15

Power Screws, Fasteners, and Connections

15.1 Introduction

This chapter is devoted to the analysis and design of power screws, threaded fasteners, bolted joints in shear, and permanent connectors such as rivets and weldments. Adhesive bonding, brazing, and soldering are also discussed briefly. Power screws are threaded devices used mainly to move loads or accurately position objects. They are employed in machines for obtaining motion of translation and also for exerting forces. The kinematics of power screws is the same as that for nuts and screws, the only difference being the geometry of the threads. Power screws find applications as motion devices.

The success or failure of a design can depend on proper selection and use of its fasteners. A fastener is a device to connect or join two or more members. Many varieties of fasteners are available commercially. The threaded fasteners are used to fasten the various parts of an assembly together. We limit our consideration to detachable threaded fasteners such as bolts, nuts, and screws (Figure 15.1). General information for threaded fasteners as well as for other methods of joining is presented in some references listed at the end of this chapter and at the websites www.americanfastener.com and www.machinedesign.com. Listings of a variety of nuts, bolts, and washers are found at www.nutty.com. For bolted joint technology, see the website at www.boltscience.com.

Analysis of riveted, welded, and bonded connections cannot be made on as rigorous a basis as used for most structural and machine members. Their design is largely empirical and relies on available experimental results. As with the threaded fasteners, rivets exist in great variety. Note that while welding has replaced riveting and bonding to a considerable extent, rivets are customarily employed for certain types of joints. Often, rivets are used in joining smaller components in products associated with the automotive, business machines, appliances, and other fields. Welding speeds the manufacturing of parts and assembly of these components into structures and reduces the cost compared to casting and forging. Soldering, brazing, cementing, and adhesives are all means of bonding parts together. Other popular fastening and joining methods [20] include snap fasteners, which greatly simplify assembly of mechanical components.

15.2 Standard Thread Forms

Threads may be external on the screw or bolt and internal on the nut or threaded hole. The thread causes a screw to proceed into the nut when rotated. The basic arrangement of a helical thread cut around a cylinder or a hole, used as screw-type fasteners, power screws,

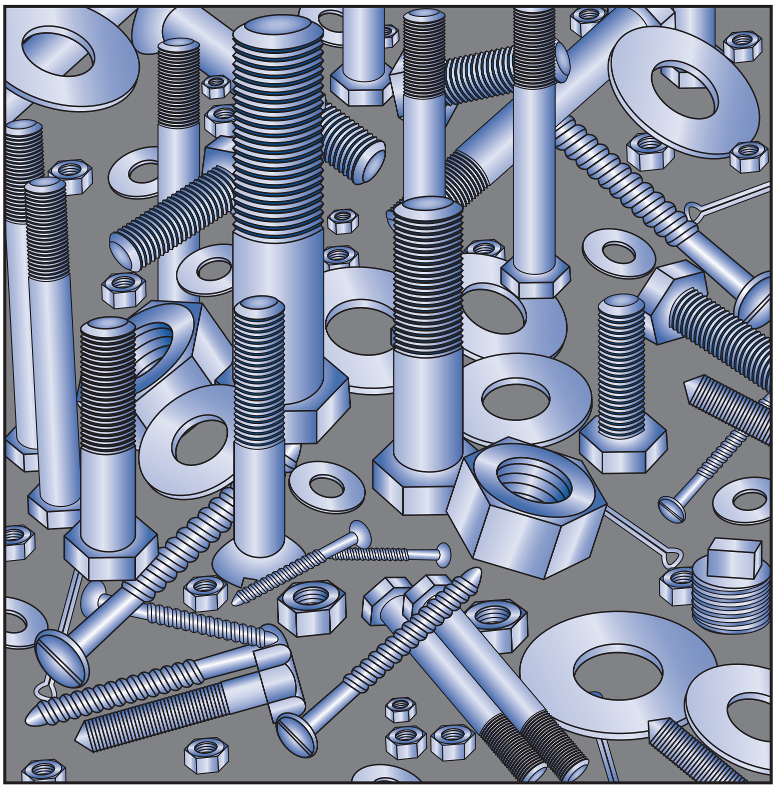


FIGURE 15.1
An assortment of threaded fasteners. (Courtesy of Clark Craft Fasteners.)

and worms, is as shown in Figure 15.2. Note that the length of unthreaded and threaded portions of shank is called the *shank* or bolt length. Also, observe the washer face, the fillet under the bolt head, and the start of the threads. Referring to the figure, some terms from geometry that relate to screw threads are defined as follows.

Pitch p is the axial distance measured from a point on one thread to the corresponding point on the adjacent thread. *Lead L* represents the axial distance that a nut moves, or advances, for one revolution of the screw. *Helix angle*, λ, also called the lead angle, may be

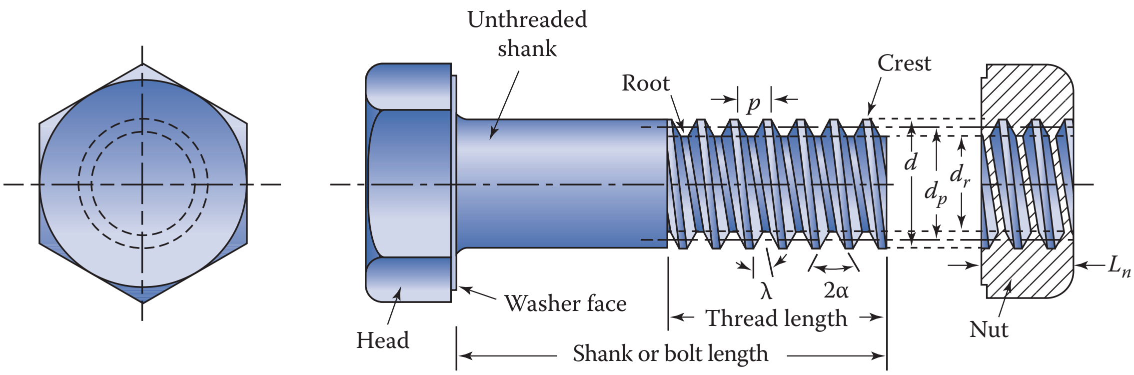


FIGURE 15.2
Hexagonal bolt and nut illustrate the terminology of threaded fasteners. *Notes*: *P*, the pitch; λ, the helix or lead angle; α, the thread angle; d, the major diameter; d_p, the pitch diameter; d_r, the root diameter; and L_n, the nut length.

cut either right handed (as in Figure 15.2) or left handed. All threads are assumed to be right handed, unless otherwise stated.

A single-threaded screw is made by cutting a single helical groove on the cylinder. For a *single thread*, the lead is the same as the pitch. Should a second thread be cut in the space between the grooves of the first (imagine two strings wound side by side around a pencil), a double-threaded screw would be formed. For a *multiple* (two or most)-*threaded screw*,

$$L = np \tag{15.1}$$

where

L = the lead
n = the number of threads
p = the pitch

We observe from this relationship that a multiple-threaded screw advances a nut more rapidly than a single-threaded screw of the same pitch. Most bolts and screws have a single thread, but worms and power screws sometimes have multiple threads. Some automotive power-steering screws occasionally use quintuple threads.

15.2.1 Unified and ISO Thread Form

For fasteners, the standard geometry of screw thread shown in Figure 15.3 is used. This is essentially the same for both the Unified National Standard (UNS), or so-called unified, and International Standards Organization (ISO) threads. The UNS (inch series) and ISO (metric series) threads are not interchangeable. In both systems, the *thread angle* is 60, and the crests and roots of the thread may be either flat (as depicted in the figure) or rounded. The major diameter d and root (minor) diameter d_r refer to the largest and smallest diameters, respectively. The diameter of an imaginary cylinder, coaxial with the screw, intersecting the thread at the height that makes width of thread equal to the width of space, is called the *pitch diameter* d_p.

Tables 15.1 and 15.2 furnish a summary of the various sizes and pitches for the UNS and ISO systems. We see from these listings that the thread size is specified by giving the number of threads per inch N for the unified sizes and giving the pitch p for the metric sizes. The tensile *stress area* tabulated is on the basis of the average of the pitch and root

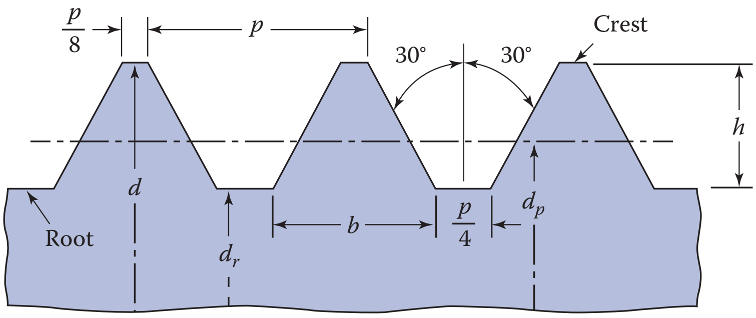


FIGURE 15.3
Unified and ISO thread forms. The portion of basic profile of the external thread is shown: h is the depth of thread and b is the thread thickness at the root.

TABLE 15.1

Dimensions of Unified Screw Threads

	Course Threads—UNC				Fine Threads—UNF		
Size	Major Diameter, d (in.)	Threads per Inch, $N = 1/p$	Minor Diameter d_r (in.)	Tensile Stress Area, A_t, (in.²)	Threads per Inch, $N = 1/p$	Minor Diameter, d_r (in.)	Tensile Stress Area, A_t (in.²)
1	0.073	64	0.0538	0.00263	72	0.0560	0.00278
2	0.086	56	0.0641	0.00370	64	0.0668	0.00394
3	0.099	48	0.0734	0.00487	56	0.0771	0.00573
4	0.112	40	0.0813	0.00604	48	0.0864	0.00661
5	0.125	40	0.0943	0.00796	44	0.0971	0.00830
6	0.138	32	0.0997	0.00909	40	0.1073	0.01015
8	0.164	32	0.1257	0.0140	36	0.1299	0.01474
10	0.190	24	0.1389	0.0175	32	0.1517	0.0200
12	0.216	24	0.1649	0.0242	28	0.1722	0.0258
1/4	0.250	20	0.1887	0.0318	28	0.2062	0.0364
3/8	0.375	16	0.2983	0.0775	24	0.3239	0.0878
1/2	0.500	13	0.4056	0.1419	20	0.4387	0.1599
5/8	0.625	11	0.5135	0.226	18	0.5368	0.256
3/4	0.750	10	0.6273	0.334	16	0.6733	0.373
7/8	0.875	9	0.7387	0.462	14	0.7874	0.509
1	1.000	8	0.8466	0.606	12	0.8978	0.663

Source: *ANSI/ASME Standards*, B1.1–2014, B1.13–2005, New York, American Standards Institute, 2005.
Note: The pitch or mean diameter $d_m \approx d - 0.65p$.

diameters. This is the area used for calculation of axial stress (P/A). Extensive information for various inch-series threads may be found in ANSI Standards [1,2].

Coarse thread (designated as UNC) is most common and recommended for ordinary applications, where the screw is threaded into a softer material. It is used for general assembly work. *Fine thread* (denoted by UNF) is more resistant to loosening, because of its smaller helix angle. Fine threads are widely employed in automotive, aircraft, and other applications where vibrations are likely to occur. In identifying threads, the letter A is used for external threads, and B is used for internal threads. The UNS defines the threads according to *fit*. Class 1 fits have the widest tolerances and so are the loosest fits. Class 2 fits are most commonly used. Class 3 fit is the one having the least tolerance and is utilized for the highest precision applications. Clearly, cost increases with higher class of fit. An example of approved identification symbols is as follows:

1 in.-12 UNF-2A-LH

This defines 1 in. diameter × 12 threads per inch, unified fine-thread series, class 2 fit, external, and left-handed thread. Metric thread specification is given in Table 15.2.

15.2.2 Power Screw Thread Forms

Figure 15.4 depicts some thread forms used for power screws. The *Acme screw* is in widespread usage. They are sometimes modified to a stub form by making the thread shorter. This results in a larger minor diameter and a slightly stronger screw. A *square thread*

TABLE 15.2

Basic Dimensions of ISO (Metric) Screw Threads

	Coarse Threads		Fine Threads	
Nominal Diameter, d (mm)	Pitch, p (mm)	Tensile Stress Area, A_t (mm²)	Pitch, p (mm)	Tensile Stress Area, A_t, (mm²)
2	0.4	2.07		
3	0.5	5.03		
4	0.7	8.78		
5	0.8	14.2		
6	1	20.1		
7	1	28.9		
8	1.25	36.6	1.25	39.2
10	1.5	58.0	1.25	61.2
12	1.75	84.3	1.25	92.1
14	2	115	1.5	125
16	2	157	1.5	167
18	2.5	192	1.5	216
20	2.5	245	1.5	272
24	3	353	2	384
30	3.5	561	2	621
36	4	817	2	915
42	4.5	1120	9	1260
48	5	1470	2	1670
56	5.5	2030	2	2300
64	6	2680	2	3030

Source: *ANSI/ASME Standards*, B1.1–2014, B1.13–2005, New York, American Standards Institute, 2005.
Notes: Metric threads are specified by nominal diameter and pitch in millimeters, for example, $M10 \times 1.5$. The letter M, which proceeds the diameter, is the clue to the metric designation; root or minor diameter $d_r \approx d - 1.227p$.

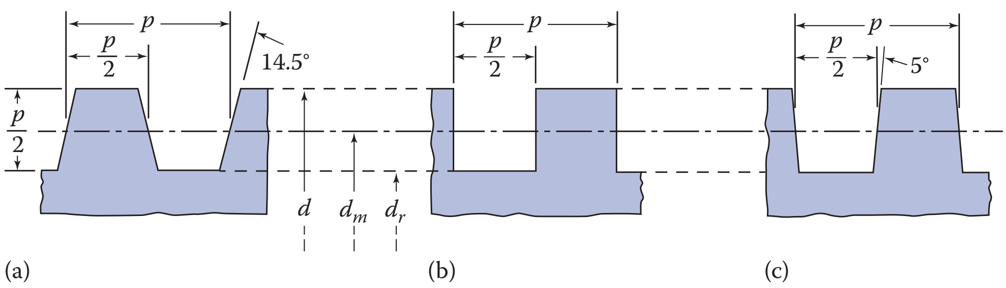


FIGURE 15.4
Typical power screw thread forms. All threads shown are external, $d_m = (d + d_r)/2$: (a) Acme, (b) square, and (c) modified square.

provides somewhat greater strength and efficiency but is rarely used, due to difficulties in manufacturing the 0° thread angle. The 5° thread angle of the *modified square thread* partially overcomes this and some other objections. Standard sizes for three power screw thread forms are listed in Table 15.3. The reader is referred to ANSI Standards for further details.

TABLE 15.3

Standard Sizes of Power Screw Threads

	Threads per Inch	
Major Diameter, *d* (in.)	**Acme, Acme Stub**	**Square and Modified Square**
$\frac{1}{4}$	16	10
$\frac{1}{2}$	10	$6\frac{1}{2}$
$\frac{5}{8}$	8	$5\frac{1}{2}$
$\frac{3}{4}$	6	5
$\frac{7}{8}$	6	$4\frac{1}{2}$
1	5	4
$1\frac{1}{4}$	5	$3\frac{1}{2}$
$1\frac{1}{2}$	4	2
$1\frac{3}{4}$	4	$2\frac{1}{2}$
2	4	$2\frac{1}{4}$
$2\frac{1}{4}$	3	$2\frac{1}{4}$
$2\frac{1}{2}$	3	2
$2\frac{3}{4}$	3	2
3	2	2
3	2	$1\frac{3}{4}$
4	2	$1\frac{1}{2}$
5	2	

Source: James. F.D. et al. eds., *Machinery's Handbook*, 23rd ed., Industrial Press, New York, 1974.

15.3 Mechanics of Power Screws

As noted previously, a power screw, sometimes called the *linear actuator* or *translation screw*, is in widespread usage in machinery to change angular motion into linear motion, to exert force, and to transmit power. Applications include the screws for vises, C-clamps, presses, micrometers, jacks (Figure 15.5), valve stems, and the lead screws for lathes and other equipment. In the usual configuration, the nut rotates in place, and the screw moves axially. In some designs, the screw rotates in place, and the nut moves axially. Forces may be large, but motion is usually slow and power is small. In all the foregoing cases, power screws operate on the same principle.

A simplified drawing of a screw jack having the Acme thread is shown in Figure 15.6. The load W can be lifted or lowered by the rotation of the nut that is supported by a washer, called a *thrust collar* (or a *thrust bearing*). It is, of course, assumed that the load and screw are prevented from turning when the nut rotates. Hence, there needs to be some friction at the

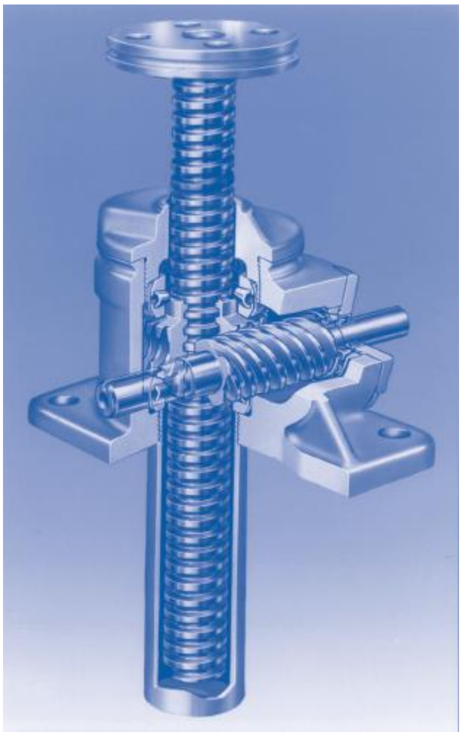


FIGURE 15.5
Worm gear screw jack. (Courtesy of Joyce/Dayton Corp.)

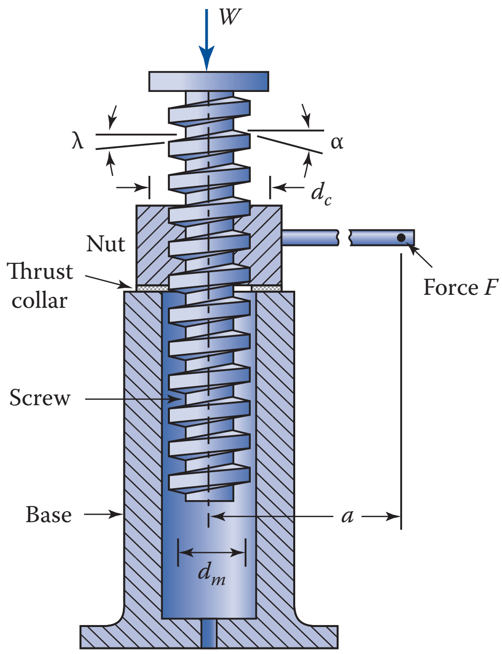


FIGURE 15.6
Schematic representation of power screw used as a screw jack. *Notes*: Only the nut rotates in this model: d_m represents the mean thread diameter and d_c is the mean collar diameter.

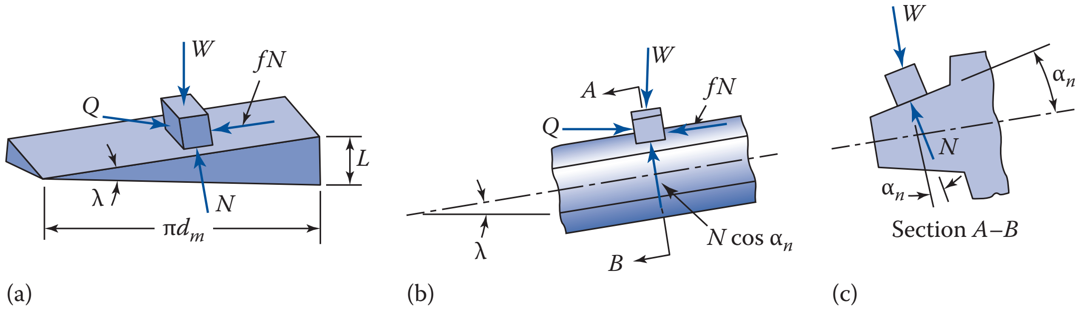


FIGURE 15.7
Forces acting on an Acme screw–nut interface when lifting load W: (a) a developed screw thread, (b) a segment of the thread, and (c) thread angle measured in the plane normal to thread, α_n.

load surface to prevent the screw from turning with the nut. Alternatively, the power screw could be turned against a nut that is prevented from turning to lift or lower the load. In either case, there is significant friction between the screw and nut as well as between the nut and the collar. Ordinarily, the screw is a hard steel, while the nut is made of a softer material (e.g., an alloy of aluminum, nickel, and bronze) to allow the parts to move smoothly.

In this section, we develop expressions for ascertaining the values of the torque needed to lift and lower the load using a jack. We see from Figure 15.6 that turning the nut forces each portion of the nut thread to climb an inclined plane. This plane is depicted by unwrapping or developing one revolution of the helix in Figure 15.7a, which includes a small block representing the nut being slid up the inclined plane of an Acme thread. The forces acting on the nut as a free-body diagram are also noted in the figure. Clearly, one edge of the thread forms the hypotenuse of the right triangle, having a base as the circumference of the mean-thread-diameter circle and as the lead. Therefore,

$$\tan \lambda = \frac{L}{\pi d_m} \tag{15.2}$$

where
λ = the helix or lead angle
L = the lead
d_m = the mean diameter of thread contact surface

The preceding notation is the same as for worms (see Section 12.9) except that unnecessary subscripts are omitted.

15.3.1 Torque to Lift the Load

The *sum* of all loads and normal forces acting on the entire thread surface in contact are denoted by W and N, respectively. To lift or raise the load, a tangential force Q acts to the right, and the friction force fN acts to oppose the motion (Figure 15.7). The quantity f represents the coefficient of sliding friction between the nut and screw or the coefficient of *thread friction.* The thread angle increases the frictional force by the wedging action of the threads. The conditions of equilibrium of the horizontal and vertical forces give

$$\begin{aligned} \sum F_h = 0: \quad & Q - N\left(f \cos\lambda + \cos\alpha_n \sin\lambda\right) = 0 \\ \sum F_v = 0: \quad & W + N\left(f \sin\lambda - \cos\alpha_n \cos\lambda\right) = 0 \end{aligned} \tag{a}$$

where α_n is the normal thread angle and the other variables are defined in the figure. Inasmuch as we are not interested in the normal force N, we eliminate it from the foregoing equations and solve the result for Q. In so doing, we have

$$Q = W = \frac{f\cos\lambda + \cos\alpha_n \sin\lambda}{\cos\alpha_n \cos\lambda - f\sin\lambda} \tag{15.3}$$

The screw torque required to move the load up the inclined plane, after dividing the numerator and denominator by cos λ, is then

$$T = \frac{1}{2}Qd_m = \frac{Wd_m}{2}\frac{f + \cos\alpha_n \tan\lambda}{\cos\alpha_n - f\tan\lambda} \tag{15.4}$$

But the thrust collar also contributes a friction force. That is, the normal reactive force acting on contact surface due to W results in an additional force f_cW. Here, f_c is the sliding coefficient of the *collar friction* between the thrust collar and the surface that supports the screw. It is assumed that this frictional force acts at the mean collar diameter d_c (Figure 15.6). The torque needed to overcome collar friction is

$$T = \frac{Wf_cd_c}{2} \tag{15.5}$$

The required total *torque T_u to lift the load* is found by addition of Equations 15.4 and 15.5:

$$T_u = \frac{Wd_m}{2}\frac{f + \cos\alpha_n \tan\lambda}{\cos\alpha_n - f\tan\lambda} + \frac{Wf_cd_c}{2} \tag{15.6}$$

15.3.2 Torque to Lower the Load

The analysis of lowering a load is exactly the same as that just described, with the exception that the directions of Q and fN (Figure 15.7b) are reversed. This leads to the equation for the total required *torque T_d to lower the load* as

$$T_d = \frac{Wd_m}{2}\frac{f - \cos\alpha_n \tan\lambda}{\cos\alpha_n + f\tan\lambda} + \frac{Wf_cd_c}{2} \tag{15.7}$$

15.3.3 Values of Friction Coefficients

When a plain *thrust collar* is used, as shown in Figure 15.6, values of f and f_c vary customarily between 0.08 and 0.20 under conditions of ordinary service, lubrication, and the common materials of steel and cast iron or bronze. The lowest value applies for good workmanship, the highest value for poor workmanship, and some in between value for other work quality. The preceding range includes both starting and running frictions. *Starting* friction can be about 4/3 times the *running* friction. Should a rolling *thrust bearing* be used, f_c would usually be low enough (about 0.008–0.02) that collar friction can be omitted. For this case, the second term in Equations 15.6 and 15.7 is eliminated.

15.3.4 Values of Thread Angle in the Normal Plane

A relationship between normal thread angle α_n, thread angle α, and helix angle λ can be obtained from a comparison of thread angles measured in axial plane and normal plane. Referring to Figures 15.6 and 15.7c, it can readily be verified that

$$\tan\alpha_n = \cos\lambda\tan\alpha \tag{15.8}$$

In most applications, λ is relatively small, and hence, $\cos\lambda \approx 1$. So, we can set $\alpha_n \approx \alpha$ and Equation 15.6 becomes

$$T_u = \frac{Wd_m}{2}\frac{f+\cos\alpha\tan\lambda}{\cos\alpha - f\tan\lambda} + \frac{Wf_c d_c}{2} \tag{15.9}$$

Obviously, for the case of the *square thread*, $\alpha = \alpha_n = 0$, and $\cos\alpha = 1$ in the preceding expressions.

15.4 Overhauling and Efficiency of Power Screws

A self-locking screw requires a positive torque to lower the load. This is a useful provision, particularly in screw jack applications. *Self-locking* refers to a condition in which the screw cannot be turned by applying an axial force of any magnitude to the nut. If collar friction is neglected, Equation 15.7 shows that the *condition* for self-locking is

$$f \ge \cos\alpha_n \tan\lambda \tag{15.10}$$

For a square thread, the foregoing equation reduces to

$$f \ge \tan\lambda \tag{15.10a}$$

In other words, self-locking is obtained when the coefficient of thread friction is equal to or greater than the tangent of the thread helix angle. Note that Equation 15.10 presumes a static situation and most power screws are self-locking.

Overhauling or back-driving screw is one that has low enough friction to enable the load to lower itself, by causing the screw to spin. In this situation, the inclined plane in Figure 15.7b moves to the right, and force Q must act to the left to preserve uniform motion. It can be shown that the *torque* T_o *of overhauling* screw is

$$T_o = \frac{Wd_m}{2}\frac{-f+\cos\alpha_n\tan\lambda}{\cos\alpha_n + f\tan\lambda} - \frac{Wf_c d_c}{2} \tag{15.11}$$

A negative external lowering torque must now be maintained to keep the load from lowering.

15.4.1 Screw Efficiency

Screw efficiency is the ratio of the torque required to raise a load without friction to the torque required with friction. Using Equation 15.6, *efficiency* is expressed in the form

$$e = \frac{d_m \tan\lambda}{d_m \dfrac{f + \cos\alpha_n \tan\lambda}{\cos\alpha_n - f\tan\lambda} + d_c f_c} \tag{15.12}$$

We observe from this equation that efficiency depends on only the screw geometry and the coefficient of friction. If the *collar friction* is *neglected*, the efficiency becomes

$$e = \frac{\cos\alpha_n - f\tan\lambda}{\cos\alpha_n + f\cot\lambda} \tag{15.13}$$

For a *square thread*, $\alpha_n = 0$ and Equation 15.13 simplifies to

$$e = \frac{1 - f\tan\lambda}{1 + f\cot\lambda} \tag{15.13a}$$

Equation 15.13 with α_n substituted from Equation 15.8 and $\alpha = 14.5°$ is plotted in Figure 15.8 for five values of *f*. We see from the curves that the power screws have very low mechanical efficiency when the helix angle is in the neighborhood of either 0° or 90°. They generally have an efficiency of 30%–90%, depending on the λ and *f*. We mention that values for square threads are higher by less than 1% over those for Acme screws in the figure.

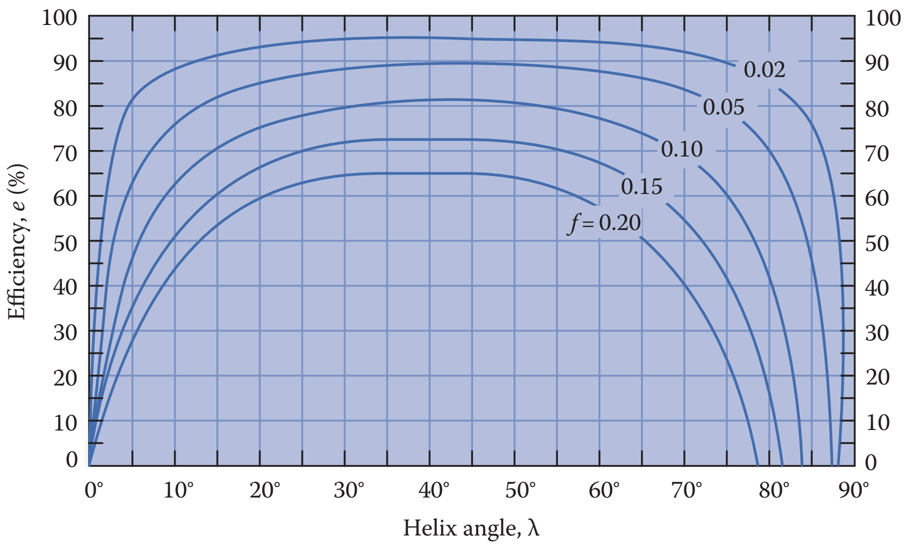


FIGURE 15.8
The efficiency of Acme screw threads (neglecting thrust collar friction).

Example 15.1: Quadruple-Threaded Power Screw

A screw jack with an Acme thread of diameter d, similar to that illustrated in Figure 15.6, is used to lift a load of W. Determine

a. The screw lead, mean diameter, and helix angle
b. The starting torque for lifting and for lowering the load
c. The efficiency of the jack when lifting the load, if collar friction is neglected
d. The length of a crank required, if F = 150 N is exerted by an operator

Design Assumptions: The screw and nut are lubricated with oil. Coefficients of friction are estimated as $f = 0.12$ and $f_c = 0.09$.

Given: d = 30 mm and W = 6 kN. The screw is quadruple threaded having a pitch of p = 4 mm. The mean diameter of the collar is d_c = 40 mm.

Solution

a. From Figure 15.4, $d_m = d - p/2 = 30 - 2 = 28$ mm. Through the use of Equations 15.1 and 15.2, we have

$$L = np = 4(4) = 16\,\text{mm}$$

$$\lambda = \tan^{-1}\frac{16}{\pi(28)} = 10.31°$$

b. The coefficients of friction for starting are $f = \frac{4}{3}(0.12) = 0.16$ and $f_c = \frac{4}{3}(0.09) = 0.12$. For an Acme thread, $\alpha = 14.5°$ (Figure 15.4a), by Equation 15.8.

$$\alpha_n = \tan^{-1}(\cos\lambda\tan\alpha)$$
$$= \tan^{-1}(\cos 10.31°\tan 14.5°) = 14.28°$$

Then, application of Equations 15.6 and 15.7 results in

$$T_u = \frac{6(28)}{2}\frac{0.16 + \cos 14.28°\tan 10.31°}{\cos 14.28° - (0.16)\tan 10.31°} + \frac{6(0.12)40}{2}$$
$$= 30.05 + 14.4 = 44.45\,\text{N}\cdot\text{m}$$

$$T_d = \frac{6(28)}{2}\frac{0.16 - \cos 14.28°\tan 10.31°}{\cos 14.28° + (0.16)\tan 10.31°} + 14.4$$
$$= -1.37 + 14.4 = 13.03\,\text{N}\cdot\text{m}$$

Comments: The minus sign in the first term of T_d means that the screw alone is not self-locking and would rotate under the action of the load, except that the collar friction must be overcome too. Since T_d is positive, the screw does not overhaul.

c. The running torque needed to lift the load is based on $f = 0.12$. Using Equation 15.13, we have

$$e = \frac{\cos 14.28° - (0.12)\tan 10.31°}{\cos 14.28° + (0.12)\cot 10.31°}$$
$$= 0.582 = 58.2\%$$

d. The length of the crank arm is

$$a = \frac{T_n}{F} = \frac{44.45}{150} = 0.296 \text{ m} = 296 \text{ mm}$$

Example 15.2: Single-Threaded Power Screw

Given: The screw jack (Figure 15.6) discussed in the previous example has a single-threaded Acme screw instead of a quadruple thread.

Find: The torque required for lifting the load and efficiency of the jack.

Solution

Refer to Example 15.1.
Now the lead is equal to the pitch, $L = p = 4$ mm. The helix angle is therefore

$$\lambda = \tan^{-1}\left(\frac{1}{\pi d_m}\right) = \tan^{-1}\left(\frac{4}{28\pi}\right) = 2.604°$$

Through the use of Equation 15.6, we have

$$\alpha_n = \tan^{-1}(\cos\lambda \tan\alpha) = \tan^{-1}(\cos 2.604° \tan 14.5°) = 14.49°$$

Then, applying Equation 15.6, the torque required to raise the load is equal to

$$T_u = \frac{6(28)}{2} \frac{0.16 + \cos 14.49° \tan 2.604°}{\cos 14.49° - (0.16)\tan 2.604°} + \frac{6(0.12)40}{2}$$
$$= 17.84 + 14.4 = 32.24 \text{ N} \cdot \text{m}$$

Equation 15.13 results in the efficiency in lifting the load as follows:

$$e = \frac{\cos 14.49° - (0.12)\tan 2.604°}{\cos 14.49° + (0.12)\cot 2.604°}$$
$$= 0.267 = 26.7\%$$

Comments: A comparison of the results obtained here with those of Example 15.1 shows that to lift the load, the single-treaded screw requires lower torque than the quadruple. However, the former is less efficient than the latter by 54.1%.

Example 15.3: Self-Locking of Quadruple- and Single-Threaded Screws

Given: The quadruple-threaded and single-threaded screws discussed in the preceding two examples.

Find: The tread coefficient of friction necessary to ensure that self-locking takes place.

Assumption: Rolling element bearings have been installed at the collar, so that the collar friction can be disregarded.

Solution

Refer to Examples 15.1 and 15.2.
Coefficient of friction for self-locking is specified by Equation 15.10 as

$$f \geq \cos\alpha_n \tan\lambda$$

Therefore, for the quadruple-threaded screw self-locking does not occur since

$$0.12 < \cos 14.28° \tan 10.31° = 0.176$$

But, for the single-threaded screw, self-locking occurs since

$$0.12 > \cos 14.49° \tan 2.604° = 0.044$$

Comment: The foregoing results indicate that the quadruple-threaded screw requires four times the friction coefficient of friction of the single-threaded screw.

15.5 Ball Screws

A *ball screw*, or so-called ball-bearing screw, is a linear actuator that transmits force or motion with minimum friction. A cutaway illustration of a ball screw and two of its precision assemblies supported by ball bearings at the ends are shown in Figure 15.9. Note that

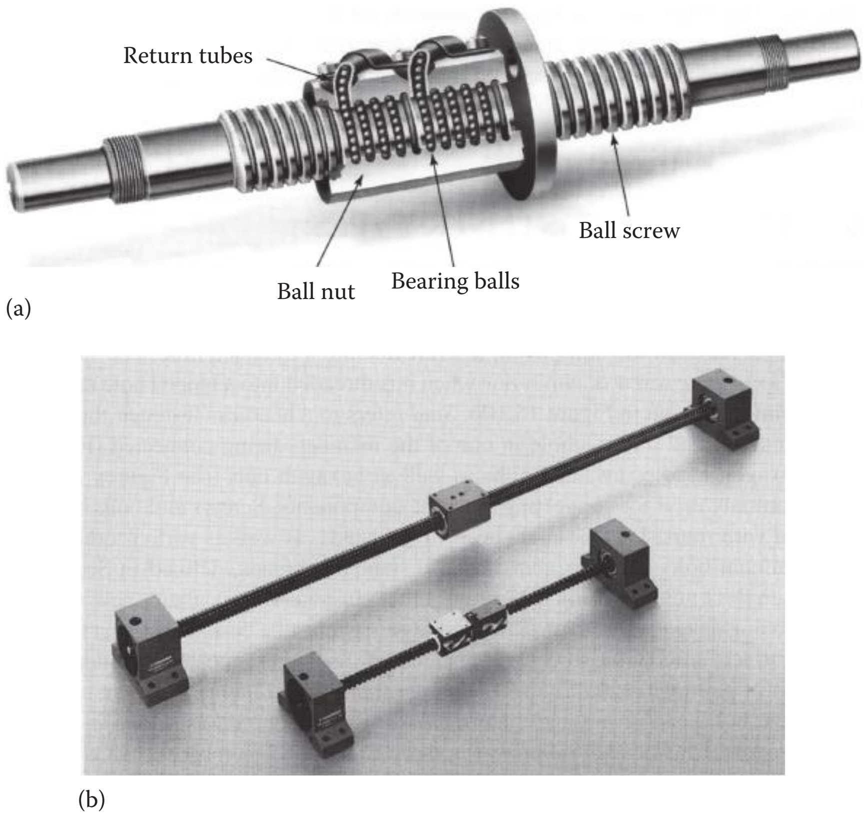


FIGURE 15.9
Ball screw used as a positioning device: (a) cutaway of a ball screw and (b) two assemblies. (Courtesy of Thomson Industries, Inc.)

a circular groove is cut to proper conformity with the balls. The groove has a thread helix angle matching the thread angle of the groove within the nut. The *balls* are contained within the *nut* to produce an approximate rolling contact with the screw threads. The rotation of the screw (or nut) is converted into a linear motion and force with very little friction torque. During the motion, the balls are diverted from one end and middle of the nut and carried by two ball-return tubes (or ball guides) located outside of the nut to the middle and opposite end of the nut. Such recirculation allows the nut to travel the full length of the screw.

A ball screw can support greater load than that of ordinary power screws of identical diameter. The smaller size and lighter weight are usually an advantage. A thin film lubricant is required for these screws. Certain dimensions of ball screws have been standardized by ANSI [3,4] but mainly for use in machine tools. Capacity ratings for ball screws are obtained by methods and equations identical to those for ball bearings, which can be found in manufacturers' catalogs.

Efficiencies of 90% or greater are possible with ball screws over a wide range of helix angles when converting rotary into axial motion. Ball screws may be preferred by the designers if higher screw efficiencies are required. As a positioning device, these screws are used in many applications. Examples include the steering mechanism of automobiles, hospital bed mechanisms, automatic door closers, antenna drives, the aircraft control (e.g., a ball or jack screw and gimbal nut assembly as an actuator on a linkage for extending and retracting the wing flaps) and landing-gear actuator, jet aircraft engine thrust reverser actuators, and machine tool controls. Because of the low friction of ball screws, they are not self-locked. An auxiliary brake is required to hold a load driven by a ball screw for some applications.

15.6 Threaded Fastener Types

The common element among screw fasteners used to connect or join two or more parts is their thread. Screws and *bolts* are the most familiar threaded fastener types. The only difference between a screw and a bolt is that the bolt needs a *nut* to be used as a fastener (Figure 15.10a). On the other hand, a screw fits into a threaded hole. The same fastener is termed a *machine screw* or *cap screw* when it is threaded into a tapped hole rather than used

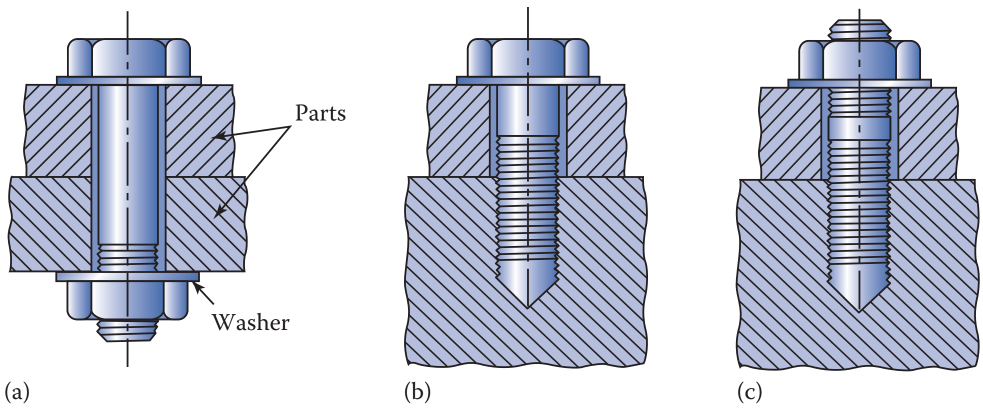


FIGURE 15.10
Typical threaded fasteners: (a) bolt and nut, (b) cap screw, and (c) stud.

with a nut, as shown in Figure 15.10b. *Stud* refers to a headless fastener, threaded on both ends, and screwed into the hole in one of the members being connected (Figure 15.10c).

Hexagon-head screws and bolts as well as hexagon nuts (see Figures 15.1 and 15.2) are commonly used for connecting machine components. Screws and bolts are also manufactured with round heads, square heads, oval heads, and various other head styles. Conventional bolts and nuts generally use standard threads, defined in Section 15.2. An almost endless number of threaded (and other) fasteners exist; many new types are constantly being developed [5–8]. Threaded fasteners must be designed so that they are lighter in weight, less susceptible to corrosion, and more resilient to loosening under vibration.

Flat or plain *washers* (Figure 15.1) are often used to increase the area of contact between the bolt head or nut and clamped part in a connection, as shown in Figure 15.10. They prevent stress concentration by the sharp edges of the bolt holes. Flat washer sizes are standardized to bolt size. A plain washer also forestalls marring of the clamped part surface by the nut when it is tightened. Belleville washers, discussed in Section 14.12, provide a controlled axial force over changes in bolt length.

Lock washers help prevent spontaneous loosening of standard nuts. The split lock washer acts as a spring under the nut. *Lock nuts* prevent too-spontaneous loosening of nuts due to vibration. Simply, two nuts jammed together on the bolt or a nut with a cotter pin serve for this purpose as well. The cotter pin is a wire that fits in diametrically opposite slots in the nut and passes through a drilled hole in the bolt. Lock nuts are considered to be more effective in preventing loosening than lock washers.

15.6.1 Fastener Materials and Strengths

A fastener is classified according to a grade or property class that defines its strength and material. Most fasteners are made from steel of specifications standardized by the SAE, ASTM, and ISO. The SAE grade (inch series) and SAE class (metric series) of steel-threaded members are numbered according to tensile strength. The *proof strength* S_p corresponds to the axial stress at which the bolt or screw begins to develop a permanent set. It is close to but lower than the material yield strength. The *proof load* F_p is defined by

$$F_p = S_p A_t \tag{15.14}$$

Here, the *tensile stress area, A*, represents the minimum radial plane area for fracture through the threaded part of a bolt or screw. Numerical values of A_t are listed in Tables 15.1 and 15.2. The proof strength is obtained from Tables 15.4 or 15.5. For other materials, an approximate value is about 10% less than for yield strength, that is, $S_p = 0.9S_y$ based on a 2% offset.

Threads are generally formed by *rolling* and *cutting* or grinding. The former is stronger than the latter in fatigue and impact because of cold working. Hence, high-strength screws and bolts have rolled threads. The rolling should be done subsequent to hardening the bolt. The material of the nut must be selected carefully to match that of the bolt. The washers should be of hardened steel, where the bolt or nut compression load needs to be distributed over a large area of clamped part.

A soft washer bends rather than uniformly distributes the load. Fasteners are also made of a variety of materials, including aluminum, brass, copper, nickel, Monel, stainless steel, titanium, beryllium, and plastics. Appropriate coatings may be used in special applications

TABLE 15.4

SAE Specifications and Strengths for Steel Bolts

SAE Grade	Size Range Diameter, *d* (in.)	Proof Strength,[a] S_p (ksi)	Yield Strength,[b] S_y (ksi)	Tensile Strength,[b] S_u (ksi)	Material Carbon Content
1	$\frac{1}{4}-1\frac{1}{2}$	33	36	60	Low or medium
2	$\frac{1}{4}-\frac{3}{4}$	55	57	74	Low or medium
2	$\frac{7}{8}-1\frac{1}{2}$	33	36	60	Low or medium
5	$\frac{1}{4}-1$	85	92	120	Medium, CD
5	$1\frac{1}{8}-1\frac{1}{2}$	74	81	105	Medium, CD
7	$\frac{1}{4}-1\frac{1}{2}$	105	115	133	Medium, alloy, Q&T
8	$\frac{1}{4}-1\frac{1}{2}$	120	130	150	Medium, alloy, Q&T

Source: Society of Automotive Engineers Standard J429k, 2011.
[a] Corresponds to permanent set not over 0.0001 in.
[b] Offset of 0.2%.
Q&T, quenched and tempered.

TABLE 15.5

Metric Specifications and Strengths for Steel Bolts

Class Number	Size Range Diameter, *d* (mm)	Proof Strength, S_p (MPa)	Yield Strength, S_y (MPa)	Tensile Strength, S_u (MPa)	Material Carbon Content
4.6	*M5–M36*	225	240	400	Low or medium
4.8	*M1.6–M16*	310	340	420	Low or medium
5.8	*M5–M24*	380	420	520	Low or medium
8.8	*M3–M36*	600	660	830	Medium, Q&T
9.8	*M1,6–M* 16	650	720	900	Medium, Q&T
10.9	*M5–M36*	830	940	1040	Low, martensite, Q&T
12.9	*M1.6–M36*	970	1100	1220	Alloy, Q&T

Source: Society of Automotive Engineers Standard J429k, 2011.

in place of a more expensive material, for corrosion protection and to reduce thread friction and wear. Obviously, a designer has many options in selecting the fastener's material to suit the particular application.

15.7 Stresses in Screws

Stress distribution of the thread engagement between the screw and the nut is nonuniform. In reality, inaccuracies in thread spacing cause virtually all the load to be taken by the first pair of contacting threads, and a large stress concentration is present here. While the stress concentration is to some extent relieved by the bending of the threads and the expansion of the nut, most bolt failures occur at this point. A concentration of stress also exists in the screw where the load is transferred through the nut to the adjoining member. Obviously, factors such as fillet radii at the thread roots and surface finish have significant effects on the actual stress values. For ordinary threads, the *stress concentration factor* K_t varies between 2 and 4 [9].

Note that the screws should always have enough ductility to permit local yielding at thread roots without damage. For *static loading*, it is commonly assumed that the load carried by a screw and nut is about *uniformly* distributed throughout thread engagement. The stress distribution for threads with steady loads is usually determined by photoelastic analysis. A variety of methods are used to obtain a more nearly equal distribution of loads among the threads, including increasing the flexibility of the nut (or bolt), making the nut from a softer material than the bolt, and cutting the thread of the nut on a very small taper. A rule of thumb for the length of full thread engagement is 1.0*d* in steel, 1.5*d* in gray cast iron, and 2.0*d* in aluminum castings, where *d* is the nominal thread size.

The following expressions for stresses in power screws and threaded fasteners are obtained through the use of the elementary formulas for stress. They enable the analyst to achieve a reasonable design for a static load. When bolts are subjected to fluctuating loads, stress concentration is very important.

15.7.1 Axial Stress

Power screws may be under tensile or compressive stress; threaded fasteners normally carry only tension. The axial stress σ is then

$$\sigma = \frac{P}{A} \tag{15.15}$$

where

P represents the tensile or compressive load

$$A \text{ is the } \begin{cases} A_t \text{ from Tables 15.1 and 15.2} & \text{(threaded fasteners)} \\ \pi d_r^2, \quad d_r \text{ is the root diameter} & \text{(power screws)} \end{cases}$$

15.7.2 Torsional Shear Stress

Power screws in operation and threaded fasteners during tightening are subject to torsion. The shear stress τ is given by

$$\tau = \frac{Tc}{J} = \frac{16T}{\pi d_r^3} \tag{15.16}$$

In the foregoing, we have

$$T = \begin{cases} \text{applied torque, for } f_c = 0 & \text{(power screws)} \\ \text{half the wrench torque} & \text{(threaded fastenrs)} \end{cases}$$

$$d = \begin{cases} \text{from Figure 15.4} & \text{(power screws)} \\ \text{from Tables 15.1 and 15.2} & \text{(threaded screws)} \end{cases}$$

15.7.3 Combined Torsion and Axial Stress

The combined stress of Equations 15.15 and 15.16 can be treated as in Section 6.7, with the energy of distortion theory employed as a criterion for yielding.

15.7.4 Bearing Stress

The direct compression or bearing stress σ_b is the pressure between the surface of the screw thread and the contacting surface of the nut:

$$\sigma_b = \frac{P}{\pi d_m h n_e} = \frac{Pp}{\pi d_m h L_n} \tag{15.17}$$

where
P = the load
d_m = the pitch or mean screw thread diameter
h = the depth of thread (Figure 15.3)
n_e = the number of threads in engagement and the Ln/p
L_n = the nut length
p = the pitch

Exact values of σ_b are given in ANSI B 1.1-1989 and various handbooks.

15.7.5 Direct Shear Stress

The screw thread is considered to be loaded as a cantilevered beam. The load is assumed to be uniformly distributed over the mean screw diameter. Hence, both the threads on the screw and the threads on the nut experience a transverse shear stress $\tau = 3P/2A$ at their roots. Here, A is the cross-sectional area of the built-in end of the beam: $A = \pi d_r b n_e$ for the screw and $A = \pi d b n_e$ for the nut. Therefore, shear stress, for the *screw*, is

$$\tau = \frac{3P}{2\pi d_r b n_e} \tag{15.18}$$

and, for the *nut*, is

$$\tau = \frac{3P}{2\pi d b n_e} \tag{15.19}$$

in which
d_r = the diameter of the screw
d = the major diameter of the screw
b = the thread thickness at the root (Figure 15.3)

The remaining terms are as defined earlier.

The *design formulas* for screw threads are obtained by incorporating K_t and replacing σ or σ_b by S_y/n and τ by S_{ys}/n in the preceding equations. For the nut, for example, Equations 15.17 and 15.19 with $n_e = L_n/p$ may be written as follows:

$$\frac{S_y}{n} = \frac{K_t Pp}{\pi d_m h L_n} \tag{15.17a}$$

and

$$\frac{S_{ys}}{n} = \frac{3K_t Pp}{2\pi dbL_n} \tag{15.19a}$$

Here, S_y, S_{ys}, and n represent the yield strength in tension, yield strength in shear, and safety factor, respectively. Application of such formulas is illustrated in Case Study 18.7.

15.7.6 Buckling Stress for Power Screws

For the case in which the unsupported screw length is equal to or larger than about eight times the root diameter, the screw must be treated as a column. So, critical stresses are obtained as discussed in Sections 5.10 and 5.13.

15.8 Bolt Tightening and Preload

Bolts are commonly used to hold parts together in opposing to forces likely to pull, or sometimes slide, them apart. Typical examples include connecting rod bolts and cylinder head bolts. *Bolt tightening* is prestressing at assembly. In general, bolted joints should be tightened to produce an *initial tensile force*, usually the so-called preload F_i. The advantages of an initial tension are especially noticeable in applications involving fluctuating loading, as demonstrated in Section 15.12, and in making a leakproof connection in pressure vessels. An increase of fatigue strength is obtained when initial tension is present in the bolt. The parts to be joined may or may not be separated by a gasket. In this section, we consider the situation when no gasket is used.

The *bolt strength* is the main factor in the design and analysis of bolted connections. Recall from Section 15.6 that the proof load F_p is the load that a bolt can carry without developing a permanent deformation. For both static and fatigue loading, the preload is often prescribed by

$$F_i = \begin{cases} 0.75F_p & \text{(reused connections)} \\ 0.9F_p & \text{(permanent connections)} \end{cases} \tag{15.20}$$

where the proof load $F_p = S_p A_t$ from Equation 15.14. The amount of initial tension is clearly a significant factor in bolt design. It is usually maintained fairly constant in value.

15.8.1 Torque Requirement

The most important factor determining the preload in a bolt is the *torque required* to *tighten* the *bolt*. The torque may be applied manually by means of a wrench that has a dial attachment indicating the magnitude of the torque being enforced. Pneumatic or air wrenches give more consistent results than a manual torque wrench and are employed extensively.

An expression relating applied torque to initial tension can be obtained using Equation 15.6 developed for power screws. Observe that load W of a screw jack is equivalent to F_i, for a bolt and that collar friction in the jack corresponds to friction on the flat surface of the nut or under the screwhead. It can readily be shown that [7], for standard screw threads, Equation 15.6 has the form

$$T = KdF_i \tag{15.21}$$

where
  T = the tightening torque
  d = the nominal bolt diameter
  K = the torque coefficient
  F_i = the initial tension or preload

For dry surface and *unlubricated* bolts or *average* condition of thread friction, taking $f = f_c = 0.15$, Equation 15.6 results in $K = 0.2$. It is suggested that, for *lubricated* bolts, a value of 0.15 be used for torque coefficient. For various-plated bolts, see [10].

Note that Equation 15.21 represents an approximate relationship between the induced initial tension and applied torque. Tests have shown that a typical joint loses about 5% or more of its preload owing to various relaxation effects. The exact tightening torque needed in a particular situation can likely be best ascertained experimentally through calibration. That is, a prototype can be built and accurate torque testing equipment used on it. Interestingly, bolts and washers are available with built-in sensors indicating a degree of tightness. Electronic assembly equipment is available [2,7].

15.9 Tension Joints under Static Loading

A principal utilization of bolts and nuts is clamping parts together in situations where the applied loads put the *bolts in tension.* Attention here is directed toward preloaded tension joints under static loading. We treat the case of two plates or parts fastened with a bolt and subjected to an external separating load P, as depicted in Figure 15.11a. The preload F_i, an initial tension, is applied to the bolt by tightening the nut prior to the load P. Clearly, the bolt axial load and the *clamping force* between the two parts F_p are both equal to F_i.

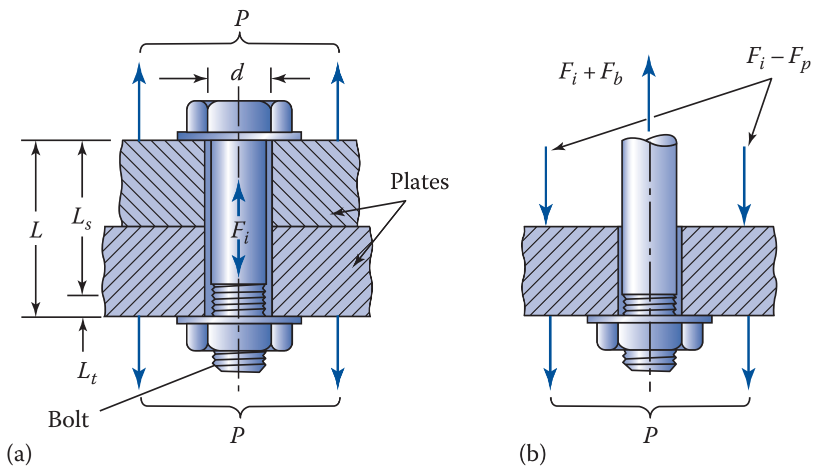


FIGURE 15.11
A bolted connection: (a) complete joint with preload F_i and external load P and (b) isolated portion depicting increased bolt force F_b and decreased force on parts or plates F_p.

To determine what portion of the externally applied load is carried by the bolt and what portion by the connected parts in the assembly, refer the free-body diagram shown in Figure 15.11b. The equilibrium condition of forces requires that

$$P = F_b + F_p \tag{a}$$

The quantity F_b is the *increased* bolt (tensile) force, and F_p represents the *decreased* clamping (compression) force between the parts. It is taken that the parts have not been separated by the application of the external load. The deformation of the bolt and the parts are defined by

$$\delta_b = \frac{F_b}{k_b}, \quad \delta_p = \frac{F_p}{k_p} \tag{b}$$

Here, k_b and k_p represent the *stiffness constants* for the bolt and parts, respectively.

Because of the setup of the members in Figure 15.11a, the deformations given by Equation (b) are equal. The *compatibility condition* is then

$$\frac{F_b}{k_b} = \frac{F_p}{k_p} \tag{c}$$

Combining Equations (a) and (c) yields

$$F_b = \frac{k_b}{k_b + k_p} P = CP, \quad F_p = \frac{k_p}{k_b + k_p} P = (1 - C)P \tag{d}$$

The term C, called the *joint's stiffness factor* or simply the *joint constant*, is defined in Equation (d) as

$$C = \frac{k_b}{k_b + k_p} \tag{15.22}$$

Note that, typically, k_b is small in comparison with k_p and C is a small fraction.

The total forces on the bolt and parts are, respectively,

$$F_b = CP + F_i \quad (\text{for } F_p < 0) \tag{15.23}$$

$$F_p = (1 - C)P - F_i \quad (\text{for } F_p < 0) \tag{15.24}$$

where

F_b = the bolt axial tensile force
F_p = the lamping force on the two parts
F_i = the initial tension or preload

A graphical representation of Equations 15.23 and 15.24 is given in Figure 15.12a. Clearly, if load P is sufficient to bring the clamping force F_p to zero (point A), we have bolt force $F_b = P$ (point B). As indicated in the expressions, the foregoing results are valid only as long as some clamping force prevails on the parts: with no preload (loosened joint),

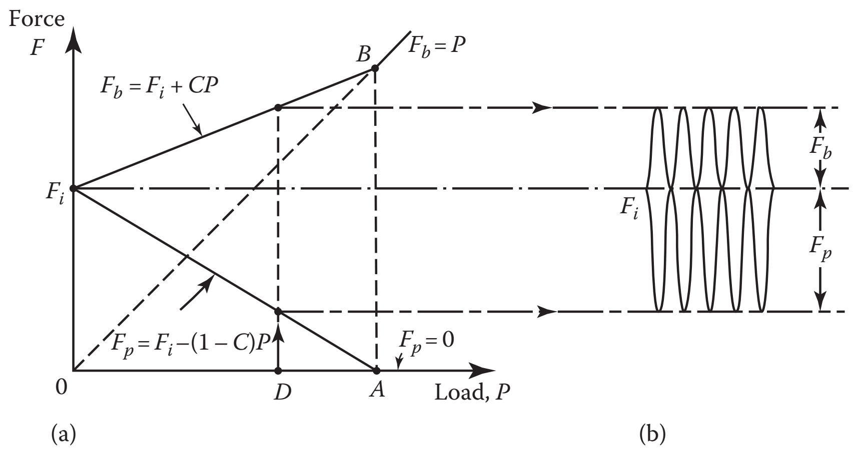


FIGURE 15.12
Preload in a bolted connection: (a) force relationship and (b) variations in F_b and F_p related to variations in P between 0 and D.

$C = 1$, $F_i = 0$. We see that the ratios C and $1 - C$ in Equations 15.23 and 15.24 describe the proportions of the external load carried by the bolt and the parts, respectively. In all situations, the *parts* take a *greater portion* of the external load. This is significant when *fluctuating* loading is present, where variations in F_b and F_p are readily found from Figure 15.12a and b, as indicated. We shall discuss this loading situation in detail in Section 15.12.

15.9.1 Deflections due to Preload

Figure 15.13 illustrates the load–deflection behavior of both bolt and parts on force (F)–deflection (δ) axes. Observe that the slope of the bolt line is positive, since its length increases with increasing force. On the contrary, the slope of the parts is negative, because its length decreases with the increasing force. As is often the case, the figure shows that $k_p > k_b$. It is obvious that the force in both bolt and parts is identical as long as they remain in contact. A preload force F_i is applied by tightening the bolt and $F_b = F_p = F_i$. The deflections of the bolt δ_b and parts δ_p are controlled by the spring rates of reach points A and B on their respective load–deflection lines.

For the case in which an external load P is applied to the joint, there will be an additional deflection added to both bolt and parts. Although the quantitative amount is the same,

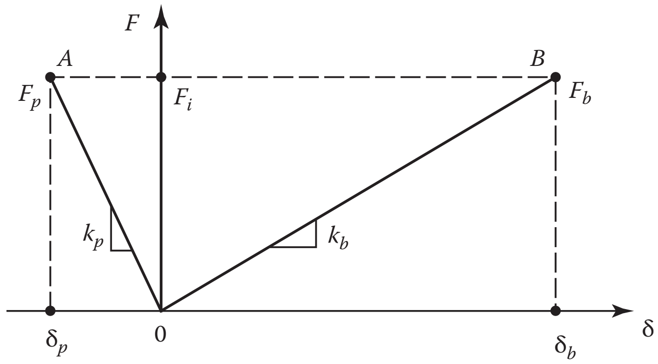


FIGURE 15.13
Preload versus initial deflections.

$\Delta\delta$, for the bolt, the deflection is an *increased elongation* while for the parts the *contraction is decreased*. The deflection $\Delta\delta$ causes a new load situation in both bolt and parts. As a result, the applied load is split into two components, one taken by the parts and one taken by the bolt. It will be seen in Section 15.12 that the preload effect is even greater for joints under dynamic loads than for statically loaded joints.

15.9.2 Factors of Safety for a Joint

The tensile stress σ_b in the bolt can be found by dividing both terms of Equation 15.24 by the tensile stress area A_t:

$$\sigma_b = \frac{CP}{A_t} + \frac{F_i}{A_t} \tag{15.25}$$

A means of ensuring a safe joint requires that the external load be smaller than that needed to cause the joint separate. Let nP be the value of external load that would cause bolt failure and the limiting value of σ_b be the proof strength S_p. Substituting these, Equation 15.25 becomes

$$\frac{CPn}{A_t} + \frac{F_i}{A_t} = S_p \tag{15.26}$$

It should be mentioned that the factor of safety is not applied to the preload. The foregoing can be rewritten to give the *bolt safety factor:*

$$n = \frac{S_p A_t - F_i}{CP} \tag{15.27}$$

As noted earlier, the tensile stress area A_t is furnished in Tables 15.1 and 15.2, and S_p is listed in Tables 15.4 and 15.5.

15.9.3 Joint-Separating Force

Equation 15.27 suggests that the safety factor n is maximized by having no preload on the bolt. We also note that for $n > 1$, the bolt stress is smaller than the proof strength. Separation occurs when in $F_p = 0$ in Equation 15.24:

$$P_s = \frac{F_i}{(1-C)} \tag{15.28a}$$

Therefore, the *load safety factor* guarding against joint separation is

$$n_s = \frac{P_s}{P} = \frac{F_i}{P(1-C)} \tag{15.28b}$$

Here, P is the *maximum* load applied to the joint.

Example 15.4: Load-Carrying Capacity of a Bolted Joint

Given: A ½ in.-13UNC grade 5 steel bolt clamps two steel plates and loaded as shown in Figure 15.11a.

Find: The maximum load based on a safety factor of 2.

Assumption: The connection will be permanent. Joint stiffness is taken as $C = 0.35$ (a detailed discussion about this is in Section 15.11).

Solution

For the ½-13UNC grade 5 steel bolt, we have

$A_t = 0.1419$ in^2. (by Table 15.1)

$S_p = 85$ ksi (from Table 15.4)

Applying Equation 15.27, the maximum load that the bolt can safely support is then

$$P_{\max,b} = \frac{S_p A_t - F_i}{nC} = \frac{(85)(0.1419) - 10.86}{2(0.35)} = 1.716 \text{ kips}$$

By Equation 15.28a, the maximum load before separation takes place equals

$$P_{\max,p} = \frac{F_i}{n(1-C)} = \frac{10.86}{2(1-0.35)} = 8.35 \text{ kips}$$

Comment: Failure owing to separation of art will not take place prior to bolt failure.

15.10 Gasketed Joints

Sometimes, a sealing or gasketing material must be placed between the parts connected. Gaskets are made of materials that are soft relative to other joint parts. Obviously, the stiffer and thinner is the gasket, the better. The stiffness factor of a gasketed joint can be defined as

$$C = \frac{k_b}{k_b + k_c} \tag{15.29a}$$

The quantity k_c represents the combined constant found from

$$\frac{1}{k_c} = \frac{1}{k_g} + \frac{1}{k_p} \tag{15.29b}$$

where k_g and k_p are the spring rates of the gasket and connected parts, respectively.

When a full gasket extends over the entire diameter of a joint, the gasket pressure is

$$p = \frac{F_p}{A_g} \tag{a}$$

in which A_g is the gasket area per bolt and F_p represents the clamping force on parts. For a load factor n_s, Equation 15.24 becomes

$$F_p = (1 - C)n_s P - F_i \tag{b}$$

Carrying Equation (a) into (b), *gasket pressure* may be expressed in the form

$$p = \frac{1}{A_g}\left[F_i - n_s P(1 - C)\right] \tag{15.30}$$

We point out that to maintain the uniformity of pressure, bolts should not be spaced more than *six* bolt diameters apart.

15.11 Determining the Joint Stiffness Constants

Application of the equations developed in Section 15.9 requires a determination of the spring rates of bolt and parts or at least a reasonable approximation of their relative values. Recall from Chapter 4 that the axial deflection is found from the equation $\delta = PL/AE$ and spring rate by $k = P/\delta$. Thus, we have for the bolt and parts, respectively,

$$k_b = \frac{A_b E_b}{L} \tag{15.31a}$$

$$k_p = \frac{A_p E_p}{L} \tag{15.31b}$$

where
- k_b = the stiffness constant for bolt
- k_p = the stiffness constant for parts
- A_b = the cross-sectional area of bolt
- A_p = the effective cross-sectional area of parts
- E = the modulus of elasticity
- L = the grip, which represents approximate length of clamped zone

15.11.1 Bolt Stiffness

When the thread stops immediately above the nut as shown in Figure 15.11, the gross cross-sectional area of the bolt must be used in approximating k_b, since the unthreaded portion is stretched by the load. Otherwise, a bolt is treated as a spring in series when considering the threaded and unthreaded portions of the shank. For a bolt of axially loaded *thread length* L_t and the unthreaded *shank length* L_s (Figure 15.11a), the spring constant is

$$\frac{1}{k_b} = \frac{L_t}{A_t E_b} + \frac{L_s}{A_b E_b} \tag{15.32}$$

in which A_b is the gross cross-sectional area and A_t represents the tensile stress area of the bolt. Note that, ordinarily, a bolt (or cap screw) has as little of its length threaded as practicable to maximize bolt stiffness. We then use Equation 15.31a in calculating the bolt spring rate k_b.

We note that a bolt or cap screw ordinarily has as little of its length threaded as practicable to increase bolt stiffness. Nevertheless, for standardized threads, the thread length is prescribed, as shown in the following expressions:

Metric threads (in mm)

$$L_t = \begin{cases} 2d + 6 & L \le 125 \\ 2d + 12 & 125 < L \le 200 \\ 2d + 25 & L > 200 \end{cases} \tag{15.33a}$$

Inch series

$$L_t = \begin{cases} 2d + 0.25\,\text{in.} & L \le 6\,\text{in.} \\ 2d + 0.50\,\text{in.} & L > 6\,\text{in.} \end{cases} \tag{15.33b}$$

Here (see Figure 15.11a), L_t is the threaded length, L represents the total bolt length, and d is the diameter.

15.11.2 Stiffness of Clamped Parts

The spring constant of clamped parts is seldom easy to ascertain and frequently approximated by employing an empirical procedure. Accordingly, the stress induced in the joint is assumed to be *uniform* throughout a region surrounding the bolt hole [11,12]. The region is often represented by a double-cone-shaped *barrel* geometry of a half-apex angle 30°, as depicted in Figure 15.14. The stress is taken to be 0 outside the region. The effective cross-sectional area A_p is equal to about the average area of the shaded section shown in the figure:

$$A_p = \frac{\pi}{4}\left[\left(\frac{d_w + d_2}{2}\right)^2 - d^2\right]$$

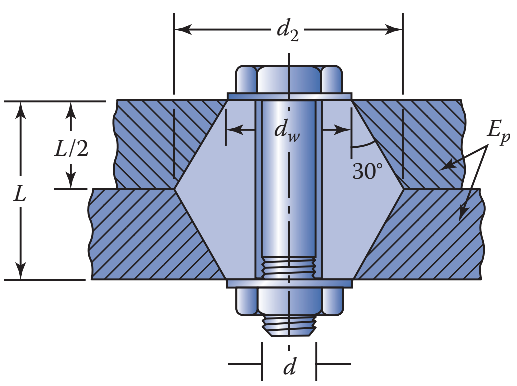


FIGURE 15.14
A method for estimating the effective cross-sectional area of clamped parts A_p.

The quantities $d_2 = d_w + L \tan 30°$ and d_w represent the washer (or washer face) diameter. Note that $d_w = 1.5d$ for *standard* hexagon-headed bolts and cap screws. The preceding expression of A_p is used for estimating k_p from Equation 15.31. It can be shown that [11], for connections using standard hexagon-headed bolts, the *stiffness constant for parts* is given by

$$k_p = \frac{0.58\pi E_p d}{2\ln\left(5\dfrac{0.58L + 0.5d}{0.58L + 2.5d}\right)} \tag{15.34}$$

where
d = the bolt diameter
L = the grip
E_p = the modulus of elasticity of the single or two identical parts

We should mention that the spring rate of clamped parts can be determined with good accuracy by experimentation or finite element analysis [13]. Various handbooks list rough estimates of the stiffness constant *ratio* k_p/k_b for typical gasketed and ungasketed joints. Sometimes, $k_p = 3k_b$ is used for ungasketed *ordinary* joints.

Example 15.5: Preloaded Bolt Connecting the Head and Cylinder of a Pressure Vessel

Figure 15.15 illustrates a portion of a cover plate bolted to the end of a thick-walled cylindrical pressure vessel. A total of N_b bolts are to be used to resist a separating force P. Determine

a. The joint constant
b. The number N_b for a permanent connection
c. The tightening torque for an average condition of thread friction

Given: The required joint dimensions and materials are shown in the figure. The applied load P = 55 kips.

Design Assumptions: The effects of the flanges on the joint stiffness are omitted. The connection is permanent. A bolt safety factor of n = 1.5 is used.

Solution

a. Referring to Figure 15.15, Equation 15.34 gives

$$k_p = \frac{0.58\pi(E_s/2)(0.75)}{2\ln\left[5\dfrac{0.58(2) + 0.5(0.75)}{0.58(2) + 2.5(0.75)}\right]} = 0.368E_s$$

Through the use of Equation 15.31a,

$$k_b = \frac{AE_s}{L} = \frac{\pi d^2 E_s}{4L} = \frac{\pi(0.75)^2 E_s}{4(2)} = 0.221E_s$$

Equation 15.22 is therefore

$$C = \frac{k_b}{k_b + k_p} = \frac{0.221}{0.221 + 0.368} = 0.375$$

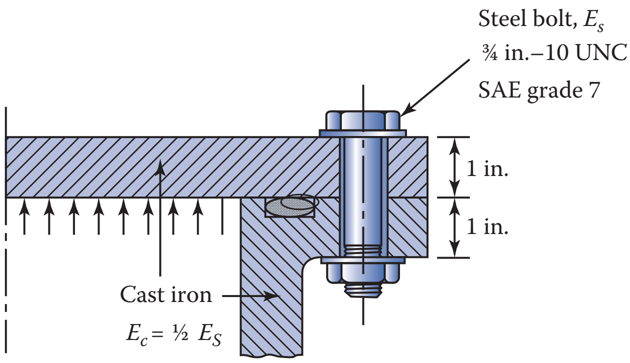


FIGURE 15.15
Example 15.5. Portion of a bolted connection subjected to pressure.

b. From Tables 15.1 and 15.4, we have A_t = 0.334 in.^{2} and S_p = 105 ksi. Applying Equation 15.20,

$$F_i = 0.9S_pA_t = 0.90(105)(0.334) = 31.6 \text{ kips}$$

For N_b bolts, Equation 15.26 can be written in the form

$$\frac{C(P/N_b)n}{A_t} + \frac{F_i}{A_t} = S_p$$

from which

$$N_b = \frac{CPn}{S_pA_t - F_i}$$

Substituting the numerical values, we have

$$N_b = \frac{(0.375)(55)(1.5)}{105(0.334) - 31.6} = 8.92$$

Comment: Nine bolts should be used.

c. By Equation 15.21,

$$T = 0.2dF_i = 0.2(0.75)(31.6) = 4.74 \text{ kips} \cdot \text{in.}$$

Example 15.6: Preloaded Bolt Clamping of a Cylinder under External Load

A steel bolt-and-nut clamps a steel cylinder of known cross section and length subjected to an external load P, as illustrated in Figure 15.16.

Given: D = 20 mm, L = 65 mm, d = 10 mm, $E = E_b = E_p$ = 200 GPa

$$P = 8 \text{ kN} \qquad A_t = 58 \text{ mm}^2 \text{ (from Table 15.2)}$$

$$S_p = 380 \text{ MPa} \quad \text{and} \quad S_y = 420 \text{ MPa (by Table 15.5)}$$

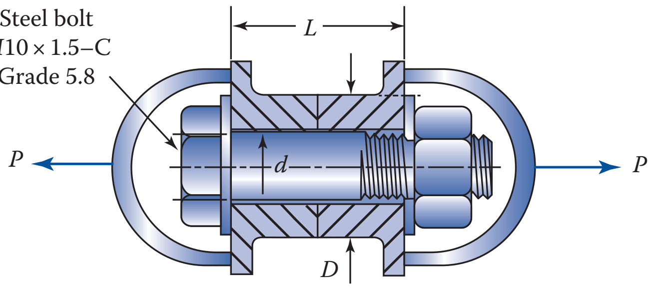


FIGURE 15.16
Example 15.6. A bolted connection carries an axial load.

Find:
a. Preload and bolt tightening torque
b. Joint stiffness factor
c. Maximum tensile stress in the bolt
d. Factors of safety against yielding and separation

Assumptions: Connection is reused. The effects of the flanges on the joint stiffness will be omitted.

Solution

See Figures 15.11 and 15.16.
The cross-sectional area of the parts is equal to $A_p = \pi(D^2 - d^2)/4 = \pi(20^2 - 10^2)/4 = 235.6\,\text{mm}^2$.

a. Through the use of Equation 15.20, the preload is

$$F_i = 0.75F_p = 0.75S_pA_t = 0.75(380)(58) = 16.53\text{ kN}$$

This corresponds to an estimated bolt tightening torque (see Section 15.8) of

$$T = 0.2F_id = 0.2(16.53)(10) = 33.06\text{ N}\cdot\text{m}$$

b. From Equation 15.33a, the lengths of thread L_t and shank L_s of the bolt (Figure 15.12) are

$$L_t = 2d + 6 = 2(10) + 6 = 26\text{ mm}$$

$$L_s = L - L_t = 65 - 26 = 39\text{ mm}$$

The stiffness constant for the bolt, by Equation 15.32, is

$$\frac{1}{k_b} = \frac{L_t}{A_tE} + \frac{L_s}{A_sE} = \frac{1}{200\left(10^6\right)}\left[\frac{26}{58} + \frac{39(4)}{\pi(10)^2}\right], \quad k_b = 2.117\left(10^8\right)\text{ N/m}$$

By Equation 15.31b, the stiffness constant for the parts is

$$k_p = \frac{A_pE}{L} = \frac{235.6\times10^{-6}\left(200\times10^9\right)}{65\times10^{-3}} = 7.249\left(10^8\right)\text{ N/m}$$

The joint stiffness factor, using Equation 15.22, is therefore

$$C = \frac{k_b}{k_p + k_b} = \frac{2.117}{7.249 + 2.117} = 0.226$$

Comment: The results indicate that $k_p \approx 3.4\, k_b$.

c. From Equations 15.23 and 15.24, the forces on the bolt and parts are

$$F_b = F_i + CP = 16.53 + 0.226(8) = 18.34 \text{ kN}$$

$$F_p = F_i - (1 - C)P = 16.53 - (1 - 0.226)(8) = 10.34 \text{ kN}$$

The largest tensile stress in the bolt equals

$$\sigma_b = \frac{F_b}{A_t} = \frac{18.34\left(10^3\right)}{58\left(10^{-6}\right)} = 316 \text{ MPa}$$

Comment: No stress-concentration factor applies for a statically loaded ductile material.

d. The factor of safety with respect to onset of yielding is equal to

$$n = \frac{S_y}{\sigma_b} = \frac{420}{316} = 1.33$$

Applying Equation 15.28, the load required to separate the joint and factor of safety against joint separation are

$$P_s = \frac{F_i}{(1 - C)} = \frac{16.53}{(1 - 0.226)} = 21.36 \text{ kN}$$

$$n_s = \frac{P_s}{P} = \frac{21.36}{8} = 2.67$$

Comment: Both safety factors found against yielding and separation are acceptable.

15.12 Tension Joints under Dynamic Loading

Bolted joints with preload and subjected to fatigue loading can be analyzed directly by the methods discussed in Chapter 7. Since failure owing to fluctuating loading is more apt to occur to the bolt, our attention is directed toward the bolt in this section. As previously noted, the use of initial tension is important in problems for which the *bolt carries cyclic loading*. The maximum and minimum loads on the bolt are higher because of the initial tension. Consequently, the mean load is greater, but the alternating load component is reduced. Therefore, the fatigue effects, which depend primarily on the variations of the stress, are likewise reduced.

Reconsider the joint shown in Figure 15.11a, but let the applied force P vary between some minimum and maximum values, both positive. The mean and alternating loads are given by

$$P_m = \frac{1}{2}(P_{\max} + P_{\min}), \quad P_a = \frac{1}{2}(P_{\max} - P_{\min})$$

Substituting P_m and P_a in place of P in Equation 15.23, the mean and alternating forces felt by the bolt are

$$F_{bm} = CP_m + F_i \tag{15.35a}$$

$$F_{ba} = CP_a \tag{15.35b}$$

The mean and range stresses in the bolt are then

$$\sigma_{bm} = \frac{CP_m}{A_t} + \frac{F_i}{A_t} \tag{15.36a}$$

$$\sigma_{ba} = \frac{CP_a}{A_t} \tag{15.36b}$$

in which C represents the joint constant and A_t is the tensile stress area. We observe from Equation 15.36 that as long as separation does not occur, the alternating stress experienced by the bolt is reduced by the joint stiffness rate C. The mean stress is increased by the bolt preload.

For the bolted joints, the Goodman criterion given by Equation 7.16 may be written as follows:

$$\frac{\sigma_{ba}}{S_e} + \frac{\sigma_{bm}}{S_u} = 1$$

As before, the safety factor is not applied to the initial tension. Hence, introducing Equation 15.36 into this equation, we have

$$\frac{CP_a n}{A_t S_e} + \frac{CP_m n + F_i}{A_t S_u} = 1$$

The preceding is solved to give the *factor of safety* guarding against *fatigue failure* of the *bolt:*

$$n = \frac{S_u A_t - F_i}{C\left[P_a\left(\frac{S_u}{S_e}\right) + P_m\right]} \tag{15.37}$$

TABLE 15.6

Fatigue Stress Concentration Factors K_f for Steel-Threaded Members

SAE Grade (Unified Thread)	Metric Grade (ISO Thread)	Rolled Threads	Cut Threads	Fillet
0–2	3.6–5.8	2.2	2.8	2.1
4–8	6.6–10.9	3.0	3.8	2.3

Alternatively,

$$n = \frac{S_u - \sigma_i}{C\left[\sigma_a\left(\frac{S_u}{S_e}\right) + \sigma_m\right]} \tag{15.38}$$

Here, $\sigma_a = P_a/A_t$, $\sigma_m = P_m/A_t$, and $\sigma_i = F_i/A_t$. Recall from Section 7.9 that this equation represents the Soderberg criterion when ultimate strength S_u is replaced by the yield strength S_y.

The modified endurance limit S_e is obtained from Equation 7.6. For threaded finishes having good quality, a surface factor of $C_f = 1$ may be applicable. The size factor $C_s = 1$ (see Section 7.7), and by Equation 7.3, we have $S'_e = 0.45S_u$ for reversed axial loading. As a result,

$$S_e = C_r C_t\left(\frac{1}{K_f}\right)(0.45S_u) \tag{15.39}$$

where C_r and C_t are the reliability and temperature factors. Table 15.6 gives average stress-concentration factors for the fillet under the bolt and also at the beginning of the threads on the shank [9]. Cutting is the simplest method of producing threads. Rolling the threads provides a smoother thread finish than cutting. The fillet between the head and the shank reduces the K_f, as shown in the table. Unless otherwise specified, the threads are usually assumed to be rolled.

A very *common case* is that the fatigue loading fluctuates between 0 and some maximum value, such as in a bolted pressure vessel cycled from 0 to a maximum pressure. In this situation, the minimum tensile loading $P_{min} = 0$. The effect of initial tension with regard to fatigue loading is illustrated in the solution of the following sample problem.

Example 15.7: Preloaded Fasteners in Fatigue Loading

Figure 15.17a illustrates the connection of two steel parts with a single ⅝ in.–11UNC grade 5 bolt having rolled threads. Determine

a. Whether the bolt fails when no preload is present
b. If the bolt is safe with preload
c. The fatigue factor of safety n when preload is present
d. The static safety factors n and n_s

Design Assumptions: The bolt may be reused when the joint is taken apart. Survival rate is 90%. Operating temperature is normal.

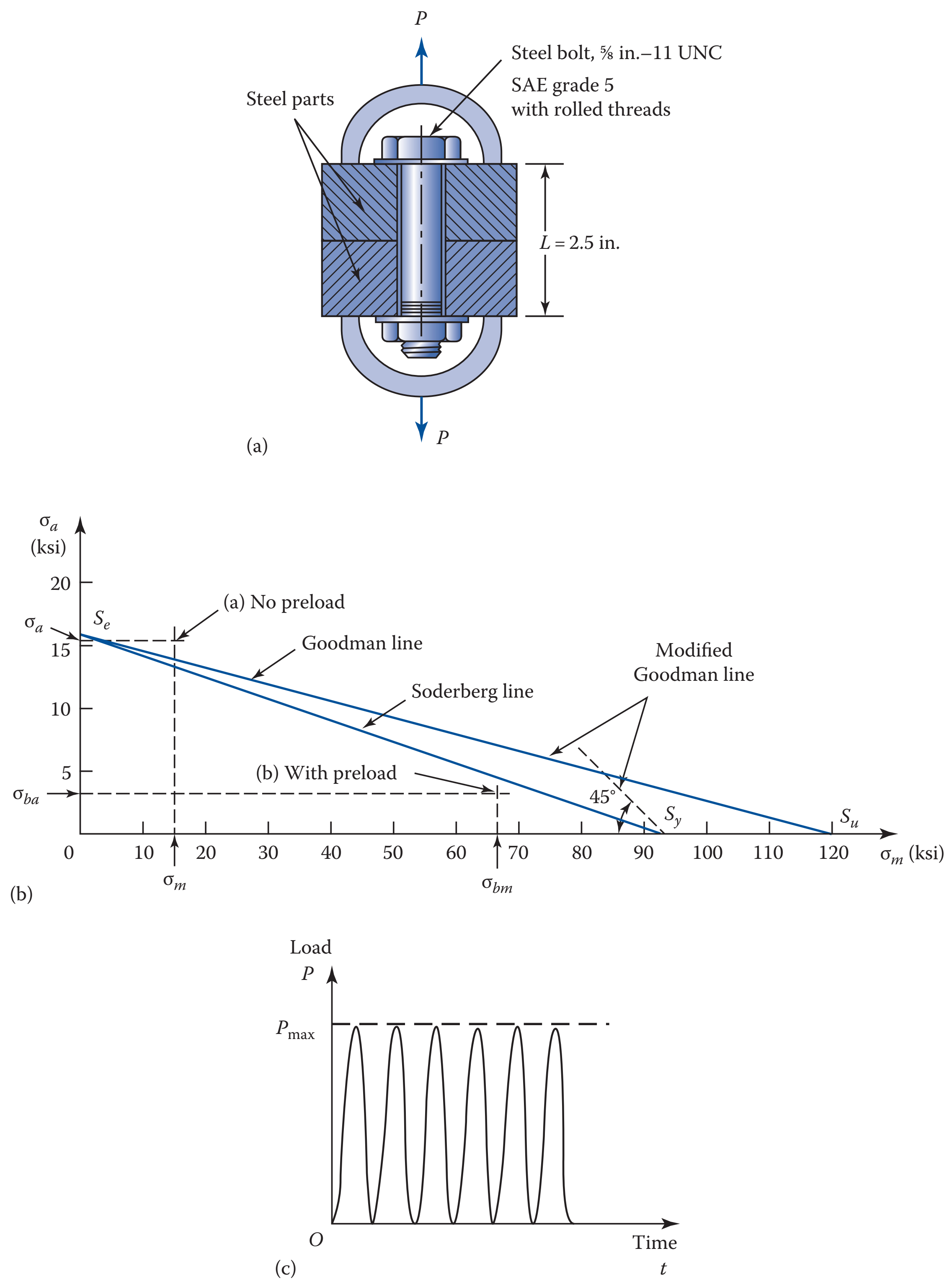


FIGURE 15.17
Example 15.7. (a) bolted parts carrying fluctuating loads, (b) alternating separating load as function of time, and (c) fatigue diagram for bolts.

Given: The joint is subjected to a load P that varies continuously between 0 and 7 kips (Figure 15.17b).

Solution

See Figure 15.17.

From Table 7.3, the reliability factor is $C_r = 0.89$. The temperature factor is $C_t = 1$ (Section 7.7). Also, $S_p = 85$ ksi, $S_y = 92$ ksi, $S_u = 120$ ksi (from Table 15.4), $K_f = 3$ (by Table 15.6), and

$$A_t = 0.226 \text{ in.}^2 \text{ (from Table 15.1)}$$

Equation 15.39 results in

$$S_e = (0.89)(1)\left(\frac{1}{3}\right)(0.45 \times 120) = 16 \text{ ksi}$$

The Soderberg and Goodman fatigue failure lines are shown in Figure 15.17c.

a. For loosely held parts, when $F_i = 0$, load on the bolt equals the load on parts:

$$P_m = \frac{1}{2}(7+0) = 3.5 \text{ kips}, \quad P_a = \frac{1}{2}(7-0) = 3.5 \text{ kips}$$

$$\sigma_a = \sigma_m = \frac{3.5}{0.226} = 15.5 \text{ ksi}$$

A plot of the stresses shown in Figure 15.17c indicates that failure will occur.

b. Through the use of Equation 15.20,

$$F_i = 0.75 S_p A_t = 0.75(85)(0.226) = 14.4 \text{ kips}$$

The grip is $L = 2.5$ in. By Equations 15.31a and 15.34 with $E_b = E_p = E$, we obtain

$$k_b = \frac{\pi d^2 E}{4L} = \frac{\pi(0.625)^2 E}{4(2.5)} = 0.123E$$

$$k_p = \frac{0.58\pi E(0.625)}{2\ln\left[5\dfrac{0.58(2.5)+0.5(0.625)}{0.58(2.5)+2.5(0.625)}\right]} = 0.53E$$

The joint constant is then

$$C = \frac{k_b}{k_b + k_p} = \frac{0.123}{0.123 + 0.53} = 0.188$$

Comment: The foregoing means that only about 20% of the external load fluctuation is felt by the bolt and hence about 80% goes to decrease clamping pressure.

Applying Equations 15.35 and 15.36,

$$F_{bm} = CP_m + F_i$$

$$= 0.188(3.5) + 14.4 = 15.1 \text{ kips}$$

$$\sigma_{bm} = \frac{15.1}{0.226} = 66.8 \text{ ksi}$$

$$F_{ba} = CP_a = 0.188(3.5) = 0.66 \text{ kips}$$

$$\sigma_{ba} = \frac{0.66}{0.226} = 2.92 \text{ ksi}$$

A plot on the fatigue diagram shows that *failure* will *not occur* (Figure 15.17c).

c. Equation 15.37 with $P_a = P_m$ becomes

$$n = \frac{S_u A_t - F_i}{CP_a\left[\left(\frac{S_u}{S_e}\right) + 1\right]} \tag{15.40}$$

Introducing the given numerical values,

$$n = \frac{(120)(0.226) - 14.4}{(0.188)(3.5)\left[\left(\frac{120}{16}\right) + 1\right]}$$

from which $n = 2.27$.

Comment: This is the factor of safety guarding against the fatigue failure. Observe from Figure 15.17c that the Goodman criteria led to a less conservative (higher) value for n.

d. Substitution of the given data into Equations 15.27 and 15.28 gives

$$n = \frac{85(0.226) - 14.4}{(0.188)(7)} = 3.66$$

$$n_s = \frac{14.4}{7(1 - 0.188)} = 2.53$$

Comments: The factor of 3.66 prevents the bolt stress from becoming equal to proof strength. On the other hand, the factor of 2.53 guards against joint separation and the bolt taking the entire load.

15.13 Riveted and Bolted Joints Loaded in Shear

A rivet consists of a cylindrical body, known as the *shank*, usually with a rounded end called the *head*. The purpose of the rivet is to join together two plates while securing proper strength and tightness. If the rivet is heated prior to being placed in the hole, it is referred to as a hot-driven rivet, while if it is not heated, it is referred to as a cold-driven rivet.

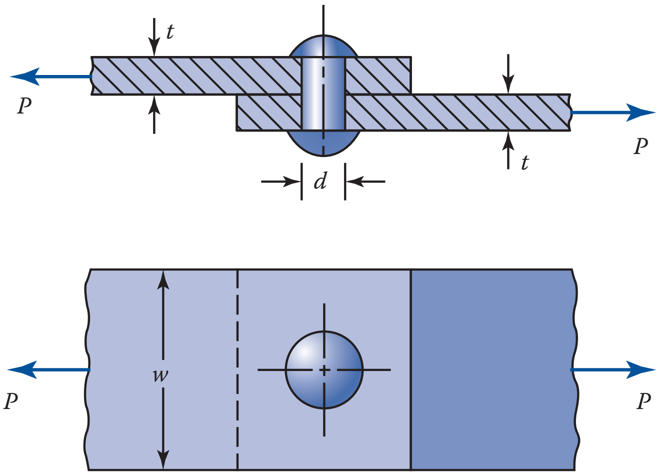


FIGURE 15.18
Riveted connection loaded in shear.

Rivets and bolts are ordinarily used in the construction of buildings, bridges, aircraft, and ships. The design of riveted and bolted connections is governed by construction codes formulated by such societies as the AISC [14] and the ASME.

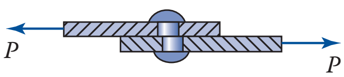
Riveted and bolted joints loaded in shear are treated *exactly alike* in design and analysis. Figure 15.18 illustrates a simple riveted connection loaded in shear. It is obvious that the loading is eccentric and an unbalanced moment Pt exists. Hence, bending stress will be present. However, the usual procedure is to ignore the bending stress and compensate for its presence by a larger factor of safety. Table 15.7 lists various types of failure of the connection shown in the figure.

The *effective diameters* in a riveted joint are defined as follows. For a drilled hole, $d_e = d + \frac{1}{16}$ in (about 1.5 mm), and for a *punched hole*, $d_e = d + \frac{1}{8}$ in (about 3 mm). Here, d represents the diameter of the rivet. Unless specified otherwise, we *assume* that the holes have

TABLE 15.7

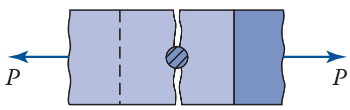
Types of Failure for Riveted Connections (Figure 15.18)

A. Shearing Failure of Rivet



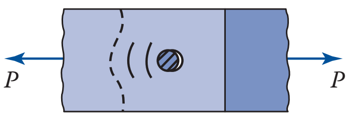
$\tau = 4P/\pi d^2$

B. Tensile Failure of Plate



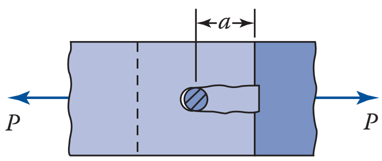
$\sigma_t = P/(w - d_e)t$

C. Bearing Failure of Plate or Rivet



$\sigma_b = P/dt$

D. Shearing Failure of Edge of Plate



$\tau_t = P/2at$

Notes: P, applied shear load; d, diameter of rivet; w, width of plate; t, thickness of the thinnest plate; d_e, effective hole diameter; a, the closest distance from rivet to the edge of plate.

been *punched.* Usually, shearing, or tearing, failure is avoided by spacing the rivet at least 1.5d away from the plate edge. To sum up, essentially, three modes of failure must be considered in determining the capacity of a riveted or bolted connection: *shearing* failure of the rivet, *bearing* failure of the plate or rivet, and *tensile* failure of the plate. The associated normal and shear equations are given in the table.

Example 15.8: Capacity of a Riveted Connection

The standard AISC connection for the W310 × 52 beam consists of two 102 × 102 × 6.4 mm angles, each 215 mm long, 22 mm rivets spaced 75 mm apart are used in 24 mm holes (Figure 15.19). Calculate the maximum load that the connection can carry.

Design Decisions: The allowable stresses are 100 MPa in shear and 335 MPa in bearing of rivets. Tensile failure cannot occur in this connection; only shearing and bearing capacities need to be investigated.

Solution

The web thickness of the beam is $t_w = 7.6$ mm (from Table A.6), and the cross-sectional area of one rivet is $A_r = \pi(22)^2/4 = 380\ \text{mm}^2$.

Bearing on the web of the beam:

$$P_b = 3(7.6)(22)(335) = 168\ \text{kN} \quad \text{(governs)}$$

Shear of six rivets:

$$P_b = 6(380)(100) = 228\ \text{kN}$$

Bearing of six rivets on angles:

$$P_b = 6(22)(6.4)(335) = 283\ \text{kN}$$

Comment: The capacity of this connection, the smallest of the forces obtained in the foregoing, is 168 kN.

15.13.1 Joint Types and Efficiency

Most connections have many rivets or bolts in a variety of models. Riveted or bolted connections loaded in shear are of two types: *lap joints* and *butt joints.* In a lap joint, sometimes called a *single-shear joint,* the two plates to be jointed overlap each other (Figure 15.20a). On the other hand, in a butt, also termed a *double-shear joint,* the two plates to be connected (main plates) butt against one other (Figure 15.20b). *Pitch* is defined as the distance between

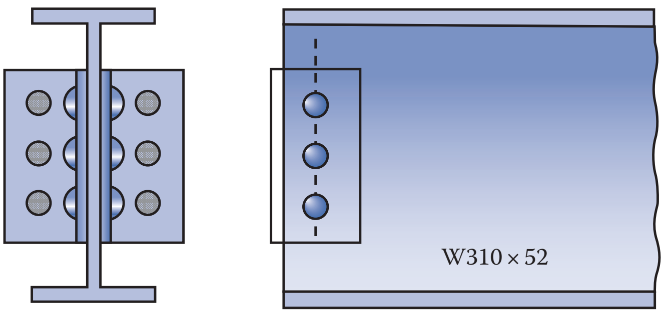


FIGURE 15.19
Example 15.8.

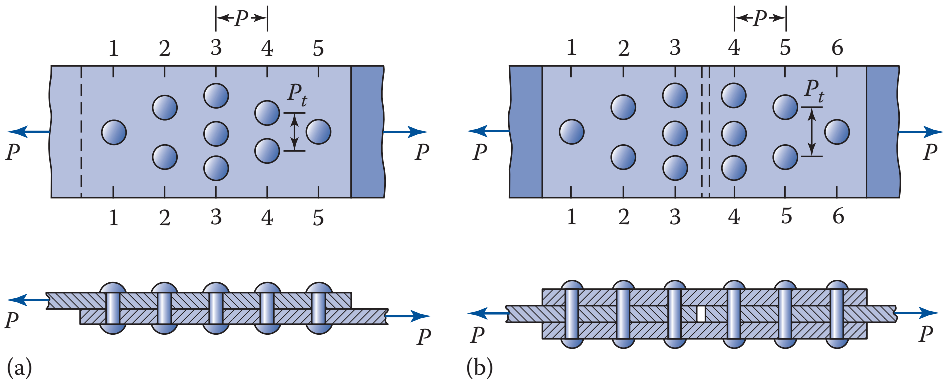


FIGURE 15.20
Types of riveted connections: (a) lap joint and (b) butt joint.

adjacent rivet centers. It represents a significant geometric property of a joint. The *axial* pitch p for rivets is measured along a line parallel to the edge of the plate, while the corresponding distance along a line perpendicular to the edge of the plate is known as the *transverse* pitch p_t. Both kinds of pitch are depicted in the figure. The smallest symmetric group of rivets that repeats itself along the length of a joint is called a *repeating section*. The strength analysis of a riveted connection is based on its repeated section (see Example 15.9).

The *efficiency of joints* is defined as follows:

$$e = \frac{P_{\text{all}}}{P_t} \tag{15.41}$$

In the foregoing equation, P_{all} is the smallest of the allowable loads in shear, bearing, and tension; P_t represents the static tensile yield load (strength) of plate with no hole. The most efficient joint would be as strong in tension, shear, and bearing as the original plate to be joined is in tension. This can never be realized, since there must be at least one rivet hole in the plate: The allowable load of joint in tension therefore always is less than the strength of the plate with no holes.

For centrally applied loads, it is often assumed that the rivets are about equally stressed. In many cases, this cannot be justified by elastic analysis; however, ductile deformations permit an equal redistribution of the applied force, before the ultimate capacity of connection is reached. Also, it is usually taken that the row of rivets immediately *adjacent* to the load carries the full load. Thus, the maximum load supported by such a row occurs when there is only one rivet in that row. The *actual load* carried by an interior row can be obtained from

$$P_i = \frac{n - n'}{n} P \tag{15.42}$$

where

P = the externally applied load
P_i = the actual load, or portion of P, acting on a particular row i
n = the total number of rivets in the joint
n' = the total number of rivets in the row between the row being checked and the external load

For instance, load on row 3 of the joint in Figure 15.20a equals $P_3 = (9 - 3)P/9 = 2P/3$. Likewise, load on row 2 or 5 of the joint in Figure 15.20b is $P_2 = P_5 = (12 - 1)P/12 = 11P/12$.

Example 15.9: Strength Analysis of a Multiple-Riveted Lap Joint

Figure 15.21a shows a multiple-riveted lap joint subjected to an axial load P. The dimensions are given in inches. Calculate the allowable load and efficiency of the joint.

Given: All rivets are ¾ in. in diameter.

Design Assumptions: The allowable stresses are 20 ksi in tension, 15 ksi in shear, and 30 ksi in bearing.

Solution

The analysis is on the basis of the repeating section, which has four rivets and $L = 6$ in. (Figure 15.21b).

The plate in tension, without holes:

$$P_t = 20\left(6 \times \frac{1}{2}\right) = 60 \text{ kips}$$

The rivet shear:

$$P_s = 4\left[\frac{\pi}{4}\left(\frac{3}{4}\right)^2\right](15) = 26.51 \text{ kips} \quad \text{(governs)}$$

The plate bearing:

$$P_b = 4\left(\frac{1}{2} \times \frac{3}{4}\right)(30) = 45 \text{ kips}$$

The tension across sections 1–1 through 3–3 of the *bottom plate*, using Equation 15.42:

$$\frac{4-3}{4}P_1 = \frac{1}{2}\left[6 - \left(\frac{3}{4} + \frac{1}{8}\right)\right](20); \quad P_1 = 205 \text{ kips}$$

$$\frac{4-1}{4}P_2 = \frac{1}{2}\left[6 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\right](20); \quad P_2 = 56.7 \text{ kips}$$

$$P_3 = \frac{1}{2}\left[6 - \left(\frac{3}{4} + \frac{1}{8}\right)\right](20); \quad P_3 = 51.25 \text{ kips}$$

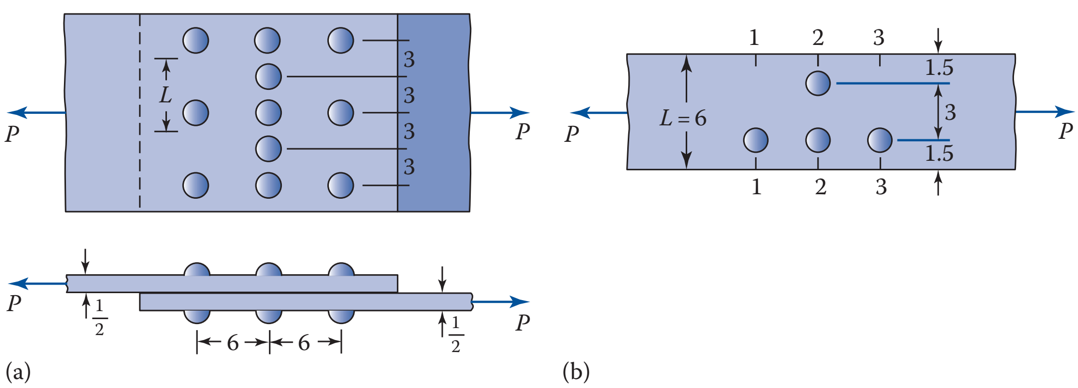


FIGURE 15.21
Example 15.9. (a) a riveted lap joint and (b) enlarged view of a repeating group of rivets.

The maximum allowable force that the joint can safely carry is the smallest of the force obtained in the preceding, P_{all} = 26.51 kips. The efficiency of this joint, from Equation 15.41, is then

$$e = \frac{26.51}{60} \times 100 = 44.2\%$$

15.14 Shear of Rivets or Bolts due to Eccentric Loading

For the case in which the load is applied eccentrically to a connection having a group of bolts or rivets, the effects of the torque or moment, as well as the direct force, must be considered. A typical structural problem is the situation that occurs when a horizontal beam is supported by a vertical column (Figure 15.22a). In this case, each bolt is subjected to a twisting moment $M = Pe$ and a direct shear force P. An enlarged view of bolt group with loading (P and M) acting at the centroid C of the group and the reactional shear forces acting at the cross section of each bolt are shown in Figure 15.22b.

Let us assume that the reactional *tangential force* due to moment, the so-called moment load or secondary shear, on a bolt varies directly with the distance from the centroid C of the group of bolts and is directed perpendicular to the centroid. As a result,

$$\frac{F_1}{r_1} = \frac{F_2}{r_2} = \frac{F_3}{r_3} = \frac{F_4}{r_4}$$

In the preceding, F_i and r_i ($i = 1,\ldots,4$) are the tangential force and radial distance from C to the center of each bolt, respectively. The externally applied moment and tangential forces are related as follows:

$$M = Pe = F_1 r_1 + F_2 r_2 + F_3 r_3 + F_4 r_4$$

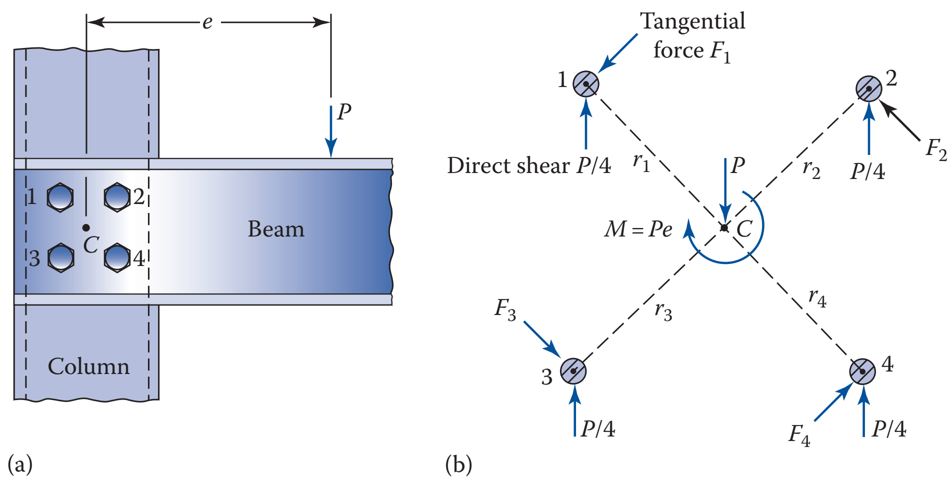


FIGURE 15.22
(a) Bolted joint with eccentric load. (b) Bolt group with loading and reactional shear forces (15.43).

Solving these equations simultaneously, we obtain

$$F_1 = \frac{Per_1}{r_1^2 + r_2^2 + r_3^2 + r_4^2}$$

This expression can be written in the following general form:

$$F_i = \frac{Mr_j}{\sum_{j=1}^{n} r_j^2} \tag{15.43}$$

where
F_i = the tangential force
$M = Pe$, externally applied moment
n = the number of bolts in the group
i = the particular bolt whose load is to be found

It is customary to assume that the reactional *direct force F/n* is the same for all bolts of the joint. The vectorial sum of the tangential force and direct force is the resultant shear force on the bolt (Figure 15.22b). Clearly, only the bolt having the maximum resultant shear force needs to be considered. An inspection of the vector force diagram is often enough to eliminate all but two or three bolts as candidates for the worst-loaded bolt.

Example 15.10: Bolt Shear Forces due to Eccentric Loading

A gusset plate is attached to a column by three identical bolts and vertically loaded, as shown in Figure 15.23a. The dimensions are in millimeters. Calculate the maximum bolt shear force and stress.

Assumption: The bolt tends to shear across its major diameter.

Solution

For the bolt group, point C corresponds to the centroid of the triangular pattern, as shown in Figure 15.23b. This free-body diagram illustrates the bolt reactions and the

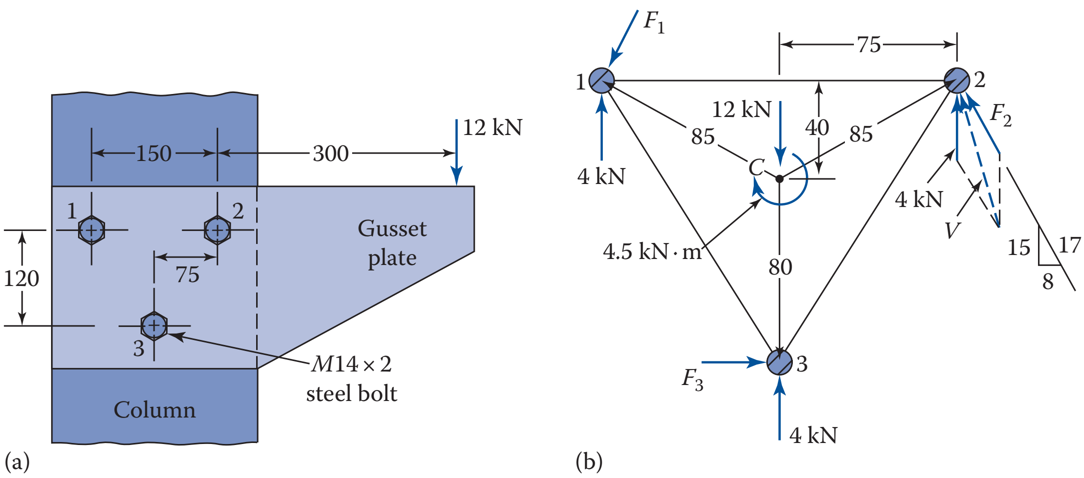


FIGURE 15.23
Example 15.10. (a) bolted connection and (b) bolt shear force and moment equilibrium.

external loading replaced at the centroid. Each bolt supports one-third of the vertical shear load, 4 kN, plus a tangential force F_i. The distances from the centroid to bolts are

$$r_1 = r_2 = \sqrt{(40)^2 + (75)^2} = 85.0 \text{ mm}, \quad r_3 = 80 \text{ mm}$$

Equation 15.43 then results in

$$F_1 = F_2 = \frac{Mr_1}{r_1^2 + r_2^2 + r_3^2} = \frac{4500(85)}{2(85)^2 + (80)^2}$$

$$= \frac{382{,}500}{20{,}850} = 18.35 \text{ kN}$$

$$F_3 = \frac{4{,}500(80)}{20{,}850} = 17.27 \text{ kN}$$

The vector sum of the two shear forces, obviously greatest for the bolt 2, can be obtained algebraically (or graphically):

$$V_2 = \left[\left(\frac{15}{17} \times 18.35 + 4\right)^2 + \left(\frac{8}{17} \times 18.35\right)^2\right]^{1/2} = 21.96 \text{ kN}$$

The bolt shear stress area is $A_s = \pi d^2/4 = \pi(14)^2/4 = 153.9$ mm^2. Hence,

$$\tau = \frac{V_2}{A_s} = \frac{21{,}960}{153.9(10^{-6})} = 142.7 \text{ MPa}$$

Example 15.11: Shear Stress in Rivets Owing to Eccentric Loading

A riveted joint is under an inclined eccentric force P, as indicated in Figure 15.24a. Calculate the maximum shear stress in the rivets.

Given: The rivets are 1 in. in diameter. $P = 10$ kips.

Solution

For simplicity in computations, the applied load P is first resolved into horizontal and vertical components. Each rivet carries one-half of the load. The centroid of the

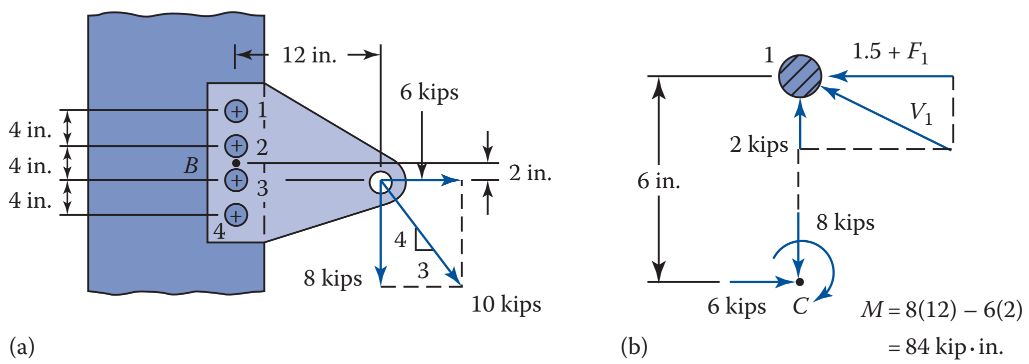


FIGURE 15.24
Example 15.11. (a) riveted connection and (b) enlarged view showing loading acting at the centroid and reactions on rivet 1.

rivet group is between the top and bottom rivets at C. An inspection of Figure 15.24a shows that the top rivet 1 is under the highest stress (Figure 15.24b). Through the use of Equation 15.43,

$$F_1 = \frac{Mr_1}{r_1^2 + r_2^2 + r_3^2 + r_4^2} = \frac{84(6)}{2(6)^2 + 2(2)^2} = 6.3 \text{ kip} \cdot \text{in}$$

Vector sum of the shear forces is

$$V_1 = \left[(1.5 + 6.3)^2 + 2^2\right]^{1/2} = 8.052 \text{ kips}$$

We then have

$$\tau = \frac{V_1}{\pi d^2/4} = \frac{4(8.052)}{\pi(1)^2} = 10.25 \text{ ksi}$$

15.15 Welding

A *weld* is a joint between two surfaces produced by the application of localized heat. Here, we briefly discuss only welding between metal surfaces: thermoplastics can be welded much like metals. A *weldment* is fabricated by welding together a variety of metal forms cut to particular configuration. Nearly all wielding is by fusion processes. Establishment of metallurgical bond between two parts by melting together the base metals with a filler metal is called the *fusion process.* Heat is brought about usually by an electric arc, electric current, or gas flame. Metals and alloys to arc and gas welding must be properly selected. Properties of welding filler material must be matched with those of base metal when possible. The joint strength would then be equal to the strength of the base metal, giving an efficiency of almost 100% for static loads.

15.15.1 Welding Processes and Properties

Metallic arc welding, the so-called shielded metal arc welding (SMAW), refers to a process where the heat is applied by an arc passing between an electrode and the work. The electrode is composed of suitable filler material with coating ordinarily similar to that of base metal. It is melted and fed into the joint as the weld is being formed. The coating is vaporized to provide a shielding gas-preventing oxidation at the weld as well as acts as a flux and directs the arc. Either direct or alternating current can be used with this process. A weld thickness greater than about ⅜ in. is often produced on successive layers. In *metal–inert gas arc welding* or gas–metal arc welding (GMAW), heat is applied by a gas flame. In this process, a bare or plated wire is continuously fed into the weld from a large spool. The wire serves as electrode and becomes the filler in the union. Uniform-quality welds are attainable with metal–gas welding.

Resistance welding uses electric-current-generated heat that passes through the parts to be welded while they are clamped together firmly. Filler material is not ordinarily

TABLE 15.8

Typical Weld-Metal Properties

	Ultimate Strength		Yield Strength		
AWS Electrode Number	**ksi**	**(MPa)**	**ksi**	**(MPa)**	**Percent Elongation**
E6010	62	(427)	50	(345)	22
E6012	67	(462)	55	(379)	17
E6020	62	(427)	50	(345)	25
E7014	72	(496)	60	(414)	17
E7028	72	(496)	60	(414)	22

Source: American Welding Society Code AWSD.1.77, American Welding Society, Miami, FL.

employed. Usually, thin metal parts may be connected by spot or continuous resistance welding. A spot weld is made by a pair of electrodes that apply pressure to either side of a lap joint and devise a complete circuit. Laser beam welding, plasma arc welding, and electron beam welding are utilized for special applications. The suitability of several metals and alloys to arc and gas welding is very important.

Materials and symbols for welding have been standardized by the ASTM and the American Welding Society (AWS). Numerous different kinds of electrodes have been standardized to fit a variety of conditions encountered in the welding of machinery and structures. Table 15.8 presents the characteristics for some E60 and E70 electrode classes. Note that the AWS *numbering system* is based on the use of an *E* prefix followed by four digits. The first two numbers on the left identify the approximate strength in ksi. The last digit denotes a group of welding technique variables, such as current supply. The next to last digit refers to a welding position number (1 for all and 2 for horizontal positions, respectively). Welding electrodes are available in diameters from $\frac{1}{16}$ to $\frac{5}{16}$ in. It should be mentioned that the *electrode* material is often the strongest material present in a joint [15,16].

15.15.2 Strength of Welded Joints

Among numerous configurations of welds, we consider only two common butt and fillet types. The geometry of a typical *butt weld* loaded in tension and shear is shown in Figure 15.25. The equations for the stresses due to the loading are also given in the figure. Note that the height *h* for a butt weld does not include the bulge or reinforcement used to compensate for voids or inclusions in the weld metal. Plates of ¼ in. and heavier should be beveled before welding as indicated.

Figure 15.26 illustrates two *fillet welds* loaded in shear and transverse tension. The corresponding average stress formula is written under each figure. Now the size of the fillet weld is defined as the leg length *h.* Normally, the two legs are of the same length *h.* In welding design, stresses are calculated for the *throat section:* minimum cross-sectional area A_t located at 45° to the legs. We have $A_t = tL = 0.707hL$, where t and L represent the throat length and length of weld, respectively (Figure 15.26a). We note that actual stress distribution in a weld is somewhat complicated and design depends on the stiffness of the base material and other factors that have been neglected. Particularly, for stress situation on the throat area in Figure 15.26b, no exact solutions are available.

The foregoing average results are valid for design, however, because weld strengths are on the basis of tests on joints of these types. Having the material strengths available for a

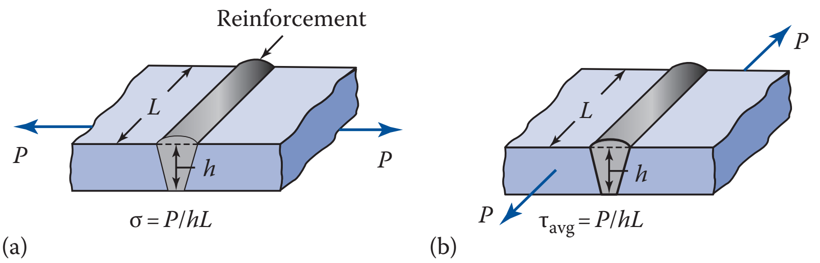


FIGURE 15.25
Butt weld: (a) tension loading and (b) shear loading.

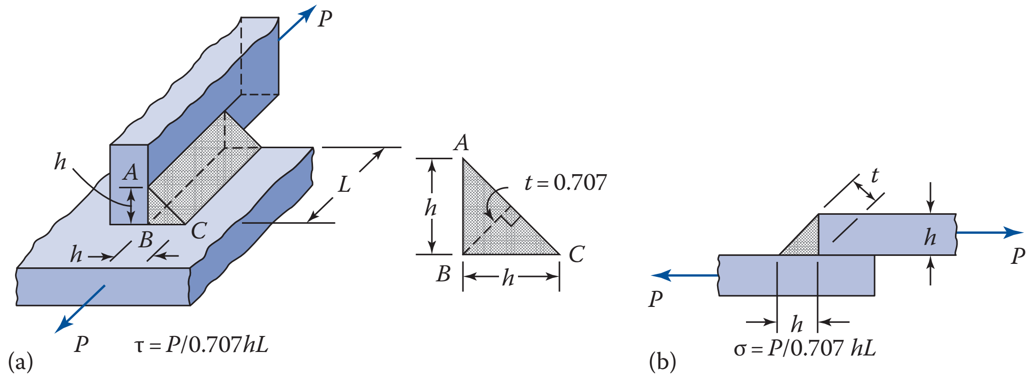


FIGURE 15.26
Fillet weld: (a) shear loading and (b) transverse tension loading. *Notes*: h is the length of weld leg, t is the throat length, and L is the weld length.

welded joint, the required weld size h can be obtained for a prescribed safety factor. The usual equation of the *factor of safety n* applies for *static loads:*

$$n = \frac{S_{ys}}{\tau} = \frac{0.5S_y}{\tau} \tag{15.44}$$

The quantities S_y and S_{ys} represent tensile yield and shear yield strengths of weld material, respectively.

15.15.3 Stress Concentration and Fatigue in Welds

Abrupt changes in geometry take place in welds, and hence, stress concentrations are present. The weld and the plates at the base and reinforcement should be thoroughly blended together (Figure 15.25). The stresses are highest in the immediate vicinity of the weld. Sharp corners at the *toe* and *heel*, points A and B in Figure 15.26, should be rounded. Since welds are ductile materials, stress concentration effects are ignored for static loads. As has always been the case, when the loading fluctuates, a stress concentration factor is applied to alternating component. Approximate values for fatigue strength reduction factors are listed in Table 15.9.

Under *cyclic loading,* the welds fail long before the welded members. The fatigue factor of safety and working stresses in welds are defined by the AISC as well as AWS codes for buildings and bridges [2,17]. The codes allow the use of a variety of ASTM structural steels. For ASTM steels, tensile yield strength is one-half of the ultimate strength in tension, $S_y = 0.5S_u$ for static or fatigue loads. Unless otherwise specified, an *as-forged surface* should always be used for weldments. Also, prudent design would suggest taking the size factor

TABLE 15.9

Fatigue Stress Concentration Factors K_f for Welds

Type of Weld	K_f
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0

Source: American Welding Society Code AWSD.1.77, American Welding Society, Miami, FL.

$C_s = 0.7$. Design calculations for fatigue loading can be made by the methods described in Section 7.11, as illustrated in the following sample problem.

Example 15.12: Design of a Butt Welding for Fatigue Loading

The tensile load P on a butt weld (Figure 15.25a) fluctuates continuously between 20 and 100 kN. Plates are 20 mm thick. Determine the required length L of the weld, applying the Goodman criterion.

Assumptions: Use an E6010 welding rod with a factor of safety of 2.5.

Solution

By Table 15.8 for E6010, S_u = 427 MPa. The endurance limit of the weld metal, from Equation 7.6, is

$$S_e = C_f C_r C_s C_t (1/K_f) S'_e$$

Referring to Section 7.7, we have

$C_r = 1$ (based on 50% reliability)
$C_s = 0.7$ (lacking information)
$C_f = AS^b = 272(427)^{-0.995} = 0.657$ (by Equation [7.7])
$C_t = 1$ (normal room temperature)
$K_f = 1.2$ (from Table 15.9)
$S'_e = 0.5S_u = 0.5(427) = 213.5$ MPa

Hence,

$$S_e = (1)(0.7)(0.657)(1)(1/1.2)(213.5) = 81.82 \text{ MPa}$$

The mean and alternating loads are given by

$$P_m = \frac{100+20}{2} = 60 \text{ kN}, \quad P_a \frac{100-20}{2} = 40 \text{ kN}$$

Corresponding stresses are

$$\sigma_m = \frac{60{,}000}{20L} = \frac{3000}{L}$$

$$\sigma_a = \frac{40{,}000}{20L} = \frac{2000}{L}$$

Through the use of Equation 7.16, we have

$$\frac{S_u}{n} = \sigma_m + \frac{S_u}{S_e}\sigma_a; \qquad \frac{427}{2.5} = \frac{3000}{L} + \frac{427}{81.82}\left(\frac{2000}{L}\right)$$

Solving,

$$L = 78.67 \text{ mm}$$

Comment: A 79 mm long weld should be used.

15.16 Welded Joints Subjected to Eccentric Loading

When a welded joint is under eccentrically applied loading, the effect of torque or moment must be taken into account as well as the direct load. The exact stress distribution in such a joint is complicated. A detailed study of both the rigidity of the parts being joined and the geometry of the weld is required. The following procedure, which is based on simplifying assumptions, leads to reasonably accurate results for most applications.

15.16.1 Torsion in Welded Joints

Figure 15.27 illustrates an eccentrically loaded joint, with the centroid of all the weld areas or weld group at point *C*. The load *P* is applied at a distance *e* from *C*, in the plane of the group.

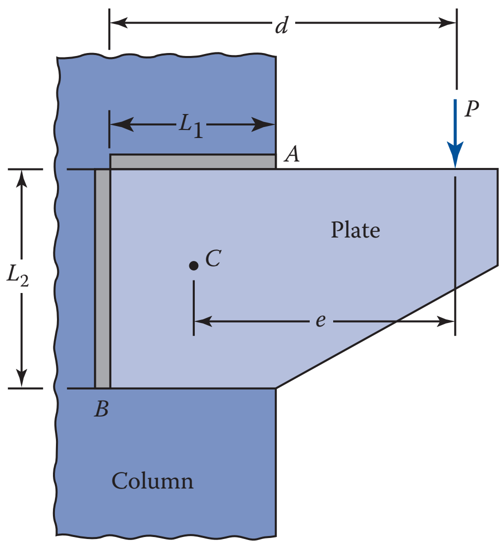


FIGURE 15.27
Welded joint in plane eccentric loading. *Notes*: *C* is the centroid of the weld group.

As a result, the welded connection is under torsion $T = Pe$ and the direct load P. The latter force causes a direct shear stress in the welds:

$$\tau_d = \frac{P}{A} \tag{15.45}$$

in which P is the applied load and A represents the throat area of all the welds. The preceding stress is taken to be uniformly distributed over the length of all welds. The torque causes the following torsional shear stress in the welds:

$$\tau_t = \frac{Tr}{J} \tag{15.46}$$

where
- T = the torque
- r = the distance from C to the point in the weld of interest
- J = the polar moment of inertia of the weld group about C (based on the throat area)

Resultant shear stress in the weld at radius r is given by the vector sum of the direct shear stress and torsional stress:

$$\tau = \left(\tau_d^2 + \tau_t^2\right)^{1/2} \tag{15.47}$$

Note that r usually represents the *farthest* distance from the centroid of the weld group.

15.16.2 Bending in Welded Joints

Consider an angle welded to a column, as depicted in Figure 15.28. Load P acts at a distance e, out of plane of the weld group, producing bending in addition to direct shear. We again take a linear distribution of shear stress due to moment $M = Pe$ and a uniform distribution of direct shear stress. The latter stress τ_d is given by Equation 15.45. The moment causes the shear stress:

$$\tau_m = \frac{Mc}{I} \tag{15.48}$$

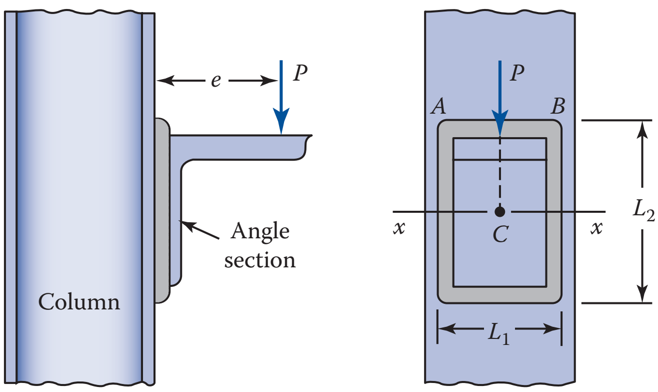


FIGURE 15.28
Welded joint under out-of-plane loading.

Here, the distance c is measured from C to the farthest point on the weld. As in the previous case, the resultant shear stress τ in the weld is estimated by the vector sum of the direct shear stress and the moment-induced stress:

$$\tau = \left(\tau_d^2 + \tau_m^2\right)^{1/2} \tag{15.49}$$

On the basis of the geometry and loading of Figure 15.28, we note that τ_d is downward and τ_m along edge AB is outward.

15.16.2.1 Centroid of the Weld Group

Let A_i denote the weld segment area and x_i and y_i the coordinates to the centroid of any (straight line) segment of the weld group. Then, the centroid C of the weld group is located at

$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i}, \quad \bar{y} = \frac{\sum A_i y_i}{\sum A_i} \tag{15.50}$$

in which $i = 1. 2,\ldots, n$ for n welds. In the case of symmetric weld group, the location of the centroid is obvious.

15.16.2.2 Moments of Inertia of a Weld (Figure 15.29)

For simplicity, we assume that the effective weld width in the plane of the paper is the same as throat length $t = 0.707h$, shown in Figure 15.26a. The parallel axis theorem can be applied to find the moments of inertia about x and y axes through the centroid of the weld group:

$$\begin{aligned} I_x &= I_{x'} + Ay_1^2 = \frac{tL^3}{12} + Lty_1^2 \\ I_y &= I_{y'} + Ax_1^2 = \frac{Lt^3}{12} + Ltx_1^2 = Ltx_1^2 \end{aligned} \tag{15.51}$$

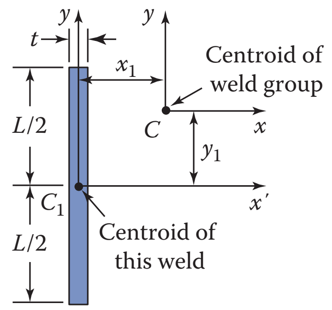


FIGURE 15.29
Moments of inertia of a weld parallel to the y axis.

Note that t is assumed to be *very small* in comparison with the other dimensions and hence $I_{y'} = Lt^3/12 = 0$ in the second of the preceding equations. The polar moment of inertia about an axis through C perpendicular to the plane of the weld is then

$$J = I_x + I_y = \frac{tL^3}{12} + Lt\left(x_1^2 + y_1^2\right) \tag{15.52}$$

The values of I and J for each weld about C should be calculated by using Equations 15.51 and 15.52: the results are added to obtain the moment and product of inertia of the entire joint. It should be mentioned that the moment and polar moment of inertias for the most common fillet welds encountered are listed in some publications [11]. The detailed procedure is illustrated in Case Study 18.9 and in the following sample problem.

Example 15.13: Design of a Welded Joint under Out-of-Plane Eccentric Loading

A welded joint is subjected to out-of-plane eccentric force P (Figure 15.28). What weld size is required?

Given: $L_1 = 60$ mm, $L_2 = 90$ mm, $e = 50$ mm, $P = 15$ kN

Assumption: An E6010 welding rod with factor of safety $n = 3$ is used.

Solution

By Table 15.8, for E6010, $S_y = 345$ MPa. The centroid lies at the intersection of the two axes of symmetry of the area enclosed by the weld group. The moment of inertia is

$$I_x = 2(60)t(45)^2 + \frac{2(90)^3 t}{12} = 364,500t \text{ mm}^4$$

The total weld area equals $A = 2(60t + 90t) = 300t$ mm^2. Moment is $M = 15(50) = 750$ kN mm. The maximum shear stress, using Equation 15.49, is

$$\tau = \left[\left(\frac{15,000}{300t}\right)^2 + \left(\frac{750,000 \times 45}{364,500t}\right)^2\right]^{1/2} = \frac{105.2}{t} \text{ N/mm}^2$$

Applying Equation 15.44, we have

$$n\tau = 0.5S_y; \quad 3\left(\frac{105.2}{t}\right) = 0.5(345) \quad \text{or} \quad t = 1.83 \text{ mm}$$

Hence,

$$h = \frac{t}{0.707} = \frac{1.83}{0.707} = 2.59 \text{ mm}$$

Comment: A nominal size of 3 mm fillet welds should be used throughout.

15.17 Brazing and Soldering

Brazing and soldering differ from welding essentially in that the temperatures are always below the melting point of the parts to be united, but the parts are heated above the melting point of the solder. It is important that the surfaces initially be clean. Soldering or brazing filler material acts somewhat similar to a molten metal glue or cement, which sets directly on cooling. Brazing or soldering can thus be categorized as bonding.

15.17.1 Brazing Process

Brazing starts with heating the workpieces to a temperature above 450°C. On contact with the parts to be united, the filler material melts and flows into the space between the workpieces. The filler materials are customarily alloys of copper, silver, or nickel. These may be handheld and fed into the joint (free of feeding) or preplaced as washers, shims, rings, slugs, and the like. Dissimilar metals, cast, and wrought metals, as well as nonmetals and metals, can be brazed together. Brazing is ordinarily accomplished by heating parts with a torch or in a furnace. Sometimes, other brazing methods are used. A brief description of some processes of brazing follows. Note that, in all metals, either flux or an inert gas atmosphere is required.

Torch brazing utilizes acetylene, propane, and other fuel gas, burned with oxygen or air. It may be manual or mechanized. On the other hand, *furnace* brazing uses the heat of a gas-fired, electric, or other kind of furnace to raise the parts to brazing temperature. A technique that utilizes a high-frequency current to generate the required heat is referred to as *induction* brazing. As the name suggests, *dip* brazing involves the immersion of the parts in a molten bath. A method that utilizes resistance-welding machines to supply the heat is called *resistance* brazing. As currents are large, water cooling of electrodes is essential.

15.17.2 Soldering Process

The procedure of soldering is identical to that of brazing. However, in soldering, the filler metal has a melting temperature below 450°C and relatively low strength. Heating can be done with a torch or a high-frequency induction heating coil. Surfaces must be clean and covered with flux that is liquid at the soldering temperature. The flux is drawn into the joint and dissolves any oxidation present at the joint. When the soldering temperature is reached, the solder replaces the flux at the joint.

Cast iron, wrought iron, and carbon steels can be soldered to each other or to brass, copper, nickel, silver, Monel, and other nonferrous alloys. Nearly all solders are tin–lead alloys, but alloys including antimony, zinc, and aluminum are also employed. The strength of a soldered union depends on numerous factors, such as the quality of the solder, thickness of the joint, smoothness of the surfaces, kind of materials soldered, and soldering temperature. Some common soldering applications involve electrical and electronic parts, sealing seams in radiators and in thin cans.

15.18 Adhesive Bonding

Adhesives are substances able to hold materials together by surface attachment. Nearly all structural adhesives are thermosetting as opposed to thermoplastic or heat-softening types, such as rubber cement and hot metals. Epoxies and urethanes are versatile and in widespread use as the structural adhesives [6]. Numerous other adhesive materials are used for various applications. Some remain liquid in the presence of oxygen, but they harden in restricted spaces, such as on bolt threads or in the spaces between a shaft and hub. *Adhesive bonding* is extensively utilized in the automotive and aircraft industries. Retaining compounds of adhesives can be employed to assemble cylindrical parts formerly needing press or shrink fits. In such cases, they eliminate press-fit stresses and reduce machining costs. Ordinary engineering adhesives have shear strengths varying from 25 to 40 MPa. The website at www.3m.com/bonding includes information and data on adhesives.

The *advantages* of adhesive bonding over mechanical fastening include the capacity to bond alike and dissimilar materials of different thickness, economic and rapid assembly, insulating characteristics, weight reduction, vibration dumping, and uniform stress distribution. On the other hand, examples of the *disadvantages* of the adhesive bonding are the preparation of surfaces to be connected, long cure times, possible need for heat and pressure for curing, service temperature sensitivity, service deterioration, tendency to creep under prolonged loading, and questionable long-term durability. The upper service temperature of most ordinarily employed adhesives is restricted to about 400°F. However, simpler, cheaper, stronger, and more easy-to-apply adhesives can be expected in the future.

15.18.1 Design of Bonded Joints

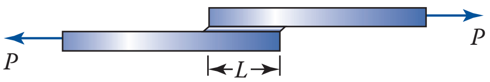
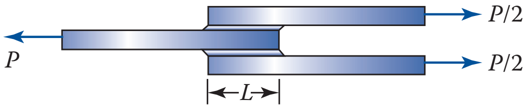
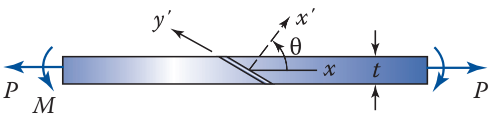
A design technique of rapidly growing significance is metal-to-metal adhesive bonding. Organic materials can be bonded as well. In cementing together metals, specific adhesion becomes important, inasmuch as the penetration of adhesive into the surface is insignificant. A number of metal-to-metal adhesives have been refined, but their use has been confined mainly to lap or spot joints of relatively limited area. Metal-to-metal adhesives, as employed in making plymetal, have practical applications.

Three common methods of applying adhesive bonding are illustrated in Table 15.10. Here, based on an approximate analysis of joints, stresses are assumed to be uniform over the bonded surfaces. The actual stress distribution varies over the area with *aspect ratio* b/L. The highest and lowest stresses occur at the edges and in the center, respectively. Adhesive joints should be properly designed to support only shear or compression and very small tension. Connection geometry is most significant when relatively high-strength materials are united. Large bond areas are recommended, such as in a lap joint (case A of the table), particularly connecting the metals. Nevertheless, this shear joint has noteworthy stress concentration of about 2 at the ends for an aspect ratio of 1.

It should be pointed out that the lap joints may be inexpensive because no preparation is required except, possibly, surface cleaning, while the machining of a scarf joint is impractical. The exact stress distribution depends on the thickness and elasticity of the joined members and adhesives. Stress concentration can arise because of the abrupt angles and changes in material properties. Load eccentricity is an important aspect in the state of stress of a single lap joint. In addition, often, the residual stresses associated with the mismatch in coefficient of thermal expansion between the adhesive and adherents may be significant [18,19].

TABLE 15.10

Some Common Types of Adhesive Joints

Configuration	Average Stress
A. Lap 	$\tau = \dfrac{P}{bL}$
B. Double lap 	$\tau = \dfrac{P}{2bL}$
C. Scarf 	Axial Loading $\sigma_{x'} = \dfrac{P}{bt}\cos^2\theta, \quad \tau_{x'y'} = -\dfrac{P}{2bt}\sin 2\theta$ Bending $\sigma_{x'} = \dfrac{6M}{bt^2}\cos^2\theta, \quad \tau_{x'y'} = -\dfrac{3M}{bt^2}\sin 2\theta$

Notes: P, centric load; M, moment; b, width of plate; t, thickness of thinnest plate; and L, length of lap.

Problems

Sections 15.1 through 15.7

15.1 A power screw is 75 mm in diameter and has a thread pitch of 15 mm. Determine the thread depth, the thread width or the width at pitch line, the mean and root diameters, and the lead, for the case in which

a. Square threads are used

b. Acme threads are used

15.2 A 1½ in. diameter, double-thread Acme screw is to be used in an application similar to that of Figure 15.6. Determine

a. The screw lead, mean diameter, and helix angle

b. The starting torques for lifting and lowering the load

c. The efficiency, if collar friction is negligible

d. The force F to be exerted by an operator, for a = 15 in.

Given: $f = 0.1, \quad f_c = 0.08, \quad d_c = 2$ in., $\quad W = 1.5$ kips

15.3 What helix angle would be required so that the screw of Problem 15.2 would just begin to overhaul? What would be the efficiency of a screw with this helix angle, for the case in which the collar friction is negligible?

15.4 A 32 mm diameter power screw has a double-square thread with a pitch of 4 mm. Determine the power required to drive the screw.