180}{0.750}u^2\right) = u\left(2000 - 0.24u^2\right).$$

The optimum power transmitted occurs when

$$\frac{\partial h_p}{\partial u} = 2000 - (0.24)(3)u^2 = 0.$$

Solving for the velocity yields $u = 52.70$ m/s. The pulley diameter that produces the maximum power transfer is

$$\frac{\omega D}{2} = u$$

$$D = \frac{2u}{\omega} = \frac{2(52.70)}{(5000)\left(\frac{2\pi}{60}\right)} = 0.2013 \text{ m} = 201.3 \text{ mm}.$$

19.4 V-Belts

V-belts are an extremely common power transmission device, used on applications as diverse as blowers, compressors, mixers, machine tools, etc. One or more V-belts are used to drive accessories on an automobile and transfer power from the internal-combustion engine. V-belts are made to standard lengths and with standard cross-sectional sizes, as shown in Fig. 19.4a. V-belts A through E are standard shapes that were standardized as early as the 1940s, but the more modern 3V, 5V, and 8V belts shown have higher power ratings for the same cross-sectional area. V-belts have a fiber-reinforced construction, as shown in Fig. 19.4b and c, so they should be recognized as advanced composite materials. The flexible reinforcement is often a cord, not an individual fiber, and can be made from nylon, kevlar (aramid), polyester, etc. The impregnated woven jacket shown protects the interior and provides a wear- and oil-resistant surface for the belt. The compressive side of the belt is produced from a high-strength, fatigue-resistant rubber. Because their interior tension cords are stretch and creep resistant, V-belts (unlike flat belts) do not require frequent adjustment of initial tension. The grooved pulleys that V-belts run in are called **sheaves**. They are usually made of cast iron, pressed steel, or die-cast metal. Figure 19.5 shows a V-belt in a sheave groove.

V-belts can also be used in multiples as shown in Fig. 19.4c. The obvious advantage of such a belt is that it can transmit higher power; a two-belt design can transmit twice the power of a single belt, etc. The tie band construction shown ensures that the belt maintains alignment when it is outside of the sheaves, which helps the belt run over the sheave properly.

V-belts find frequent application where synchronization between shafts is not important; that is, V-belts can slip (see Section 19.2.3). V-belts are easily installed and removed, quiet in operation (but not quite as quiet as flat belts), low in maintenance, and provide shock absorption between the driver and driven shafts. V-belt drives normally operate best at belt velocities between 7.5 and 30 m/s (1500 and 6500 ft/min).

The **velocity ratio** is similar to the gear ratio given in Eq. (14.19), or

$$g_r = \frac{N_{a1}}{N_{a2}} = \frac{r_2}{r_1}. \tag{19.10}$$

V-belts can operate satisfactorily at velocity ratios up to approximately 7 to 1. V-belts typically operate at 90 to 98% efficiency, lower than that found for flat belts.

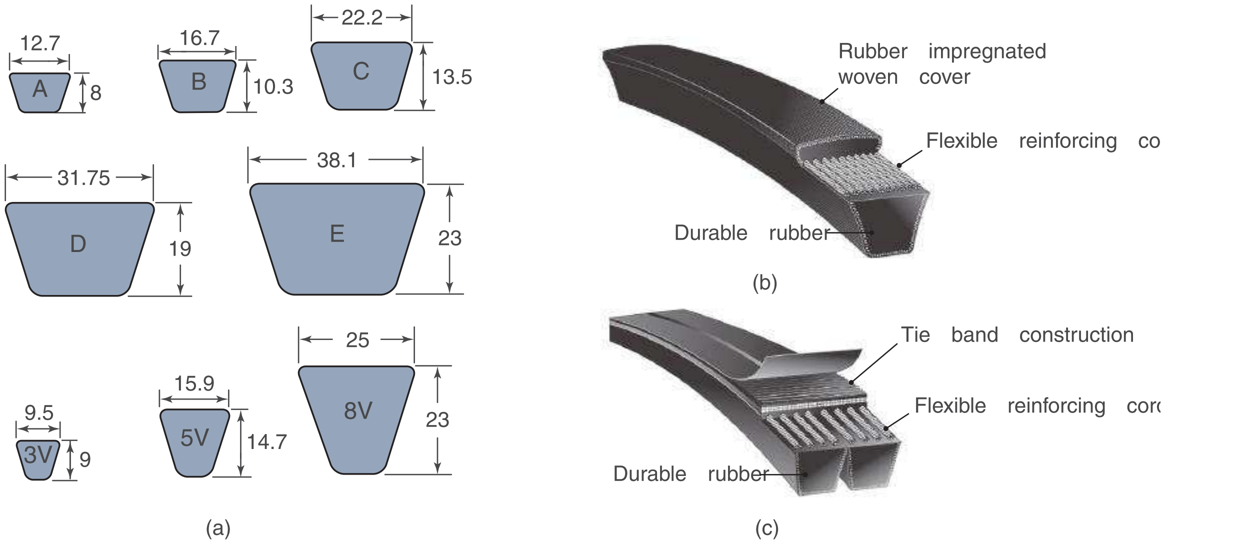

Figure 19.4: Design and construction of V-belts. (a) Standard V-belts cross sections with dimensions in millimeters; (b) typical single-belt, showing reinforcing cords and wear-resistant exterior; (c) double V-belt, used for higher power transmission than single belts. Up to five belts can be combined in this fashion.

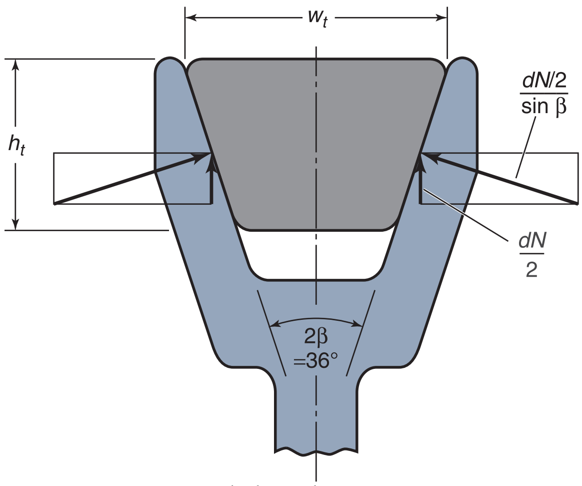

Figure 19.5: V-belt in sheave groove.

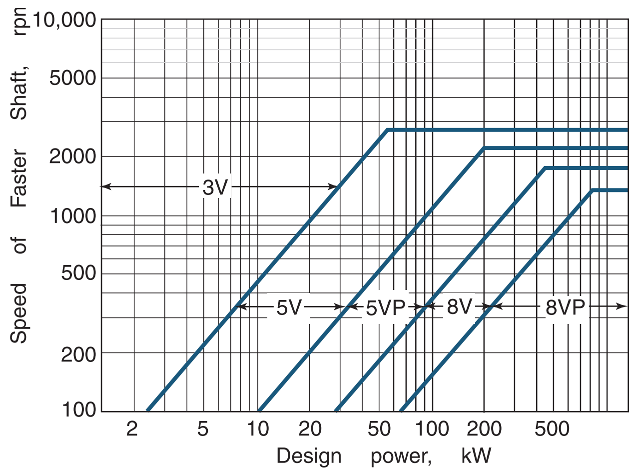

Figure 19.6: Guide to selection of belt cross section as a function of power transmitted and shaft speed.

A major advantage of V-belts over flat belts is their wedging action, which increases the normal force from dN for flat belts to $(dN/2)/\sin\beta$ (as shown in Fig. 19.5) for V-belts, where β is the sheave angle. Because the V-belt has a trapezoidal cross section, the belt rides on the side of the groove and a wedging action increases the traction. Pressure and friction forces act on the side of the belt. The force equations developed in Section 19.2 for flat belts are equally applicable for V-belts if the coefficient of friction μ is replaced with $\mu/\sin\beta$. Also, the belt length equation for flat belts given by Eq. (19.5) is applicable to V-belts.

19.4.1 Input Normal Power Rating

It is essential that the maximum possible load conditions be considered in designing a V-belt. The design power rating of the belt needs to consider the service factor, or

$$h_{pr} = f_1 h_p, \tag{19.11}$$

where f_1 is the overload service factor for various applications as given in Table 19.2, and h_p is the input power. Figure 19.6 shows the belt profiles that are commonly applied as a function of design power rating.

19.4.2 Sheave Size

The materials and construction of V-belts have improved dramatically in the past few decades. For example, a B-Section in a 175-mm sheave was rated at 3.1 kW in 1945, while a modern belt under the same conditions has a rating over 8 kW. This is due in large part to the transition from multiple reinforcing cords centrally located in the belt to a single tensile cord line located higher in the cross section. One subtle complication occurs with such belt designs: the pitch diameter (from which velocities are calculated) shifts upward when a modern belt replaces a conventional design. For this reason, the International Standards Organization introduced the term **datum line** to differentiate pitch lines associated with new and conventional belt designs; this standard was adopted in the United States in 1988. Modern sheaves are

Table 19.2: Typical overload service factors, f_1. *Source:* Courtesy of the Gates Corporation.

Driven machine	Power source: Normal[a] Service (hr/day) 3–5	8–10	16–24	Demanding[b] Service (hr/day) 3–5	8–10	16–24
Dispensing, display equipment, measuring equipment, office and projection equipment.	1.0	1.1	1.2	1.1	1.2	1.3
Liquid agitators, appliances, sewing machines, sweepers, light-duty conveyors, fans, light duty machine tools (drill presses, lathes, saws), woodworking equipment.	1.1	1.2	1.3	1.2	1.3	1.4
Semi-liquid agitators, centrifuges, centrifugal compressors, heavy-duty conveyors, dough mixers, generators, laundry equipment, heavy-duty machine tools (boring mills, grinders, mills, shapers), presses, shears, printing machinery, centrifugal and gear pumps.	1.1	1.2	1.4	1.2	1.3	1.5
Brick machinery, piston compressors, screw conveyors, bucket elevators, extractors, hammer mills, paper pulpers, pulverizers, piston pumps, extruders, rubber calendar mills, textile machinery.	1.2	1.3	1.5	1.4	1.5	1.6
Jaw crushers, hoists, ball mills, rod and tube mills, sawmill machinery.	1.3	1.4	1.6	1.5	1.6	1.8

[a] Includes normal torque, squirrel cage, synchronous and split phase AC motors; shunt wound DC motors; multiple cylinder internal combustion engines.

[b] Includes high torque, high slip, repulsion-induction, single phase, series wound AC motors; series wound, compound wound DC motors; single piston internal combustion engines.

Table 19.3: Recommended minimum datum diameters, in millimeters, of sheaves for general purpose electric motors. *Source:* Courtesy of the Gates Corporation.

Motor power, kW	Motor rpm: 575	690	870	1160	1750	3450
0.375	64	64	56	—	—	—
0.50	75	64	61	56	—	—
0.75	75	75	61	61	56	—
1.1	75	75	61	61	61	56
1.5	96	75	75	61	61	61
3.75	115	115	96	75	75	66
7.5	150	132	117	112	96	75
11.2	175	150	5.4	117	112	96
15	210	175	150	137	117	112
22.5	250	230	175	175	137	—
37.3	280	250	230	208	175	—
55	355	330	267	250	230	—
75	457	380	320	280	250	—
150	560	560	560	—	—	—

specified based on a datum diameter; the concepts of datum diameter and pitch diameter are the same, and the sheave datum diameter can be used for velocity calculations. For standard belt cross-sections, the datum diameter is nearly the same as the outside diameter of the sheave, since this is very close to the value of the reinforcing line for most belts.

The design of a V-belt drive should use the largest possible sheaves. Unfortunately, large sheaves are usually more expensive, and there is an inherently larger center distance associated with their use compared to smaller sheaves. Small sheaves are less efficient, require larger belt preload (and associated higher load on bearings and shafts), and greatly reduce belt life because of slip and extreme flexing of the belt. Table 19.3 shows minimum recommended sheave datum diameters for general purpose electric motors.

Usually, much larger sheave datum diameters are used than suggested by Table 19.3. For some applications, it may be economically justifiable to select sheave designs based on design constraints and manufacture them to the desired dimensions. However, it is often preferable to utilize standard sheaves. Table 19.4 presents some combinations of standard sheave sizes, and also provides a power correction factor, f_2, which will modify the belt power rating as discussed in Design Procedure 19.1.

19.4.3 Design Power Rating

Table 19.5 gives the rated power for selected belt types. The belts considered are the 3V and 5V cross sections, but the data for the other cross sections shown in Fig. 19.4a are readily available in manufacturers' literature. It should be noted that Table 19.5 represents a very small sampling of available V-belts; there are many more cross-sections, and many more lengths and capacities depending on specific materials and belt construction. Table 19.5 is useful for illustration purposes, but for detailed design, a wider selection of belts should be considered, as can be readily obtained from manufacturers' web sites.

Design Procedure 19.1: V-Belt Drives

It will be assumed that a belt drive will be designed for power transmission where the shaft speeds (and hence speed ratio) and desired center distance are known. The power available can be obtained from the rating of the motor, or else it can be obtained from design requirements. Based on these quantities, this design procedure provides a methodology for selecting a cross-section of a belt, choosing sheaves and number of belts required.

1. Estimate the overload service factor from Table 19.2 and use it to obtain the required belt power rating using Eq. (19.11).

2. Select a cross section of the belt from the required belt power rating and the shaft speed using Fig. 19.6.

3. Obtain the minimum allowable sheave datum diameter from Table 19.3.

Table 19.4: Center distance and power correction factor, f_2, for standard sheaves. *Source:* Courtesy of the Gates Corporation.

Belt type	Speed ratio[a]	Datum diameter, mm Small sheave	Large sheave	Belt length, mm 635	762	875	1020	1140	1250	1520
3V	1.00	67	67	211	274	338	401	465	528	655
		76	76	198	262	325	389	452	516	643
		85	85	183	246	310	373	437	500	627
		114	114	137	201	264	328	391	455	582
	1.25	64	80	206	269	333	396	460	523	650
		93	114	155	218	282	345	409	472	599
		114	142	—	178	244	307	371	434	561
		120	152	—	165	229	292	355	420	546
	1.5	71	105	178	241	305	371	434	498	625
		76	114	173	231	295	358	422	485	612
		85	127	150	213	277	340	404	467	594
		135	203	—	—	—	241	305	368	495
	2.0	67	135	155	221	284	348	411	485	333
		76	152	132	198	262	325	391	465	569
		105	203	—	—	196	262	325	391	594
		135	269	—	—	—	—	244	310	582
	2.5	67	165	—	193	257	323	386	450	577
		71	175	—	180	246	310	373	439	566
		80	203	—	—	213	279	343	409	536
		142	356	—	—	—	—	—	—	356
	3.0	67	203	—	152	221	287	353	417	546
		93	269	—	—	—	203	272	340	470
		121	356	—	—	—	—	—	—	368

Belt type	Speed ratio[a]	Datum diameter, mm Small sheave	Large sheave	Belt length, mm 1500	1770	2030	2290	2540	3175	3810
5V	1.00	180	180	478	617	732	859	986	1303	1621
		203	203	442	582	696	823	950	1267	1585
		229	229	404	544	658	785	912	1229	1547
		318	318	—	404	518	645	772	1090	1407
	1.5	191	287	384	523	640	767	894	1212	1529
		235	356	—	434	549	676	805	1123	1440
		318	475	—	—	—	516	643	963	1280
	2.0	180	356	330	472	589	716	843	1163	1608
		203	406	—	411	528	655	785	1105	1549
		356	711	—	—	—	—	—	729	1181
	3.0	203	599	—	—	—	470	607	937	1260
		180	538	—	—	411	549	681	1006	1328
		318	953	—	—	—	—	—	—	848

[a] Nominal speed ratio, actual value may vary slightly.

Power Correction Factor:

$f_2 = 0.8$

$f_2 = 0.9$

$f_2 = 1.0$

4. Locate the sheave diameter combinations in Table 19.4 that are suitable for a desired speed ratio. Disregard from consideration any candidates that are smaller than the minimum values obtained in Step 3. From the remaining candidates, select a sheave size that is consistent with space requirements.

5. From Table 19.4, locate the center distance that most closely matches design constraints, and obtain the power correction factor, f_2. Note that the belt length can be calculated from Eq. (19.5) or read directly from Table 19.4.

6. From Table 19.5, locate the proper belt cross section and center distance, and obtain the basic power rating per belt, h_1. Note that for very high speeds or small sheaves, an additional power may be required. This is usually a small amount and is neglected in this design procedure.

7. The rated power per belt is given by

$$h_{pb} = f_2 h_1. \tag{19.12}$$

8. The number of belts required can be obtained from the required power from Step 1:

$$N = \frac{h_{pr}}{h_{pb}}. \tag{19.13}$$

Example 19.3: V-Belt Design

Given: A 7.5-kW squirrel cage electric motor operating at a speed of 2000 rpm drives a centrifugal air compressor in normal service at 1000 rpm. It is desired to have a center distance around 375 mm.

Find: Determine the proper belt cross section for this application, select pulleys, and choose the number of belts.

Solution: This approach will closely follow Design Procedure 19.1.

1. Referring to Table 19.2, note from the footnote that the squirrel cage motor represents a normal power source. Therefore, an overload service factor of $f_1 = 1.2$ is selected as typical for a compressor, assuming the pump is operated during a normal 8-hr shift. Therefore, the design power is, from Eq. (19.11),

$$h_{pr} = f_1 h_p = (1.2)(7.5) = 9.0 \text{ kW}.$$

2. For 7.5 kW and a shaft speed of 2000 rpm, Fig. 19.6 suggests a 3V cross-section is proper. Note, however, that the application is very close to the limit of 3V effectiveness, so a 5V belt may also provide a reasonable design solution. However, this solution will continue using a 3V belt as recommended by Fig. 19.6.

Table 19.5: Rated power in kW per belt for selected 3V and 5V cross sections. *Source:* Courtesy of the Gates Corporation.

Belt	Speed	Small sheave outside diameter, mm											
	(rpm)	65	71	76	80	85	93	105	114	120	127	135	142
3V	200	0.20	0.23	0.26	0.29	0.32	0.37	0.46	0.52	0.56	0.60	0.66	0.71
	400	0.37	0.41	0.48	0.53	0.60	0.69	0.85	0.97	1.05	1.13	1.22	1.32
	600	0.51	0.58	0.68	0.75	0.85	0.99	1.22	1.40	1.51	1.63	1.77	1.90
	800	0.64	0.74	0.87	0.96	1.08	1.27	1.56	1.80	1.95	2.10	2.28	2.46
	1000	0.77	0.89	1.04	1.16	1.31	1.54	1.89	2.18	2.36	2.55	2.78	2.99
	1200	0.89	1.03	1.21	1.35	1.53	1.80	2.22	2.56	2.78	2.99	3.25	3.51
	1400	1.01	1.16	1.37	1.53	1.74	2.05	2.53	2.92	3.17	3.42	3.72	4.01
	1600	1.11	1.29	1.53	1.71	1.94	2.29	2.83	3.27	3.54	3.83	4.16	4.48
	1800	1.22	1.42	1.68	1.88	2.14	2.53	3.13	3.60	3.92	4.22	4.59	4.95
	2000	1.31	1.54	1.83	2.04	2.33	2.75	3.41	3.93	4.27	4.60	5.00	5.39
	2400	1.51	1.76	2.10	2.36	2.69	3.19	3.95	4.54	4.94	5.32	5.77	6.21
	3000	1.75	2.07	2.48	2.78	3.18	3.77	4.68	5.38	5.83	6.27	6.80	7.30
	4000	2.09	2.48	3.00	3.37	3.87	4.60	5.68	6.51	7.02	7.53	8.06	8.65
	5000	2.33	2.78	3.38	3.81	4.38	5.18	6.36	7.23	7.76	—	—	—
Belt	**Speed**	**Small sheave outside diameter, mm**											
	(rpm)	**180**	**190**	**200**	**215**	**230**	**235**	**250**	**260**	**287**	**317**	**355**	**405**
5V	100	1.01	1.10	1.22	1.34	1.46	1.52	1.63	1.76	1.99	2.27	2.61	3.07
	200	1.88	2.06	2.28	2.51	2.73	2.84	3.07	3.31	3.75	4.27	4.92	5.78
	300	2.69	2.95	3.28	3.60	3.93	4.10	4.42	4.77	5.41	6.17	7.12	8.36
	400	3.45	3.80	4.23	4.66	5.08	5.29	5.71	6.17	7.00	7.98	9.18	10.82
	500	4.20	4.62	5.15	5.67	6.18	6.45	6.96	7.53	8.50	9.77	11.26	13.20
	600	4.91	5.41	6.03	6.65	7.26	7.53	8.13	8.80	10.00	11.41	13.13	15.44
	800	6.27	6.92	7.68	8.50	9.33	9.70	10.44	11.34	12.83	14.62	16.79	19.62
	1000	7.53	8.36	9.33	10.29	11.19	11.71	12.61	13.65	15.44	17.53	20.14	23.35
	1500	10.37	11.49	12.83	14.10	15.44	16.04	17.31	18.65	20.96	23.65	26.71	30.29
	2000	12.76	14.02	15.67	17.23	18.72	19.47	20.81	22.31	24.84	—	—	—

3. From Table 19.3, note that the 7.5-kW motor operating at 2000 rpm will require sheaves that are slightly smaller than 96 mm.

4. In Table 19.4, there are four sheave combinations that are suitable for this application. Either the 105 mm or 135 mm smaller sheave are above the minimum sheave dimensions and will yield reasonable designs. This solution will assume that the bearing reaction forces are not as much of a concern as low center distance, so the 105- and 203-mm sheave combination will be selected.

5. For the required nominal center distance of 375 mm, a 1250-mm belt is most suitable. From the color code in Table 19.4, note that the application factor for this belt is $f_2 = 0.90$.

6. Referring to Table 19.5, a 105 mm outer diameter sheave operating at 2000 rpm has a basic power rating of $h_1 = 3.41$ kW.

7. The rated power per belt is therefore, from Eq. (19.12),

$$h_{pb} = f_2 h_1 = (0.90)(3.41) = 3.07 \text{ kW}.$$

8. The number of belts required is given by Eq. (19.13) as

$$N = \frac{h_{pr}}{h_{pb}} = \frac{9.0}{3.07} = 2.93.$$

Therefore, the number of belts are chosen as $N = 3$.

In summary, three 1.25-m-long, 3V belts operating on sheaves of 105 and 203 mm are needed for this application. Since three belts are needed, a triple belt design (see Fig. 19.4) and associated sheaves are specified.

19.5 Wire Ropes

Wire ropes are used instead of flat belts or V-belts when power must be transmitted over long distances as in hoists, elevators, ski lifts, etc. Figure 19.7 shows cross sections of selected wire ropes. The center portion (dark section) is the **core** of the rope and is often made of hemp (a tall Asiatic herb), but can also be constructed of a polymer such as polypropylene or steel strands. The purposes of the core are to elastically support the strands and to lubricate them to prevent excessive wire wear.

The **strands** are groupings of wires placed around the core. In Fig. 19.7a there are six strands, and each strand consists of 19 wires. Wire ropes are typically designated as, for example, "$1\frac{1}{8}, 6 \times 19$ hauling rope." The $1\frac{1}{8}$ gives the wire rope diameter in inches, designated by the symbol d. The 6 designates the number of strands and 19 designates the number of wires in a strand. The wire diameter is designated by the symbol d_w. The term "hauling rope" designates the application in which the wire rope is to be used. Generally, wire ropes with more but smaller wires are more resistant to fatigue, whereas ropes with fewer but larger wires have greater abrasive wear resistance.

When a wire rope is bent and unbent around a sheave, the strands and wires slide over each other at a small length scale. The sliding velocities are low, but the pressure is high, so that wire wear is a concern. For this reason, wire rope lubrication is essential for prolonged life, and lubricant needs to be periodically replaced. Lubricants need to be matched to core materials to ensure proper wetting, and are also essential for corrosion protection.

Figure 19.8 shows two **lays** of wire rope. The **regular lay** (Fig. 19.8b) has the wire twisted in one direction to form strands and the strands twisted in the opposite direction to form the rope. Visible wires are approximately parallel to the rope axis. The major advantage of the regular lay is that the rope does not kink or untwist and is easy to handle. The **Lang lay** (Fig. 19.8a) has the wires in the strands and the strands in the rope twisted in the same direction. This type of lay has

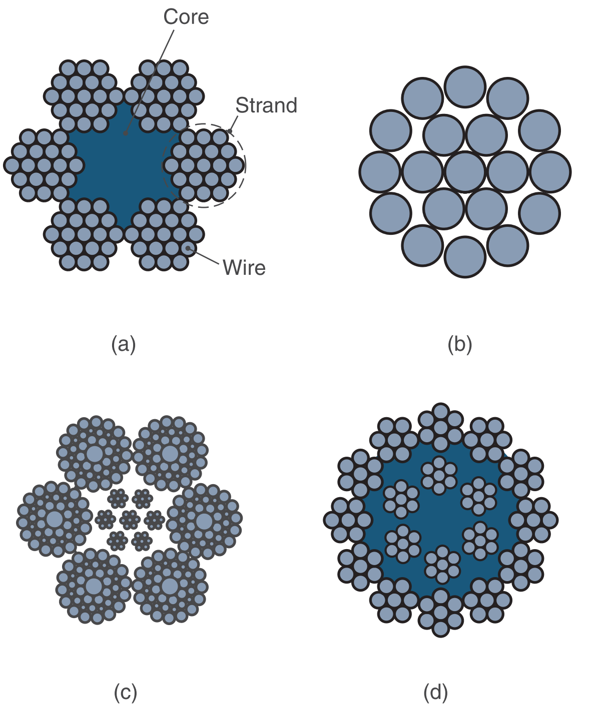

Figure 19.7: Cross sections of selected wire rope. (a) 6×19 fiber core; (b) 1×19; (c) 6×36 wire core; (d) 18×7 fiber core.

more resistance to abrasive wear and bending fatigue than the regular lay. Lang-lay ropes are, however, more susceptible to handling abuses, pinching in undersized grooves, and crushing when improperly wound on drums. Also, the twisting moment acting in the strand tends to unwind the strand, causing excessive rope rotation. Lang-lay ropes should therefore always be secured at the ends to prevent the rope from unlaying.

Although steel is most popular, wire rope is made of many kinds of metal, such as copper, bronze, stainless steel, and wrought iron. Table 19.6 lists some of the various ropes that are available, together with their characteristics and properties. The cross-sectional area of the metal strand in standard hoisting and haulage ropes is $A_m \approx 0.38d^2$.

19.5.1 Tensile Stress

The total force acting on the rope is

$$F_t = F_w + F_r + F_a + F_h, \tag{19.14}$$

where

F_w = weight being supported, N
F_r = rope weight, N
F_a = force due to acceleration, N
F_h = static load, often due to a *headache ball*, N

The static load or dead weight, F_h is essential for most applications to maintain tension in the wire rope and to prevent it from slipping off of a sheave. The tensile stress is

$$\sigma_t = \frac{F_t}{A_m}, \tag{19.15}$$

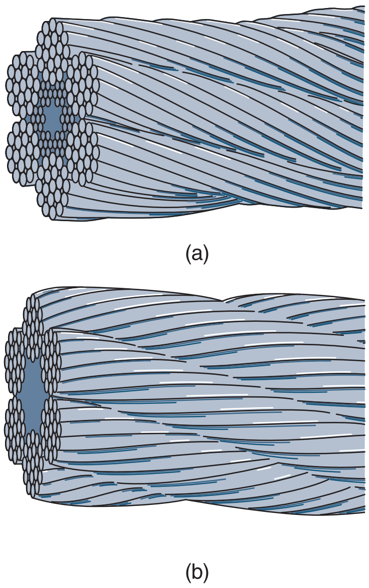

Figure 19.8: Two lays of wire rope. (a) Lang; (b) regular.

where A_m is the cross-sectional area of metal strand in standard hoisting and haulage ropes. The allowable stress is obtained from Table 19.6.

From Eq. (1.1), the safety factor is

$$n_s = \frac{\sigma_{\text{all}}}{\sigma_t}. \tag{19.16}$$

Table 19.7 gives minimum safety factors for a variety of wire rope applications. The safety factor obtained from Eq. (19.16) should be larger than the safety factor obtained from Table 19.7.

19.5.2 Bending Stress

From Eqs. (4.46) and (4.47), the bending moment applied to the wires in a rope passing over a pulley is

$$M = \frac{EI}{r} \quad \text{and} \quad M = \frac{\sigma I}{c}. \tag{19.17}$$

Equating these expressions and solving for the bending stress gives

$$\sigma_b = \frac{Ec}{r},$$

where r is the radius of curvature that the rope will experience and c is the distance from the neutral axis to the outer fiber of the wire. A rope bearing against a sheave may flatten slightly, but the radius of curvature that the rope experiences is very nearly the sheave radius $D/2$. Similarly, c can be taken as $d_w/2$, yielding

$$\sigma_b = \frac{Ed_w}{D}, \tag{19.18}$$

where d_w is the wire diameter and D is the pulley diameter. Note from Eq. (19.18) that if D/d_w is very large, the bending stress will be small. Suggested minimum sheave diameters in Table 19.6 are based on a D/d_w ratio of 400. If possible, the sheaves should be designed for a larger ratio. If the ratio D/d_w is less than 200, heavy loads will often cause a permanent set of the rope. Thus, for a safe design with very long service-free life, it is recommended that $D/d_w \geq 400$.

Table 19.6: Wire rope data. *Source:* Shigley and Mitchell [1983].

Rope	Weight per length[a], kg/m	Minimum sheave, diameter,	Rope diameter, d, mm	Material	Size of outer wires	Stiffness,[b] GPa	Strength,[c] MPa
6 × 7 Haulage	$3465d^2$	$42d$	6 – 12	Monitor steel	$d/9$	96	690
				Plow steel	$d/9$	96	600
				Mild plow steel	$d/9$	96	525
6 × 19 Standard hoisting	$3700d^2$	$26d$ – $34d$	6 – 70	Monitor steel	$d/13$ – $d/16$	83	730
				Plow steel	$d/13$ – $d/16$	83	640
				Mild plow steel	$d/13$ – $d/16$	83	550
6 × 37 Special flexible	$3580d^2$	$18d$	6 – 90	Monitor steel	$d/22$	75	690
				Plow steel	$d/22$	75	600
8 × 19 Extra flexible	$3350d^2$	$21d$ – $26d$	6 – 40	Monitor steel	$d/15$ – $d/19$	69	630
				Plow steel	$d/15$ – $d/19$	69	550
7 × 7 Aircraft	$3930d^2$	—	1.5 – 10	Corrosion-resistant steel	—	—	850
				Carbon steel	—	—	850
7 × 9 Aircraft	$4040d^2$	—	3 – 35	Corrosion-resistant steel	—	—	930
				Carbon steel	—	—	985
19-Wire aircraft	$4970d^2$	—	0.75 – 8	Corrosion-resistant steel	—	—	1140
				Carbon steel	—	—	1140

[a] Weight per length in kg/m for d in meters.
[b] The stiffness is only approximate; it is affected by the loads on the rope and, in general, increases with the life of the rope.
[c] The strength is based on the nominal area of the rope. The figures given are only approximate and are based on 25-mm rope sizes and 6-mm aircraft cable sizes.

However, these design rules are hardly ever followed for some applications, for a number of reasons. First, consider a 50-mm-diameter wire rope, such as would typically be used in the crane or in the dragline discussed in Case Study 19.1. According to the rules just stated, the sheave on the crane would have to be around 2 m in diameter. Further, since the wire rope is wound on a drum and failure can occur in the rope adjacent to the drum, such a large diameter would also be needed for the drum. Because the required motor torque is the product of the hoist rope tension and the drum radius, a very large motor would be required for lifting relatively light loads. As a result, the entire crane would be much larger and more expensive, and other design challenges would arise. For example, ground collapse below the crane would be more common, energy consumption would increase, and it would be more difficult to move cranes to different sites.

The design rules just stated are recommendations for applications where the wire rope should attain infinite life. The economic consequences of infinite-life wire rope are usually too large to bear, and smaller pulleys are usually prescribed. To prevent failures that result in property damage or personal injury, the wire ropes are periodically examined for damage. Since a broken wire will generally be easily detected and will snag a cotton cloth dragged over the rope's surface, rope life requirements are often expressed in terms of the number of broken wires allowed per length of wire rope. For example, the American Society of Mechanical Engineers [2007] requires inspections of crane wire ropes every 6 months, and if any section has more than six broken wires within one lay (revolution of a strand) of the rope, or three in any strand within a lay, the entire wire rope must be replaced. The same standard calls for pulley-to-wire diameter ratios of 12:1, which obviously results in wire rope with finite service life. Figures 19.9 and 19.10 relate the decrease in strength and service life associated with smaller pulley diameters.

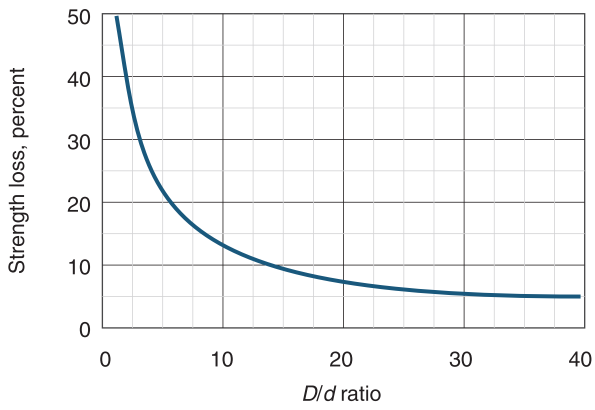

Figure 19.9: Percent strength loss in wire rope for different D/d ratios.

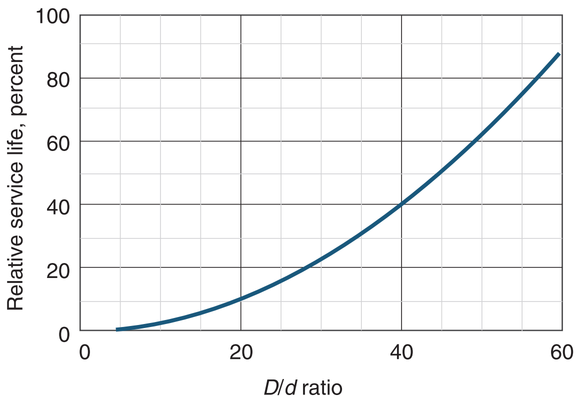

Figure 19.10: Service life for different D/d ratios.

19.5.3 Bearing Pressure

The rope stretches and rubs against the pulley, causing wear of both the rope and pulley. The amount of wear depends on the pressure on the rope in the pulley groove, or

$$p = \frac{2F_t}{dD}, \tag{19.19}$$

Table 19.7: Minimum safety factors for a variety of wire rope applications. Note that the use of these safety factors does not preclude a fatigue failure. *Source:* From Shigley and Mitchell [1983].

Application	Safety factor, n_s
Track cables	3.2
Guys	3.5
Mine shafts, m	
Up to 150	8.0
300–600	7.0
600–900	6.0
Over 900	5.0
Hoisting	5.0
Haulage	6.0
Cranes and derricks	6.0
Electric hoists	7.0
Hand elevators	5.0
Private elevators	7.5
Hand dumbwaiters	4.5
Grain elevators	7.5
Passenger elevators	
Up to 0.25 m/s	7.60
0.25–1.5	9.20
1.5–4.0	11.25
4.0–6.0	11.80
6.0–1500	11.90
Freight elevators	
Up to 0.25 m/s	6.65
0.25–1.5	8.20
1.5–4.0	10.00
4.0–6.0	10.50
6.0–7.5	10.55
Powered dumbwaiters	
Up to 0.25 m/s	4.8
0.25–1.5	6.6
1.5-4.0	8.0

where d is the rope diameter. The pressure obtained from Eq. (19.19) should be less than the maximum pressure obtained from Table 19.8 for various pulley materials and types of rope.

19.5.4 Fatigue

For the rope to have a long life, the total force, F_t, must be less than the fatigue allowable force, F_f, where

$$F_f = \frac{S_u d D}{2000}. \tag{19.20}$$

The ultimate strength given by Eq. (19.20) for three materials is

$$\begin{array}{ll} \text{Monitor plow steel} & 1650 \leq S_u \leq 1930 \text{ MPa} \\ \text{Plow steel} & 1400 \leq S_u \leq 1650 \text{ MPa} \\ \text{Mild plow steel} & 1250 \leq S_u \leq 1400 \text{ MPa} \end{array} \tag{19.21}$$

Table 19.8: Maximum allowable bearing pressures for various sheave materials and types of rope. *Source:* From Shigley and Mitchell [1983].

	Maximum bearing pressure for sheave material, MPa			
Rope	Cast iron[a]	Cast steel[b]	Chilled cast iron[c]	Manganese steel[d]
Regular lay				
6 × 7	2.07	3.80	4.48	10.1
6 × 19	3.31	6.20	7.58	16.5
6 × 37	4.04	7.41	9.13	20.7
8 × 19	4.69	8.69	10.7	24.1
Lang lay				
6 × 7	2.41	4.13	4.93	11.4
6 × 19	3.79	6.89	8.34	19.0
6 × 37	4.55	8.14	10.0	22.8

[a] On end grain of beech, hickory, orgum.
[a] For minimum HB = 125.
[b] 0.30–0.40% carbon; HB (min.) = 160.
[c] Use only with uniform surface hardness.
[d] For high speeds with balanced sheaves having ground surfaces.

Example 19.4: Analysis of Wire Rope

Given: Lighting fixtures in a large theater are to be raised and lowered by a wire rope, and need to be moved a maximum of 30 m. The maximum load to be hoisted is 12 kN at a velocity not exceeding 0.5 m/s and an acceleration of 1 m/s^2. Use 25-mm plow steel in the form of 6×19 standard hoisting ropes. Use a rope cross-sectional area of 2.41×10^-4 m^2.

Find: Determine the safety factor while considering

(*a*) Tensile stress

(*b*) Bending stress

(*c*) Bearing pressure

(*d*) Fatigue

Solution: From Table 19.6 for 6×19 standard hoisting wire rope, assuming the use of only one rope,

$$F_r = 3700 d^2 h_2 = 3700(0.025)^2(30) = 69.4 \text{ kg/rope},$$

$$F_w = W_{\max} = 12,000 \text{ N}.$$

Note that F_r is 680 N/rope. The force due to acceleration is

$$F_a = ma = \frac{W}{g}a = \frac{(12,000 + 680)(1)}{9.81} = 1290 \text{ N}$$

The total force on the rope is

$$F_t = F_w + F_r + F_a = 12,000 + 680 + 1290 = 14,000 \text{ N}.$$

(*a*) *Tensile stress:*

$$\sigma_t = \frac{F_t}{A_m} = \frac{14,000}{2.41 \times 10^{-4}} = 58.1 \text{ MPa}.$$

From Table 19.6 for 6×19 standard hoisting wire rope made of plow steel, $\sigma_{\text{all}} = 640$ MPa. The safety factor is

$$n_s = \frac{\sigma_{\text{all}}}{\sigma_t} = \frac{640}{58.1} = 11.0$$

From Table 19.7, the recommended safety factor for hoisting applications is 5.0. Thus, one rope is sufficient as far as the tensile stress is concerned.

(*b*) *Bending stress:* From Table 19.6, the minimum pulley diameter for 6×19 standard hoisting wire rope is $26d$ to $34d$. No design constraints have been given regarding the pulley, so assign the conservative value of $D = 34d = 850$ mm. Also, from the same table, the wire diameter should be between $d/13$ and $d/16$. Choose $d_w = d/16 = 1.56$ mm, so that

$$\frac{D}{d_w} = \frac{850}{1.56} = 545.$$

Permanent set should not be a concern, since $D/d_w \geq 400$. From Table 19.6, the stiffness is 83 GPa. The bending stress is, from Eq. (19.18),

$$\sigma_b = E\frac{d_w}{D} = \frac{83 \times 10^9}{544} = 153 \text{ MPa}.$$

The safety factor due to bending is

$$n_s = \frac{\sigma_{\text{all}}}{\sigma_b} = \frac{640}{153} = 4.18.$$

This safety factor is less than the 5.0 recommended, but the safety factor is for static tensile loading. Changing the material from plow steel to monitor steel would produce a safety factor of 4.81, closer to 5, but this is an indication that the wires will not fail in bending. The tensile overload factor from Table 19.7 is not really applicable to other failure modes.

Note that increasing the number of ropes will not alter the results, since the rope will still be wrapped around the same sized sheave. The obvious solution is to use an even larger pulley, but this solution has design implications, as discussed in Section 19.5.2.

(*c*) *Bearing pressure:* From Eq. (19.19),

$$p = \frac{2F_t}{dD} = \frac{2(14,000)}{(0.025)(0.85)} = 1320 \text{ kPa}.$$

From Table 19.8 for a 6×19 Lang lay for a cast steel pulley, $p_{\text{all}} = 6.20$ MPa. The safety factor is

$$n_s = \frac{p_{\text{all}}}{p} = \frac{6.20}{1.32} = 4.70.$$

(*d*) Fatigue: For monitor steel the ultimate strength is at most 1930 MPa from Eq. (19.21). The allowable fatigue force from Eq. (19.20) is

$$F_f = \frac{S_u dD}{2000} = \frac{(1930 \times 10^6)(0.025)(0.85)}{2000} = 20 \text{ kN}.$$

The safety factor is

$$n_s = \frac{F_f}{F_t} = \frac{20,000}{12,000} = 1.67.$$

Thus, fatigue failure is the most likely failure to occur, since it produced the smallest safety factor. Using four ropes instead of one would produce a safety factor greater than 5.

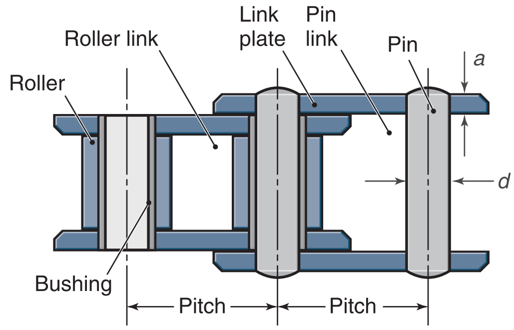

Figure 19.11: Various parts of rolling chain.

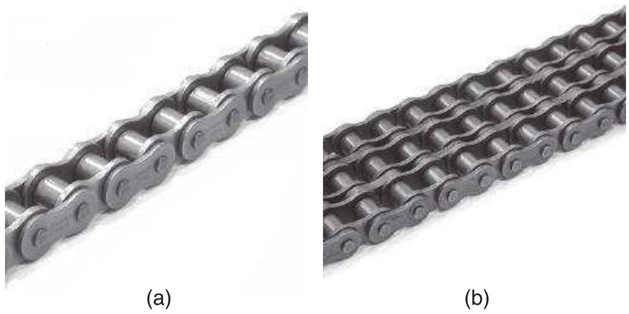

Figure 19.12: Typical rolling chain. (a) One-strand rolling chain; (b) three-strand chain.

19.6 Rolling Chains

Rolling chains are used to transmit power between two sprockets rotating in the same plane. The machine element that it most resembles is a timing belt (shown in Fig. 19.3). The major advantage of using a rolling chain compared to a belt is that rolling chains do not slip. Large center distances can be dealt with more easily with rolling chains with fewer elements and in less space than with gears. Rolling chains also have high efficiency. No initial tension is necessary and shaft loads are therefore smaller than with belt drives. The only maintenance required after careful alignment of the elements is periodic lubrication, and with proper lubrication, a long life can be attained.

19.6.1 Operation of Rolling Chains

Figure 19.11 shows the various parts of a rolling chain with pins, bushings, rollers, and link plates. The rollers turn on bushings that are press fitted to the inner link plates. The pins are prevented from rotating in the outer plates by the press-fit assembly. Table 19.9 gives dimensions for standard sizes. The size range given in Table 19.9 is large, and multiple strands can be used (see Fig. 19.12), so that chains can be used for both large and small power transmission levels. A large reduction in speed can be obtained with rolling chains if desired. The tolerances for a chain drive are larger than for gears, and the installation of a chain is relatively easy.

The minimum wrap angle of the chain on the smaller sprocket is 120°. A smaller wrap angle can be used on idler sprockets, which are used to adjust the chain slack where the center distance is not adjustable. Horizontal drives (the line connecting the axes of sprockets is parallel to the ground) are

Table 19.9: Standard sizes and strengths of rolling chains.

ANSI Chain number[a]	ISO Chain number	Pitch, p_t, mm	Roller Diameter, mm	Roller Width, mm	Pin diameter, d, mm	Link plate thickness, a, mm	Average tensile strength, S_u, kN	Weight per meter, kg/m
[b]25	—	6.350	3.302	3.175	2.30	0.762	3.89	0.12
[b]35	—	9.525	5.080	4.763	3.58	1.27	9.34	0.31
40	08A-1	12.700	7.93	7.85	3.98	1.50	13.90	0.591
50	10A-1	15.875	10.15	9.55	5.09	2.03	21.80	1.00
60	12A-1	19.050	11.91	12.65	5.96	2.42	31.13	1.36
80	16A-1	25.400	15.88	15.87	7.94	3.25	55.60	2.58
100	20A-1	31.750	19.05	19.05	9.53	4.00	87.00	3.88
120	24A-1	38.100	22.22	25.40	11.10	4.80	125.00	5.56
140	28A-1	44.450	25.40	25.40	12.70	5.60	170.00	7.44
160	32A-1	50.800	28.58	31.75	14.27	6.40	223.00	10.0
200	40A-1	63.500	39.67	38.10	19.85	8.00	347.00	16.7
240	48A-1	76.200	47.60	47.60	23.80	9.50	500.00	23.7

[a] The pitch, in inches, can be obtained from the chain number by taking the left digits and dividing by 8.
[b] Without rollers.

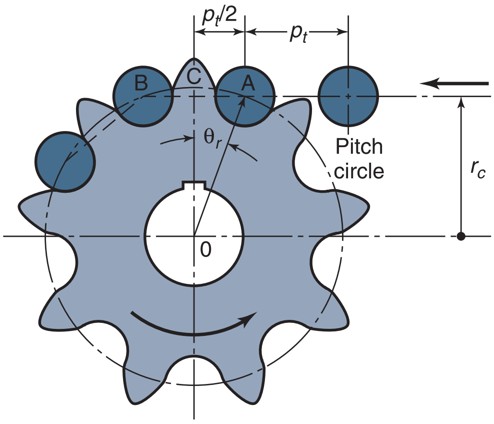

Figure 19.13: Chordal rise in rolling chains. Note that the chain link travels upwards as well as horizontally when moving from position A to position B.

recommended; vertical drives (the line connecting the axes of sprockets is perpendicular to the ground) are less desirable. Vertical drives, if used, should be used with idlers to prevent the chain from sagging and to avoid disengagement from the lower sprocket.

19.6.2 Kinematics

The velocity ratio, comparable to the gear ratio given in Eq. (14.19), is

$$g_r = \frac{d_2}{d_1} = \frac{\omega_1}{\omega_2} = \frac{N_2}{N_1}, \tag{19.22}$$

where N is the number of teeth in the sprocket, ω is the angular velocity in rad/s, and d is the diameter. For a one-step transmission it is recommended that $g_r < 7$. Values of g_r between 7 and 10 can be used at low speed (below 650 ft/min).

19.6.3 Chordal Rise

An important factor affecting the smoothness of rolling chain drive operation, especially at high speeds, is **chordal rise**, shown in Fig. 19.13. From right triangle 0CA

$$r_c = r\cos\theta_r. \tag{19.23}$$

The chordal rise while using Eq. (19.23) gives

$$\Delta r = r - r_c = r\left(1-\cos\theta_r\right) = r\left[1-\cos\left(\frac{180}{N}\right)\right], \tag{19.24}$$

where N is the number of teeth in the sprocket. Note also from triangle 0CA

$$\sin\theta_r = \frac{p_t/2}{r} \qquad \text{or} \qquad p_t = 2r\sin\theta_r = D\sin\theta_r. \tag{19.25}$$

19.6.4 Chain Length

The number of links is

$$\frac{L}{p_t} = \frac{2c_d}{p_t} + \frac{N_1+N_2}{2} + \frac{(N_2-N_1)^2}{4\pi^2\left(\dfrac{c_d}{p_t}\right)}, \tag{19.26}$$

where c_d is the center distance between sprockets. It is normally recommended that c_d/p_t lie between 30 and 50 pitches. If the center distance per pitch is not given, the designer can fix c_d/p_t and calculate L/p_t from Eq. (19.26). The next larger even integer L/p_t should be chosen. With L/p_t as an integer, the center distance per pitch becomes

$$\frac{c_d}{p_t} = A + \sqrt{A^2 - \frac{B^2}{2}}, \tag{19.27}$$

where

$$A = \frac{1}{4}\left(\frac{L}{p_t} - \frac{N_1+N_2}{2}\right), \tag{19.28}$$

$$B = \frac{N_2-N_1}{2\pi}. \tag{19.29}$$

The value of c_d/p_t obtained from Eq. (19.27) should be decreased by about 1% to provide slack in the nondriving chain strand.

The chain velocity is

$$u = N_a p_t N, \tag{19.30}$$

where

N_a = angular speed, rev/min
p_t = chain pitch, m
N = number of teeth in the sprocket

Table 19.10: Service factor, a_1, for rolling chains.

Type of driven load	Type of input power: Internal combustion engine with hydraulic drive	Electric motor or turbine	Internal combustion engine with mechanical drive
Smooth	1.0	1.0	1.2
Moderate shock	1.2	1.3	1.4
Heavy shock	1.4	1.5	1.7

Table 19.11: Multiple-strand factor for rolling chains.

Number of strands	Multiple-strand factor, a_2
1	1.0
2	1.7
3	2.5
4	3.3
5	3.9
6	4.6

19.6.5 Power Rating

The required power rating of a chain is given by

$$h_{pr} = h_p \frac{a_1}{a_2}, \tag{19.31}$$

where h_p is the power that is transmitted or input, a_1 is the service factor obtained from Table 19.10 and a_2 is the strand factor obtained from Table 19.11.

Rolling chains are available in a wide variety of sizes; Fig. 19.14 and Design Procedure 19.2 provide some guidelines for selection of rolling chains. Note that Fig. 19.14 has four scales, depending on the number of strands in the chain (see Fig. 19.12). For each type of roller chain, there is a characteristic shape to the power curve, where the power rating increases up until a certain sprocket size, and then decreases. This is attributed to a change in failure mode; at low speeds, link plate fatigue will be the dominant failure mode, while at higher speeds, wear of the pins becomes dominant.

Table 19.12 provides power ratings for selected standard-sized roller chains. Similar data can be readily obtained for other chain configurations from manufacturers' literature and web sites. The four types of lubrication given in Table 19.12 are

- Type A – Manual or drip lubrication, oil applied periodically with brush or spout can or applied between the link plate edges with a drop lubricator.
- Type B – Oil bath or oil slinger, where the oil level is maintained in a casing at a predetermined height, and the chain is immersed into the bath during at least a part of its travel.
- Type C – Oil stream lubrication, where oil is supplied by a circulating pump inside the chain loop or lower span.

Example 19.5: Power and Force in Rolling Chains

Given: A four-strand, ANSI No. 25 rolling chain transmits power from a 25-tooth driving sprocket that turns at 900 rpm. The speed ratio is 4:1.

Find: Determine the following:

(*a*) Power that can be transmitted for this drive

(*b*) Tension in the chain

(*c*) Chain length if center distance is about 250 mm.

Solution:

(*a*) From Table 19.12 for a smaller sprocket of 25 teeth and a speed of 900 rpm, the power rating is 0.9 kW and bath lubrication is required. From Table 19.11 for four strands, a_2 = 3.3. Nothing is mentioned about the type of input power or drive load, so assume that a_1 = 1. From Eq. (19.31), the power that can be transmitted is

$$h_p = h_{pr}\frac{a_2}{a_1} = (0.9)\left(\frac{3.3}{1}\right) = 2.97 \approx 3.0 \text{ kW}.$$

(*b*) From Table 19.9 for No. 25 chain, the pitch is 6.35 mm. The velocity can be calculated from Eq. (19.30) as

$$u = N_a p_t N = (900)(0.00635)(25) = 142.9 \text{ m/min}$$

or $u = 2.38$ m/s. Therefore, the tension in the chain is

$$P = \frac{h_p}{u} = \frac{3000}{2.38} = 1260 \text{ N}.$$

(*c*) The number of teeth on the larger sprocket given a speed ratio of 4 is 4(25) = 100 teeth. Note that for a center distance of 250 mm,

$$\frac{c_d}{p_t} = \frac{250}{6.35} = 39.4,$$

which is between the 30 and 50 pitches recommended. From Eq. (19.26),

$$\begin{aligned}\frac{L}{p_t} &= 2\left(\frac{c_d}{p_t}\right) + \frac{N_1+N_2}{2} + \frac{(N_2-N_1)^2}{4\pi^2\left(\frac{c_d}{p_t}\right)} \\ &= 78.8 + \frac{25+100}{2} + \frac{(100-25)^2}{4\pi^2(39.4)} = 144.9.\end{aligned}$$

Since it is good practice to specify the next largest even number of links, L is chosen as 146 pitches, or 927 mm.

19.6.6 Silent Chain

An interesting type of chain is the **inverted tooth** or **silent chain**, shown in Fig. 19.15, and made up of stacked rows of flat, tooth shaped links that mesh with sprockets having compatible tooth spaces, much the way gear teeth mesh (see Fig. 14.8). Because of the geometry advantages, these chains are much quieter than conventional roller chains, especially at high speeds, but they are somewhat more expensive. Washers or spacers may be present in some chain con-

Table 19.12: Power rating of selected standard roller chains, in kW.

ANSI Chain No.	No. of Teeth[a]	Maximum speed of small sprocket (rpm) 50	100	300	500	900	1500	2100	3000	4000	5000	6000
25	11	0.02	0.04	0.14	0.22	0.40	0.65	0.90	1.26	1.03	0.74	0.56
	12	0.03	0.05	0.15	0.25	0.43	0.71	0.98	1.37	1.17	0.84	0.64
	15	0.04	0.07	0.19	0.31	0.54	0.88	1.22	1.72	1.64	1.17	0.90
	18	0.04	0.08	0.22	0.37	0.65	1.06	1.46	2.06	2.16	1.54	1.17
	20	0.04	0.09	0.25	0.41	0.72	1.18	1.63	2.29	2.52	1.81	1.37
	22	0.05	0.10	0.28	0.45	0.79	1.29	1.79	2.51	2.91	2.08	1.58
	25	0.06	0.11	0.31	0.51	0.90	1.47	2.03	2.86	3.53	2.52	1.92
	28	0.06	0.12	0.35	0.57	1.01	1.65	2.28	3.21	4.18	2.99	2.27
	30	0.07	0.13	0.37	0.61	1.08	1.77	2.44	3.43	4.53	3.32	2.52
	35	0.08	0.16	0.44	0.72	1.26	2.06	2.84	4.01	5.28	4.18	3.18
	40	0.09	0.17	0.50	0.82	1.44	2.35	3.25	4.58	6.03	5.11	3.66
	45	0.10	0.19	0.57	0.92	1.62	2.65	3.66	5.15	6.79	6.09	1.03
35	11	0.08	0.16	0.46	0.76	1.34	2.19	3.02	2.19	1.42	1.02	0.78
	12	0.09	0.18	0.51	0.83	1.46	2.39	3.30	2.50	1.62	1.16	0.88
	15	0.11	0.22	0.63	1.04	1.83	2.98	4.12	3.49	2.27	1.62	1.23
	18	0.13	0.27	0.76	1.25	2.19	3.58	4.95	4.59	2.98	2.13	1.62
	20	0.15	0.30	0.85	1.39	2.43	3.98	5.49	5.37	3.49	2.50	1.90
	22	0.16	0.33	0.93	1.52	2.68	4.37	6.04	6.20	4.03	2.88	2.19
	25	0.19	0.37	1.06	1.73	3.04	4.97	6.86	7.51	4.88	3.49	2.66
	28	0.22	0.41	1.18	1.94	3.41	5.57	7.69	8.90	5.78	4.14	–
	30	0.23	0.44	1.27	2.08	3.66	5.97	8.24	9.87	6.41	4.59	–
	35	0.27	0.51	1.48	2.42	4.26	6.96	9.62	12.44	8.08	0.25	–
	40	0.31	0.59	1.73	2.77	4.87	7.95	10.99	15.20	8.24	–	–
	45	0.34	0.66	1.91	3.12	5.48	8.94	12.36	17.40	2.32	–	–
50	11	0.42	0.81	2.34	3.83	6.73	6.23	3.76	2.20	1.43	1.02	0.78
	12	0.52	1.02	2.93	4.78	8.41	8.71	5.26	3.08	2.00	1.43	–
	15	0.63	1.22	3.51	5.74	10.09	11.44	6.91	4.04	2.63	0.04	–
	18	0.70	1.36	3.91	6.38	11.22	13.41	8.09	4.74	3.08	–	–
	20	0.77	1.50	4.30	7.02	12.34	15.46	9.34	5.47	3.55	–	–
	22	0.87	1.70	4.89	7.97	14.02	18.73	11.31	6.62	–	–	–
	25	0.98	1.90	5.47	8.93	15.70	22.21	13.41	7.85	–	–	–
	28	1.05	2.04	5.86	9.57	16.82	24.63	14.87	8.71	–	–	–
	30	1.22	2.38	6.83	11.17	19.63	31.03	18.73	0.70	–	–	–
	35	1.40	2.72	7.81	12.76	22.43	36.64	22.89	–	–	–	–
	40	1.57	3.06	8.79	14.35	25.24	41.21	27.31	–	–	–	–
80	11	1.54	3.01	8.62	14.08	17.14	8.03	4.87	2.81	1.83	–	–
	12	1.69	3.27	9.41	15.36	19.52	9.15	5.56	3.21	2.08	–	–
	15	2.10	4.10	11.76	19.20	27.29	12.79	7.77	4.48	–	–	–
	18	2.53	4.92	14.11	23.04	35.87	16.81	10.21	5.89	–	–	–
	20	2.80	5.46	15.67	25.60	42.01	19.69	11.96	–	–	–	–
	22	3.09	6.01	17.25	28.16	48.47	22.72	13.79	–	–	–	–
	25	3.51	6.83	19.60	32.00	56.26	27.51	15.40	–	–	–	–
	28	3.93	7.65	21.95	35.84	63.01	32.62	–	–	–	–	–
	30	4.13	8.18	23.51	38.40	67.51	36.17	–	–	–	–	–
	35	4.91	9.56	27.44	44.80	78.76	45.58	–	–	–	–	–
100	11	2.95	5.75	16.52	26.97	20.49	9.60	6.18	3.36	–	–	–
	12	3.22	6.27	18.02	29.42	23.34	10.94	7.05	3.83	–	–	–
	15	4.04	7.84	22.52	36.78	32.62	15.29	9.85	–	–	–	–
	18	4.84	9.41	27.03	44.13	42.88	20.10	12.94	–	–	–	–
	20	5.38	10.46	30.03	49.03	50.22	23.54	15.16	–	–	–	–
	22	5.92	11.50	33.03	53.94	57.94	27.15	17.49	–	–	–	–
	25	6.72	13.07	37.53	61.29	70.19	32.89	–	–	–	–	–
	28	7.53	14.64	42.04	68.65	83.18	38.99	–	–	–	–	–
	30	8.06	15.69	45.04	73.55	92.28	42.20	–	–	–	–	–
	35	9.41	18.30	52.55	85.79	116.30	–	–	–	–	–	–
160	11	11.20	21.81	62.60	72.05	29.83	13.98	9.00	–	–	–	–
	12	12.23	23.78	68.29	82.06	34.00	15.93	10.26	–	–	–	–
	15	15.29	29.74	85.34	114.73	47.51	22.27	–	–	–	–	–
	18	18.34	35.68	102.43	150.84	62.46	–	–	–	–	–	–
	20	20.38	39.64	113.84	176.65	73.15	–	–	–	–	–	–
	22	22.42	43.60	125.18	203.81	84.37	–	–	–	–	–	–
	25	25.48	49.56	142.26	232.30	102.20	–	–	–	–	–	–
	28	28.53	55.50	159.35	260.20	121.15	–	–	–	–	–	–
	30	30.57	59.46	170.76	278.78	134.35	–	–	–	–	–	–
	35	35.67	69.38	199.18	325.26	84.00	–	–	–	–	–	–
200	11	20.54	39.97	114.76	86.13	35.67	16.71	–	–	–	–	–
	12	22.42	43.60	125.19	98.14	40.64	19.04	–	–	–	–	–
	15	28.02	54.51	156.50	137.16	56.79	–	–	–	–	–	–
	20	37.36	72.68	208.66	211.17	87.44	–	–	–	–	–	–
	25	46.70	90.85	260.97	295.12	122.20	–	–	–	–	–	–

[a] The use offewer than 17 teeth is possible, but should be avoided when practical.

Required Lubrication:

- Manual/drip
- Bath or disc
- Oil stream

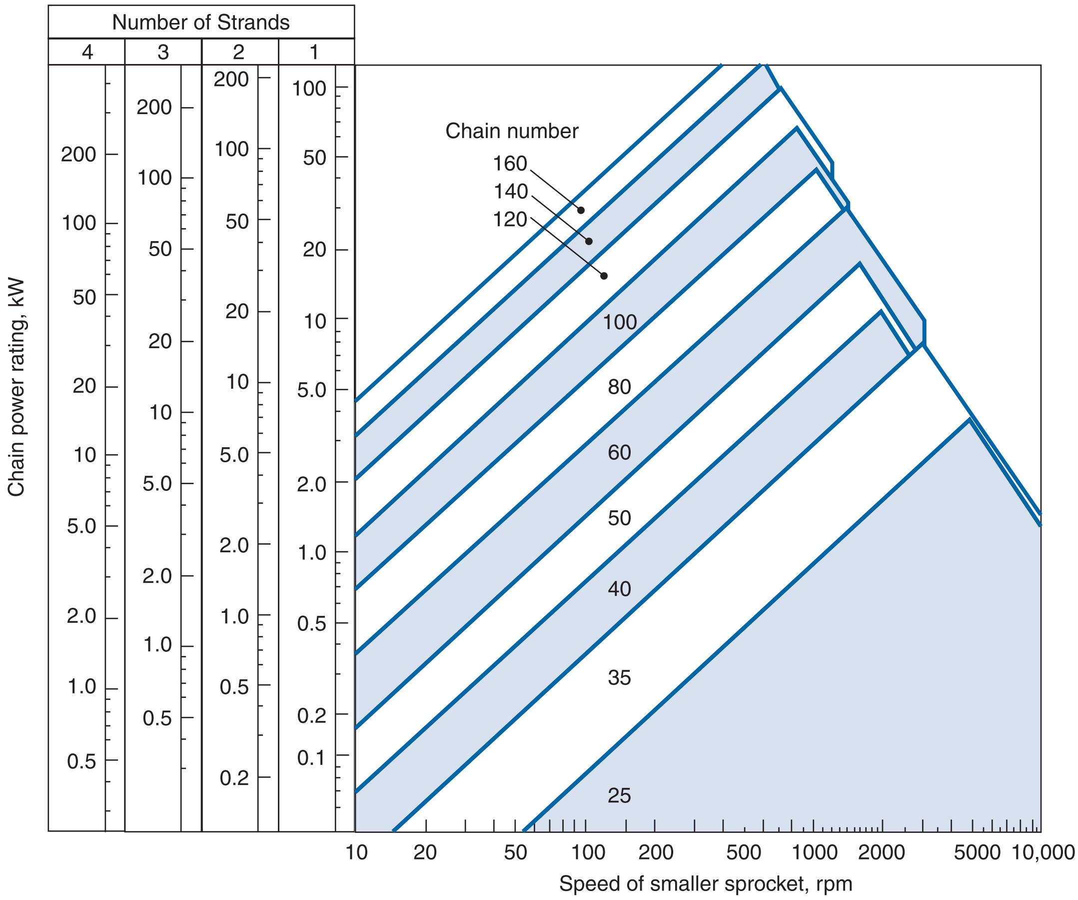

Figure 19.14: Design guideline for standard roller chains.

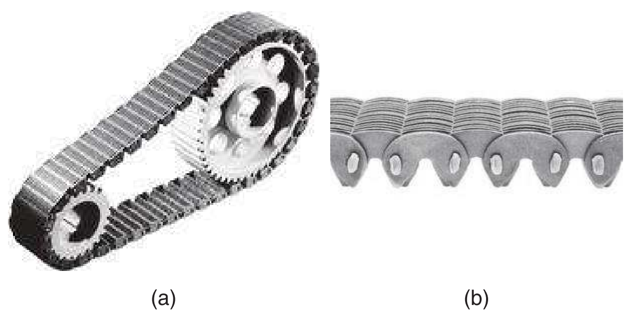

Figure 19.15: A silent chain drive. (a) Silent chain with sprockets; (b) detail of silent chain links. *Source:* Courtesy of Ramsey Products Corp.

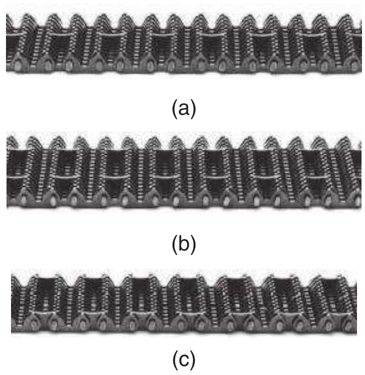

Figure 19.16: The use of guide links in silent chains. (a) One guide link in center of chain; (b) two center guide links; (c) two side guide links. *Source:* Courtesy of Ramsey Products Corp.

structions, especially for conveyor applications. All of these components are held together by riveted pins located in each chain joint. Although all silent chains have these basic features, there are still many different styles, designs, and configurations. The width of a silent chain can be varied according to horsepower constraints. Silent chains are available for power transmission, as considered in this section, but they can also be modified to incorporate links that provide a stable and wear resistant surface for conveyor applications.

Guide links are used to maintain tracking on sprockets, as shown in Fig. 19.16. The guide links may be located within the chain (center guide) or along the outer edges of the chain (side guide). A chain with two rows of guide links within the chain is referred to as *two center guide*. In many applications, any of these guide types will be satisfactory but it is essential that sprockets be selected with the same guide type as the chain.

Most silent chains have teeth that engage sprockets on only one side of the chain. However, duplex silent chain designs are available that have teeth on both sides and are designed for use in serpentine drives, where sprockets are driven from both sides of the chain.

Silent chains are able to transfer higher powers at higher speeds than roller chains. Essentially, the width of the chain can be increased to accommodate any power. Because they do not have a chordal rise, there is less vibration and longer sprocket life as well. However, the main advantage of silent chains is that they are much quieter than conventional rolling chains.

Silent chains can be made in large widths and used as conveyor belts. In such cases, it is common to use a spacer between links to produce a more open belt.

Design Procedure 19.2: Design of Chain Drives

This Design Procedure presents a method for designing power transmission devices consisting of chains and sprockets. While the approach is intended for roller chains, the Design Procedure can be used for silent chains, with caveats as noted. It will be assumed that operating conditions and design constraints are adequately described. For the purposes of this Design Procedure, the power transmitted (or chain force and speed), power source, speed ratio, and loading environment need to be known, or at least be somewhat constrained.

1. Obtain the service factor from Table 19.10. Calculate the chain's required power rating from Eq. (19.31), taking $a_2 = 1.0$.
2. Select a chain size from Fig. 19.14 using the required power rating and the small sprocket speed. Note that using the fewest number of chain strands while satisfying power requirements usually results in the most economical design.
3. Obtain the strand factor, a_2, from Table 19.11.
4. The required power rating, given by Eq. (19.31), needs to be recalculated if a multiple strand chain is to be used.
5. Referring to Table 19.12, identify the column of the table that corresponds to the small sprocket's speed. Reading down from the top, find the number of teeth in the smaller sprocket that produces the required modified power rating, h'_{pr}. This is the minimum number of teeth that are required for the application. Larger sprockets can be used if desired.
6. If a multiple strand chain is being considered, record the modified power rating from Table 19.12, and multiply by the strand factor to obtain the chain's power rating.
7. Note the required lubrication method in Table 19.12. Variation from the lubrication approach may compromise chain longevity.
8. The number of teeth on the larger sprocket can be calculated from the desired velocity ratio by using Eq. (19.22).
9. If the center distance has not been prescribed, it can be estimated by recognizing that c_d/p_t should be between 30 and 50, although larger lengths can be allowed if chain guides are incorporated into the design. If the center distance exceeds space limitations, increase the number of strands or select the next largest pitch chain and return to Step 4.
10. The number of links in the chain can be calculated from Eq. (19.26), rounded up to the next highest even integer.

Example 19.6: Chain Drive Design

Given: A chain will transmit power from a 7.5-kW motor to a drive roller on a belt conveyor. The motor is connected to a speed reducer so that the input speed is 100 rpm, and the desired conveyor drive roller speed is 25 rpm. The conveyor has some starts and stops, so it should be considered to have moderate shock. The center distance of the shafts should be around 1.25 m.

Find: Select a roller chain and sprockets for this application.

Solution: This solution will closely follow Design Procedure 19.2.

1. Referring to Table 19.11, note that for an electric motor with a driven load with moderate shock, the service factor is $a_1 = 1.3$. The required power rating, assuming $a_2 = 1$ is obtained from Eq. (19.31) as

$$h_{pr} = h_p \frac{a_1}{a_2} = (7.5)(1.3) = 9.75 \text{ kW}.$$

2. Noting that the smaller sprocket will have a speed of 100 rpm, Fig. 19.14 suggests that a single strand of standard No. 100 chain can be used for this application. Note that a No. 120 chain may also lead to a reasonable design, and may be worth investigating, but this solution will continue with a No. 100 chain.
3. From Table 19.11, the strand factor is $a_2 = 1.0$.
4. A single strand chain is being analyzed, so that the power rating of 9.75 kW can still be used. Note that the power rating would need to be recalculated if a multiple strand chain was selected.
5. Referring to Table 19.12, for a No. 100 chain with a small sprocket speed of 100 rpm, 20 teeth are required in order to exceed the modified power rating of 9.75 kW.
6. Table 19.12 also indicates that a bath-type of lubrication is required, and will need to be incorporated into the chain system.
7. From Eq. (19.22), the number of teeth on the larger sprocket can be obtained as:

$$g_r = \frac{\omega_1}{\omega_2} = \frac{N_2}{N_1},$$

$$\frac{100}{25} = \frac{N_2}{20},$$

or $N_2 = 80$.

8. Note from Table 19.9 that the pitch of No. 100 chain is 31.75 mm, so that $\frac{c_d}{p_t} = \frac{1.25}{0.03175} = 39.3$. This is within the standard range, so no further modification (changing chain size, using more strands, etc.) is required.
9. From Eq. (19.26),

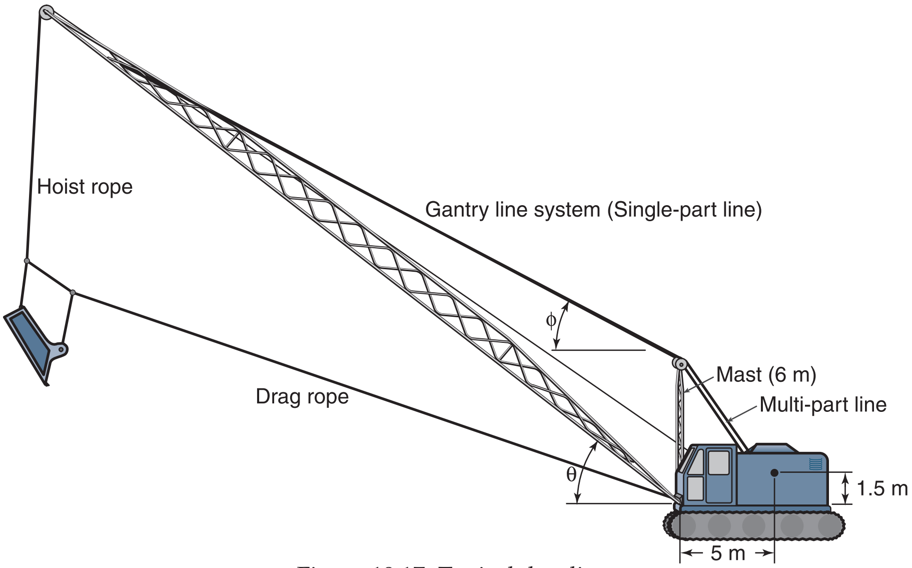

Figure 19.17: Typical dragline.

$$\frac{L}{p_t} = \frac{2c_d}{p_t} + \frac{N_1+N_2}{2} + \frac{(N_2-N_1)^2}{4\pi^2\left(\frac{c_d}{p_t}\right)}$$

$$= \frac{2(1250)}{31.75} + \frac{20+80}{2} + \frac{(80-20)^2}{4\pi^2\left(\frac{1250}{31.75}\right)} = 131.$$

or $L = 4.16$ m. Since it is good practice to specify an even integer of links, a value of $L/p_t = 132$ is selected. This would result in an actual center distance of $c_d = 1.32$ m as obtained from Eq. (19.27). Note that the center distance should be reduced by 1% to provide some slack, so that an actual center distance of around 1.3 m should be used.

Case Study 19.1: Design of a Gantry for a Dragline

A dragline, often used for mining or dredging operations, uses a large bucket to remove material and transport it elsewhere, usually into a trailer or a train boxcar. The gantry line system is the portion of the dragline that supports the boom and fixes the boom angle, as shown in Fig. 19.17. The boom typically weighs over 40 kN, is around 30 m long, and can be considered to have its center of gravity at its geometric center. The dragline tips if the moment from the hoist rope and the boom exceeds the moment from the dragline about the tipping point. Tipovers are to be avoided, of course, but it should be recognized that the load applied to the gantry cable cannot be larger than that resulting from a tipover condition.

Figure 19.17 shows the dimensions of a typical dragline. The gantry line lengths shown were selected so that the 6-m mast is vertical at a boom angle of 30°. Since draglines have booms, many people confuse them with cranes. There are significant differences, however. Dragline load is limited by the size of the bucket, whereas a crane can attempt to lift extremely large loads. Further, a dragline will operate at a set boom angle for extended periods of time and will rarely, if ever, operate at very high or very low boom angles. Therefore, the range of boom angles that must be achieved is somewhat limited.

Just as with cranes, however, the gantry line system serves to eliminate bending forces on the boom. Thus, the gantry line must attach at the tip of the boom, known as the *boom point*. Attaching the gantry line to the crane's superstructure near the boom pivot point will result in extremely large forces, as can be seen from moment equilibrium. A mast is used to offset the gantry line system and obtain reasonably low forces in the gantry line.

A number of clever design features have been incorporated into draglines in the past (Fig. 19.17). The lines closest to the crane superstructure have multiple pulleys (and hence are called a *multiple-part line*) to share the load and reduce the stress on the cable. Above the multiple-part line is a single line attached by proper couplings to the boom. Given the operating characteristics of a dragline, this single line is never wound over a pulley and its size is determined by the rated cable strength and does not have to be reduced for fatigue effects (see Fig. 19.10).

Because the mast rotates as the boom angle changes, the load supported by the gantry lines will change with the boom angle. The gantry line tension is highest at a boom angle of 0°. Draglines are not operated at such low boom angles because the lifting capacity is severely limited, but analyzing a boom angle of 0° is important. Draglines are shipped in multiple parts and assembled at the construction site, so that the boom needs to be lifted, at least for the first time that the dragline is placed into service.

A number of equalizer pulleys (a multiple-part line) are used in Fig. 19.17 so that the load in the gantry line is reduced. The load in the gantry line depends on the number of pulleys or parts in the line, each of which supports an equal share of the load. The earlier discussion (Section 19.5.2) regarding sheave sizing is relevant. If the pulley and drum are too large, the motor capacity will be excessive. Per industry standard requirements, pulley and drum diameters of $12d$ are to be chosen. These diameters reduce gantry line strength by approximately 12% (Fig. 19.9), but a maintenance procedure of checking for broken wires in the wire rope must still be followed. This leads to a very safe system, since draglines rarely change their boom angle, so the gantry line does not run over pulleys often.

19.7 Summary

Belts and ropes are machine elements (like brakes and clutches) that use friction as a useful agent, in contrast to other machine elements in which friction is to be kept as low as possible. Belts, ropes, and chains provide a convenient means for transferring power from one shaft to another. This chapter discussed flat, synchronous, and V-belts. All flat belts are subject to slip, that is, relative motion between the pulley surface and the adjacent belt. For this reason, flat belts must be kept under tension to function and require tensioning devices. One major advantage of V-belts over flat belts is that the wedging action of V-belts increases the normal force from dN to $dN/\sin\beta$, where β is the sheave angle. V-belts also can transfer much higher power than a similarly sized flat belt.

Wire ropes are used when power must be transmitted over very long center distances. Wire rope is widely used in hoisting applications including cranes and elevators. The major advantage of using rolling chains over flat or V-belts is that rolling chains do not slip. Another advantage of rolling chains over belt drives is that no initial tension is necessary and thus the shaft loads are smaller. The required length, power rating, and modes of failure were considered for belts, ropes, and rolling chains.

Key Words

core center of wire rope, mainly intended to support outer strands

flat belts power transmission device that consists of loop of rectangular cross section placed under tension between pulleys

inverted tooth chain see *silent chain*

lay type of twist in wire rope (regular or Lang); distance for strand to revolve around wire rope

rolling chains power transmission device using rollers and links to form continuous loop, used with sprockets

sheaves grooved pulleys that V-belts run in

silent chain a chain that consists of specially formed links that mesh with a gear-shaped sprocket

strands grouping of wire used to construct wire rope

synchronous belt flat belt with series of evenly spaced teeth on inside circumference, intended to eliminate slip and creep

timing belt same as *synchronous belt*

V-belt power transmission device with trapezoidal cross section placed under tension between grooved sheaves

wire rope wound collection of strands

Summary of Equations

Belts:

Belt length:

$$\begin{aligned} L &= \sqrt{(2c_d)^2 - (D_2 - D_1)^2} \\ &+ \frac{\pi}{2}(D_1 + D_2) \\ &+ \frac{\pi(D_2 - D_1)}{180} \sin^{-1}\left(\frac{D_2 - D_1}{2c_d}\right) \end{aligned}$$

Force: $\dfrac{F_1}{F_2} = e^{\mu\phi\pi/180^\circ}$

Including centrifugal force: $\dfrac{F_1 - F_c}{F_2 - F_c} = e^{\mu\phi\pi/180^\circ}$

Torque: $T = \dfrac{(F_1 - F_2)D_1}{2}$

Velocity ratio: $g_r = \dfrac{N_{a1}}{N_{a2}} = \dfrac{r_2}{r_1}$

Design power rating: $h_{pr} = f_1 h_p$

Rated power per belt: $h_{pb} = f_2 h_1$

Number of belts required: $N = \dfrac{h_{pr}}{h_{pb}}$

Wire Rope:

Total force: $F_t = F_w + F_r + F_a + F_h$

Wire stress: $\sigma = \dfrac{Ed_w}{D}$

Bearing pressure: $p = \dfrac{2F_t}{dD}$

Fatigue: $F_f = \dfrac{S_u dD}{2000}$

Rolling Chains:

Velocity ratio: $g_r = \dfrac{d_2}{d_1} = \dfrac{\omega_1}{\omega_2} = \dfrac{N_2}{N_1}$

Chordal rise: $\Delta r = r\left[1 - \cos\left(\dfrac{180}{N}\right)\right]$

Number of links: $\dfrac{L}{p_t} = \dfrac{2c_d}{p_t} + \dfrac{N_1 + N_2}{2} + \dfrac{(N_2 - N_1)^2}{4\pi^2\left(\dfrac{c_d}{p_t}\right)}$

Center distance: $\dfrac{c_d}{p_t} = A + \sqrt{A^2 - \dfrac{B^2}{2}}$

$A = \dfrac{1}{4}\left(\dfrac{L}{p_t} - \dfrac{N_1 + N_2}{2}\right)$

$B = \dfrac{N_2 - N_1}{2\pi}$

Power rating: $h_{pr} = h_p \dfrac{a_1}{a_2}$

Recommended Readings

American Chain Association, *Chains for Power Transmission and Material Handling*, Marcel Dekker.

Budynas, R.G., and Nisbett, J.K. (2011), *Shigley's Mechanical Engineering Design*, 9th ed., McGraw-Hill.

Dickie, D.E. (1985) *Rigging Manual*, Construction Safety Association of Ontario.

Juvinall, R.C., and Marshek, K.M. (2012) *Fundamentals of Machine Component Design*, 5th ed., Wiley.

Krutz, G.W., Schuelle, J.K., and Claar, P.W. (1994) *Machine Design for Mobile and Industrial Applications*, Society of Automotive Engineers.

Mott, R. L. (2014) *Machine Elements in Mechanical Design*, 5th ed., Pearson.

Rossnagel, W.E., Higgins, L.R., and MacDonald, J.A. (1988) *Handbook of Rigging*, 4th ed., McGraw-Hill.

Shapiro, H.I. et al. (1991) *Cranes and Derricks*, 2nd ed., McGraw-Hill.

Wire Rope Users Manual (1972) Armco Steel Corp.

References

ASME (2007) B30.5 *Mobile and Locomotive Cranes*, American Society of Mechanical Engineers.

Shigley, J.E., and Mitchell, L.D. (1983), *Mechanical Engineering Design*, 4th ed., McGraw-Hill.