Powertrain Calibration Optimisation

Introduction to Design of Experiments

Overview

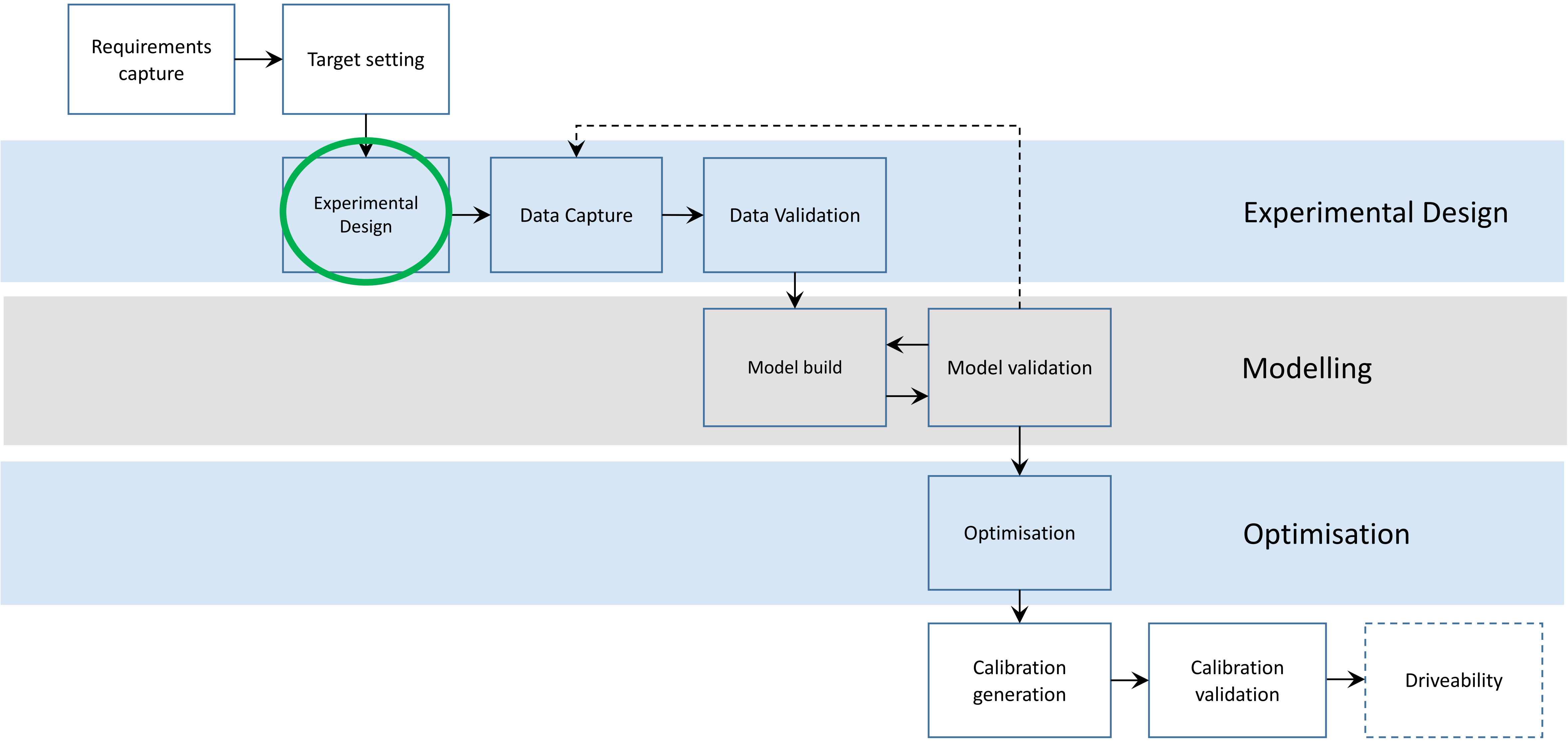
- Objectives for Design of Experiments (DOE)
  - Outline of the DOE process
    - Classical design
    - Space Filling design
    - Optimal design
  - Criteria for the DOE process

Four major steps in calibration

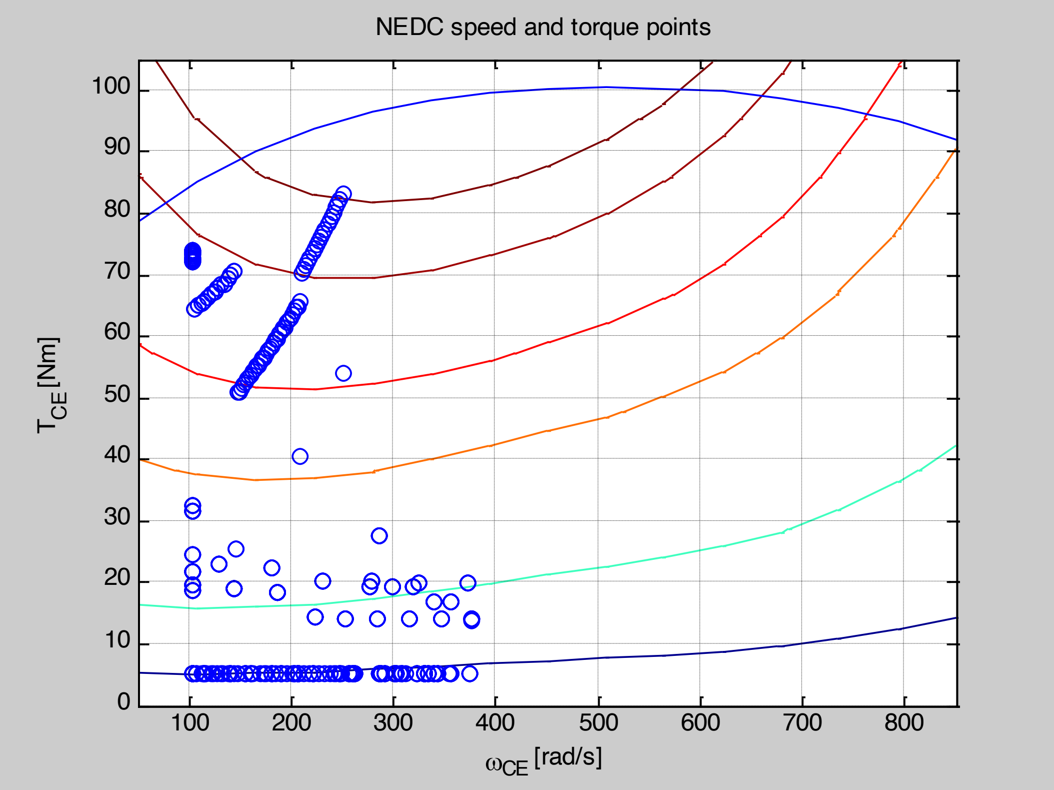
- Plan the experiments
  - With limited test bed time what is the best way to gather data? Identify modal points – plan experiments.
- Acquire the data
  - There is always a significant volume of data; automated methods are essential
- Fit models
  - Models will be quick to fit and accurate and represent engine behaviour
- Conduct optimisation
  - Using models, identify the combinations of controls that give *best* engine behaviour

High Level Overview

Powertrain Calibration Optimisation



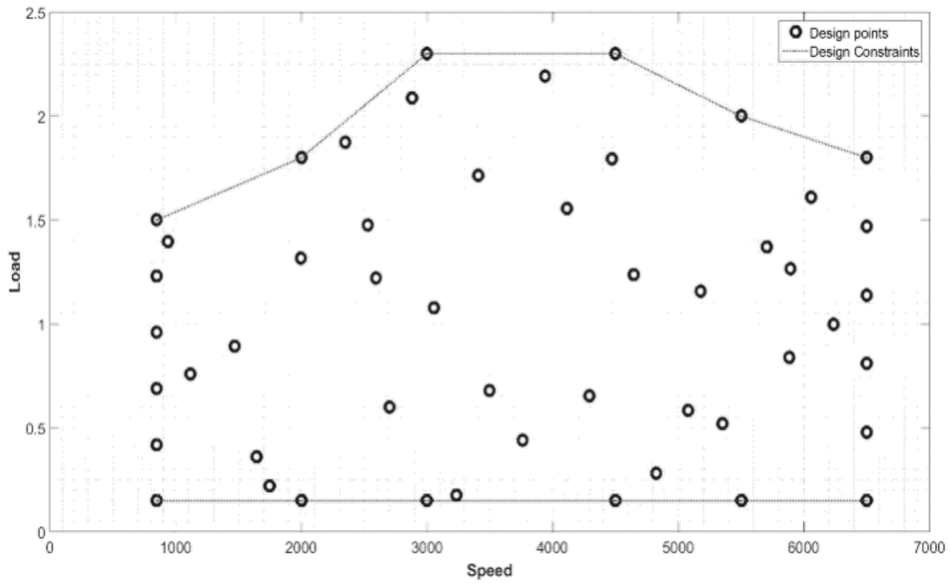
NEDC speed and torque points



Design of Experiments is used to plan engine testing

- Design of Experiments (DOE) provides efficient experimentation
- DOE is widely used in the process and medical industries

Torque experiment



Why 3 levels?

3^k factorial experiments
(i.e. 3 levels, k factors)

k	Test Points
2	9
3	27
4	81
5	243
6	729
7	2187

Design of Experiments (DOE) - What do you do?

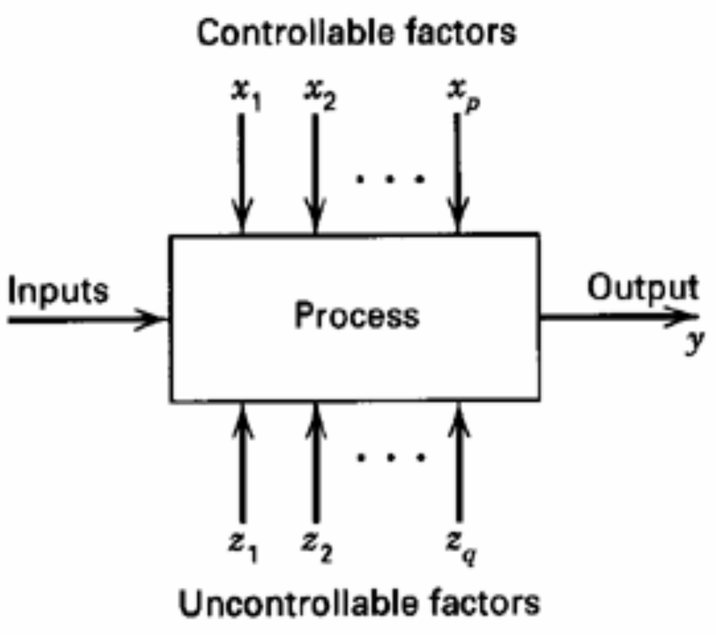
- Find the variables which influence the output (speed, load, ignition timing ..)
- Estimate the levels that are of interest (high, low ..)
- Two levels and k variables gives a 2^k design
  - 2^k is likely to be too many
  - select a fraction
- There are many ways to select a fraction
- Estimate *main effects* first - then *first order* interactions - and so on.

Quadratic surface model

$$\widehat{y_q} = \beta_0 + \underbrace{\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3}_{\text{first order}} + \underbrace{\beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3}_{\text{second order}} + \underbrace{\beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2}_{\text{higher order}}$$

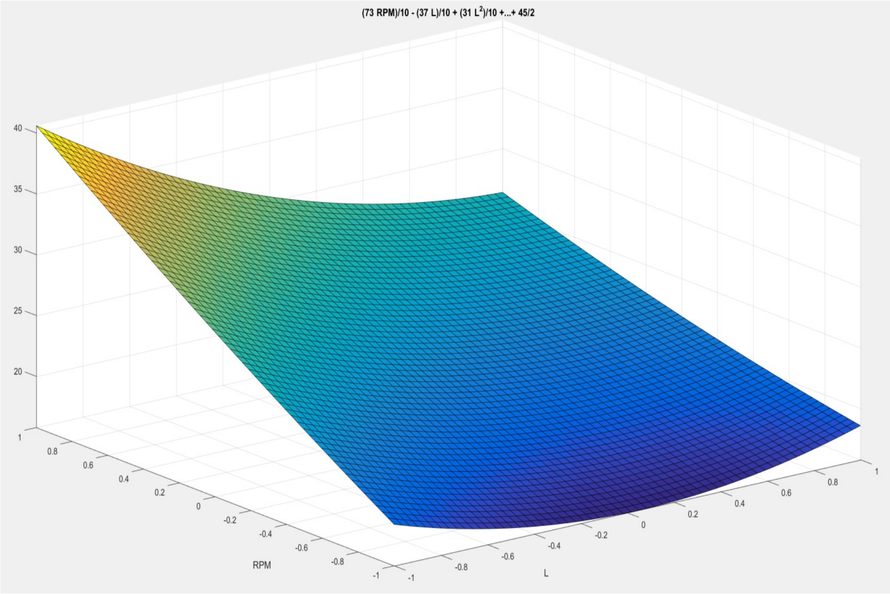
Why DOE?

- Much time required for full factorial experiments
- Characterisation of engine for optimisation
- The use of DOE improves the yield of information compared with ad-hoc experimental methods
- The result is better use of resources
- A DOE process allows the inclusion of explicit constraints: speed load, EGR limits



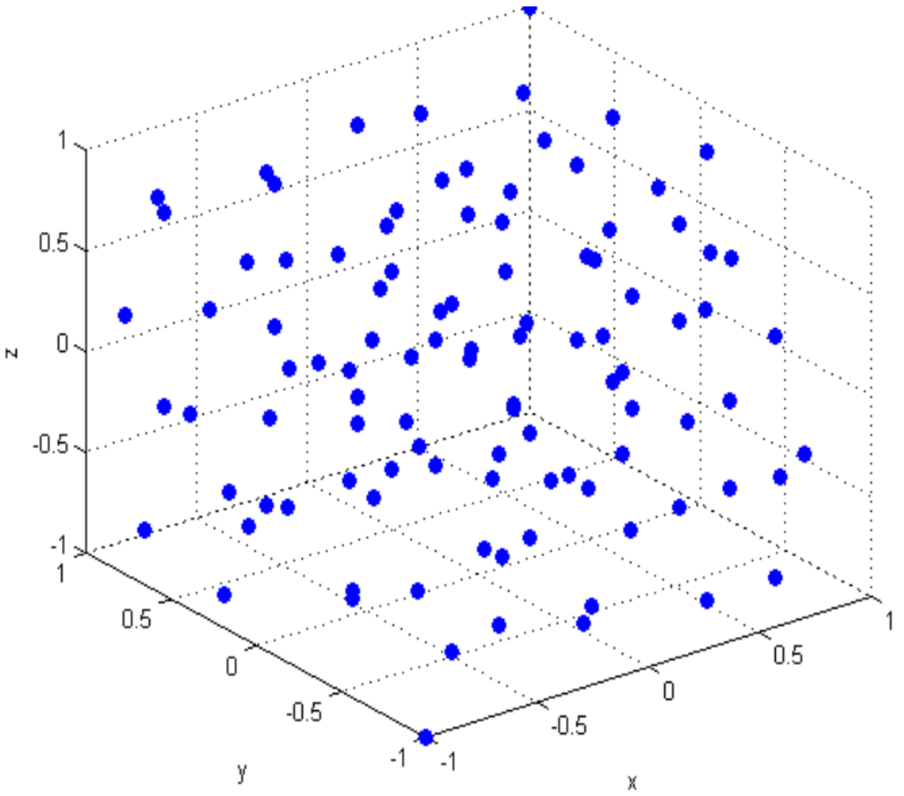
Mathematical Model

$$y = f(\boldsymbol{\beta}, \boldsymbol{X}) = \beta_0 + x_1\beta_1 + x_2\beta_2 + x_1x_2\beta_1 + x_1x_2\beta_2 + x_1^2\beta_{11} + x_2^2\beta_{22}$$



Categories of DOE

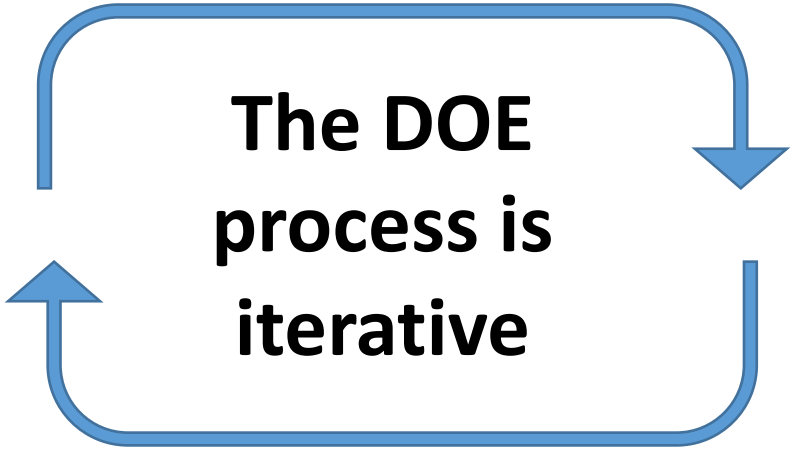
- **Classical**: Full Factorial, Fractional Factorial, Box-Behnken, Central-Composite
- **Space-filling**: Latin Hypercube, Lattice, stratified Latin Hypercube
- **Optimal**: “alphabet soup” A, D and V optimal



Space filling design

How to choose different design styles

- Decide on the aim of the experiment.
  - A/B testing
  - Factor screening
  - Response surface modelling
- Evaluate how much you already know
  - Classic designs:
    - Simple regions (linear models, quadratic models)
  - Space-filling:
    - Low system knowledge
  - Optimal designs:
    - High system knowledge



Terminology (1)

- Randomisation
  - Randomising the order of experiments so as to avoid systematic errors
- Blocking
  - Explicitly accounting for key factors in the planning of experiments – test bed, operator
- Confounding
  - Independent variable a and b are said to be confounded when they both influence dependent variable c – it is difficult to separate out their respective contributions

Terminology (2)

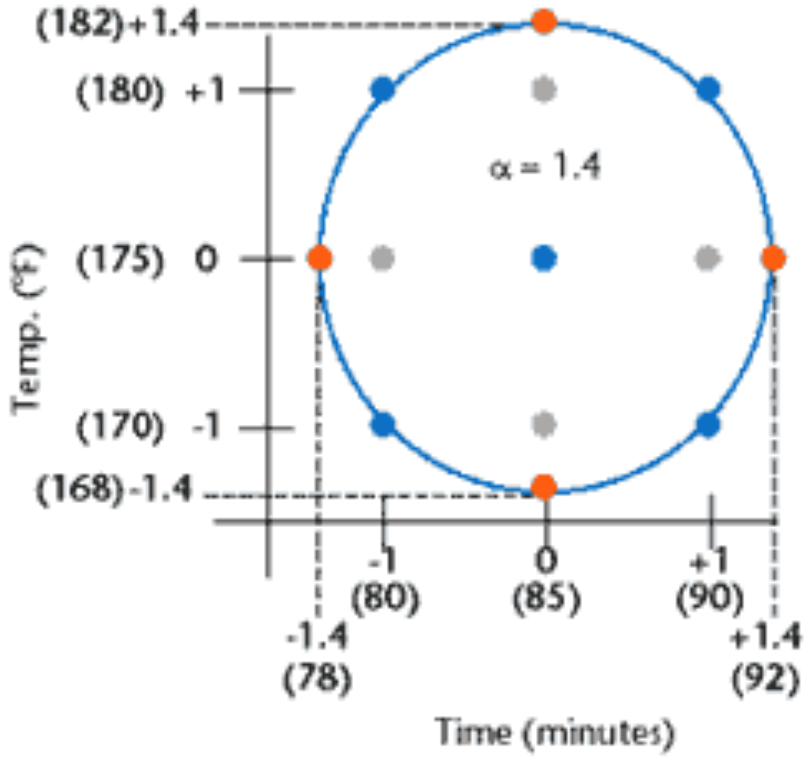
- Response variable
  - Measured output value
- Factors
  - Input variables that can be changed
    - E.g. torque, speed, voltage, frequency, current
- Interaction
  - Effect of one input factor depends on level of another input factor

Quadratic surface model

$$\widehat{y_q} = \beta_0 + \underbrace{\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3}_{\text{first order}} + \underbrace{\beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3}_{\text{second order}} + \underbrace{\beta_{11} \mathrm{x}_1^2 + \beta_{22} \mathrm{x}_2^2 + \beta_{33} \mathrm{x}_3^2}_{\text{higher order}}$$

Terminology (3)

- Levels
  - Specific values of factors (inputs)
- Replication
  - Completely re-run experiment with same input levels
  - Used to determine impact of “noise” (measurement error, random effects)
- Rotatability
  - A design is rotatable if the variance of the predicted response at any point **x** depends only on the distance of **x** from the design centre



Classical design

Key steps in designing an experiment

1. Identify factors of interest and a response variable

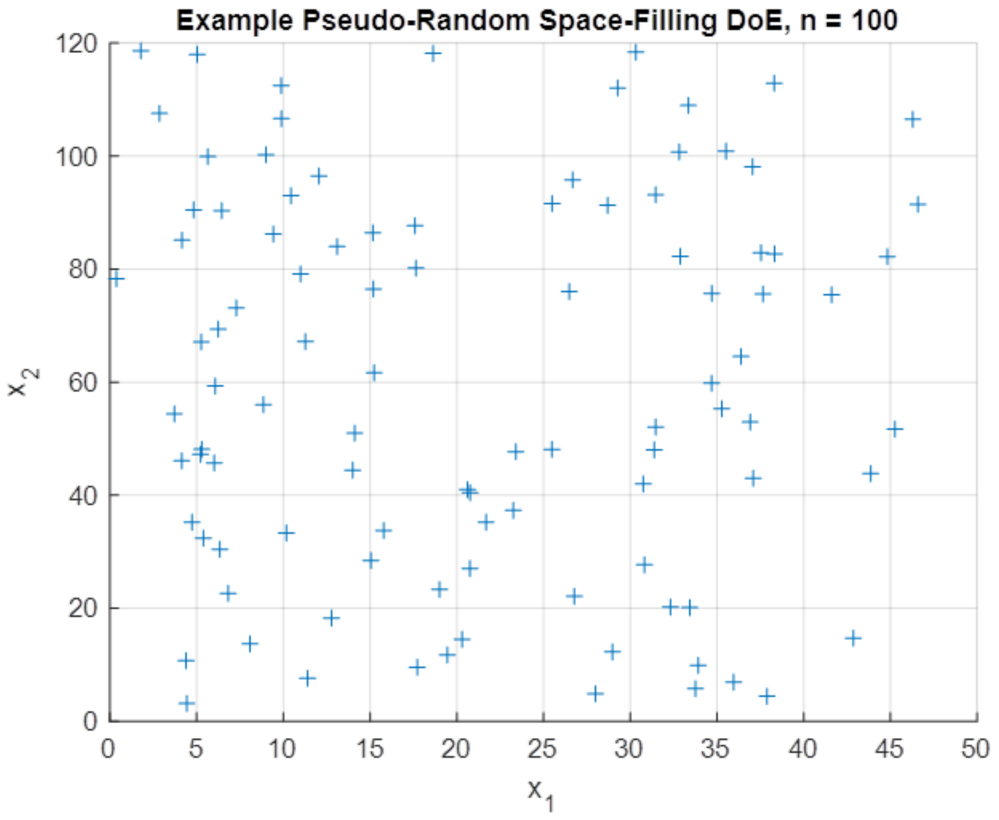
2. Determine appropriate level for each explanatory variable

3. Determine a design structure

4. Randomise (take care!!) the order in which each set of conditions is run and collect the data e.g. latin hypercube or sobol sequence

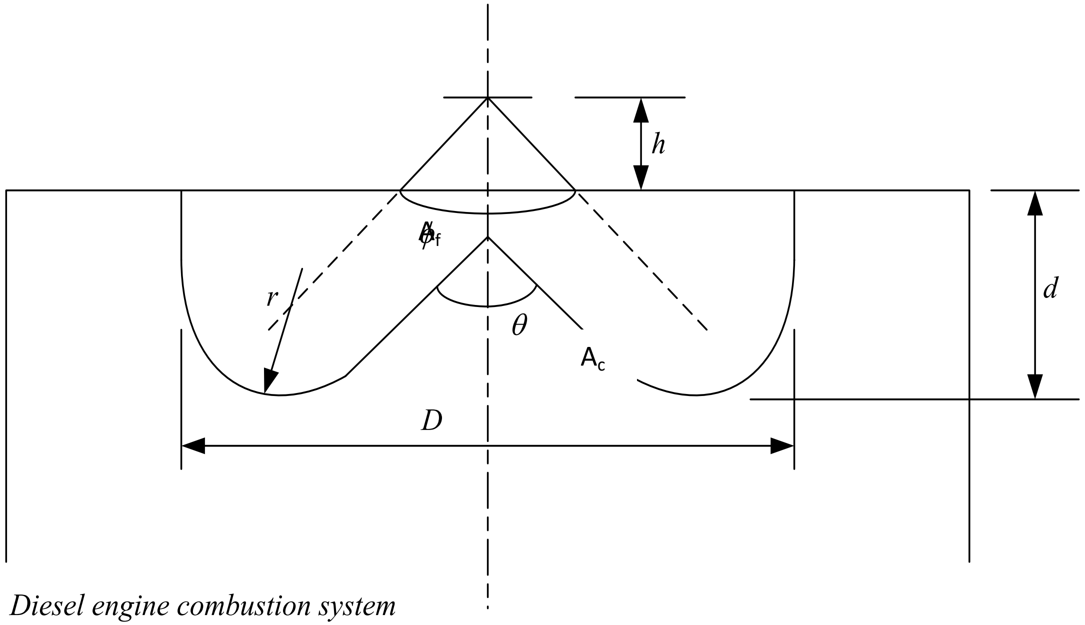
5. Organise the results in order to draw appropriate conclusions

6. Replicate to give “noise” information



Classical Design Example: Optimising a diesel engine combustion system

- □ Output/response of interest: BSFC
- □ Factors;
- □ Overall diameter D, radius r
- □ Depth d, Angle of central cone A_c
- □ Angle of fuel jets A_f, Height of injector h, Injection pressure p



Diesel engine combustion system

- □ The number of experiments at 2 levels = 2^k (k=number of variables)
- □ $k = 3$: 8 experiments
- □ $k = 4$: 16 experiments
- □ $k = 7$: 128 experiments

Classical design: Determine factors, range and variable level

- Angle of fuel jets A_f, height of injector h, injector pressure p
- Two level values, denoted by + and -
- $\mathbf{A_f}$- = 110^o, $\mathbf{A_f}$+ = 130^o
- **H**- = 2mm, H+ = 8mm
- **p-** = 800bar, p+ = 1200bar

Determine the design structure

- Full factorial design: To list each factor combination exactly once
- Structure the list
  - 1st Column – alternate every 4 rows
  - 2nd Column – alternate every 2 rows
  - 3rd Column – alternate every other row

A	Af	h	p
1	-	-	-
2	-	-	+
3	-	+	-
4	-	+	+
5	+	-	-
6	+	-	+
7	+	+	-
8	+	+	+

Experiment design

Classical design: Organise the results to draw conclusions

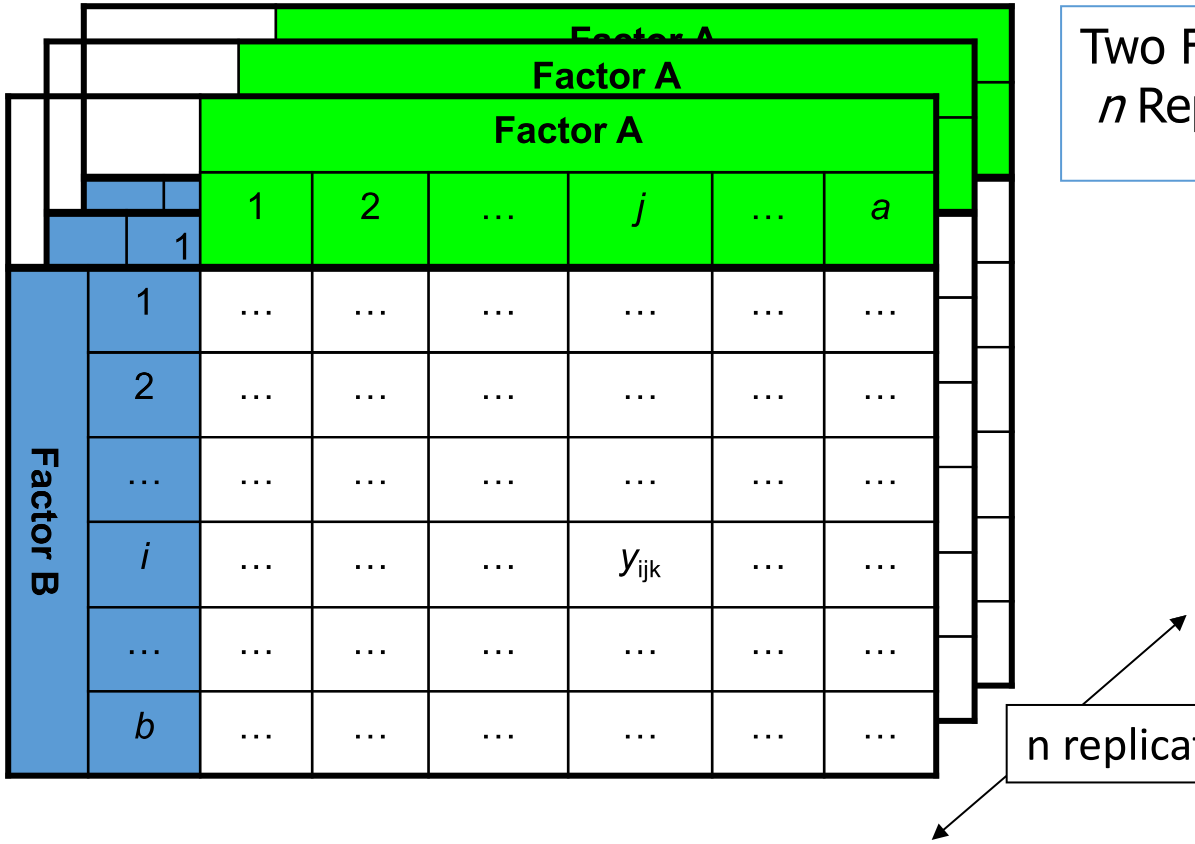
- Run the experiments according to the design
- To determine what effect changing the level of p, A_f and h has on BSFC
  - For p
    - ½ (average(-)-average(+)) = -4.5
  - For A_f
    - ½((average-) - (average+)) = -1.75
  - For h
    - ½((average-) - (average+)) = -0.25

main effect

A	Af	h	p	bsfc
1	-	-	-	218
2	-	-	+	207
3	-	+	-	220
4	-	+	+	210
5	+	-	-	216
6	+	-	+	208
7	+	+	-	212
8	+	+	+	205

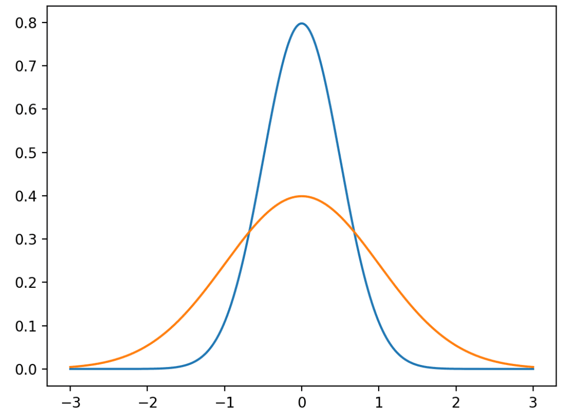
Experiment results

Classical design: Replications



Two Factors
n Replications

n replications



variance

Quadratic designs

Classical quadratic designs → Central composite

Classical quadratic designs → Box-Behnken design

e.g. MBT model, naturally aspirated

$$MBT = 22.5 + 7.3RPM - 3.7L + 0.6RPM^2 + 3.1L^2 - 0.6RPM \cdot L - 2.8RPM \cdot L$$

Quadratic surface

$$y_q = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{11} \mathrm{x}_1^2 + \beta_{22} \mathrm{x}_2^2 + \beta_{33} \mathrm{x}_3^2$$

Cubic surface

$$y_c = y_q + \beta_{123} x_1 x_2 x_3 + \beta_{112} x_1^2 x_2 + \beta_{113} x_1^2 x_3 + \beta_{122} x_1 x_2^2 + \beta_{133} x_1 x_3^2 + \beta_{223} x_2^2 x_3 + \beta_{233} x_2 x_3^2 + \beta_{111} x_1^3 + \beta_{222} x_2^3 + \beta_{333} x_3^3$$

Classical design: Box-Behnken designs

- The design is intended to fit a quadratic model, containing squared terms and products of two factors
- Suitable for small number of factors (three or less) and at least three levels (to get quadratic curvature)
- The ratio of the number of experimental points to the number of coefficients in the range of 1.5 to 2.6
  - More efficient i.e. fewer tests than full factorial

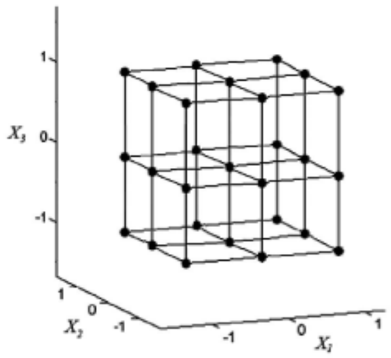
$$y = \beta_0 + x_1\beta_1 + x_2\beta_2 + \underbrace{x_1x_2\beta_1 + x_1x_2\beta_2}_{\text{products of 2 factors}} + \underbrace{x_1^2\beta_{11} + x_2^2\beta_{22}}_{\text{squared terms}}$$

Classical design: Box-Behnken designs

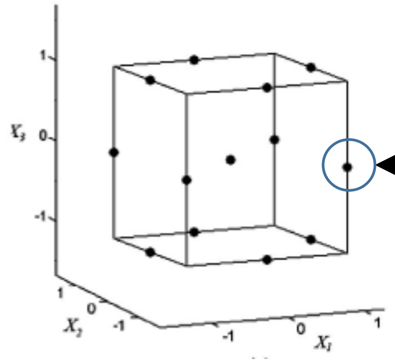
- Midpoints of edges of the input space at the centre (avoiding corner/extreme points)
- Fewer points than fractional factorial
- A combination of a two-level factorial with an *incomplete block design*.
- All combinations for the factorial design, while the other factors are kept at the central values.

Incomplete block

Factor 1	Factor 2	Factor 3
+	-	0
+	+	0
-	0	-
-	0	+
+	0	-
+	0	+
0	-	-
0	-	+
0	+	-
0	+	+
-	-	0
-	+	0
0	0	0



Full factorial (27)

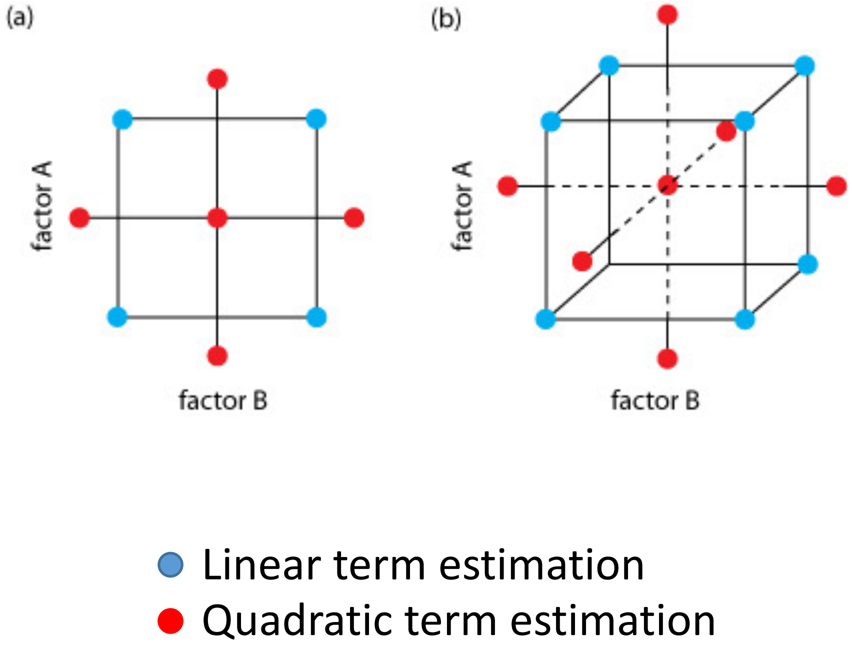


Box-Behnken (13)

$$x_1 = 1$$
$$x_2 = -1$$
$$x_3 = 0$$

Classical design: Central Composite Design (CCD)

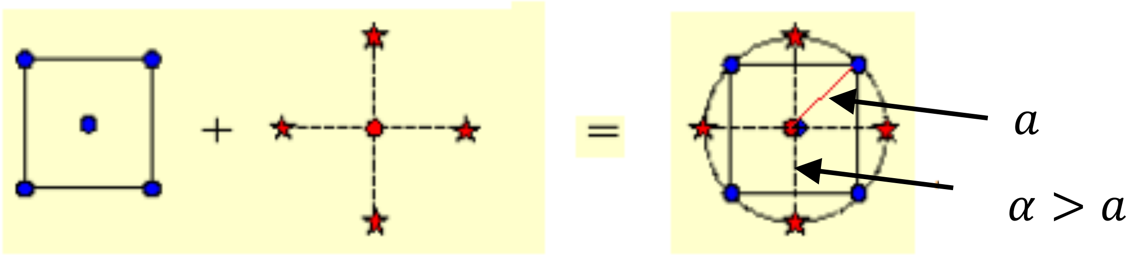
- Used when second order model is suspected in 2^k design
- Similar to Box-Behnken with corner and extreme points
- A set of centre points: the medians of the values used in the factorial portion – usually repeated
- A set of axial points, experimental runs identical to the centre points except for one factor, which will take on values both below and above the median of the two factorial levels



Classical design: Central Composite Design (CCD)

- Start with factorial design (with centre points)
- Add ‘star’ points to get an estimate of curvature

Central composite design with two factors



- Consider: star points may not be achievable

Space filling designs

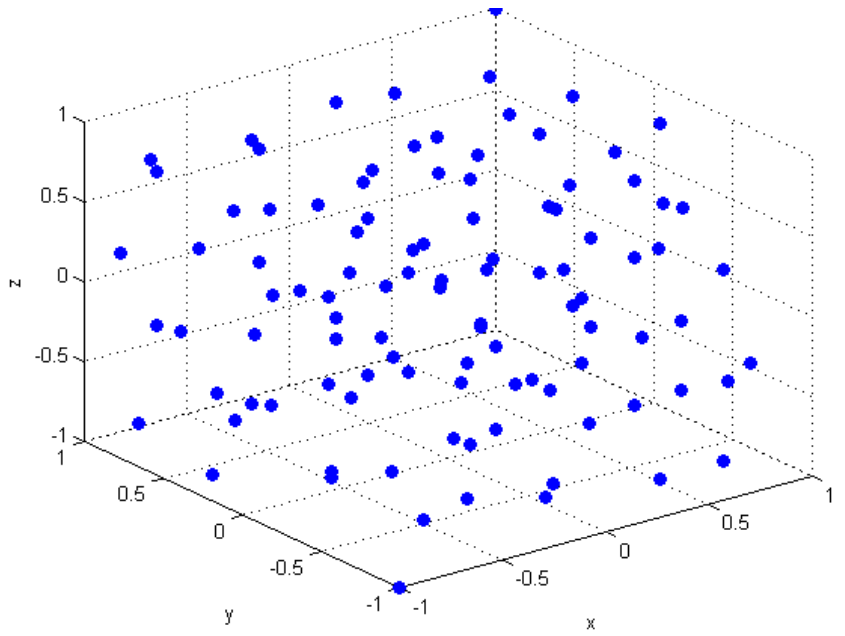
- Good when little is known about the system under study
- Distributes design points (in hyperspace) as far from each other as possible
- Various strategies for achieving this
- Fill-out the n-dimensional space that are regularly spaced

	1	2	3	4	5
1					
2					
3					
4					
5					

Space filling: Motivation

- Predictors for response are often based on interpolations
- Prediction error at any point is relative to its distance from closest design point
- Uneven designs can yield predictors that are very inaccurate in sparsely observed parts of experimental region
- Disadvantage: superfluous points may be placed in regions of the design space

So what??



Space filling design in 3 factors

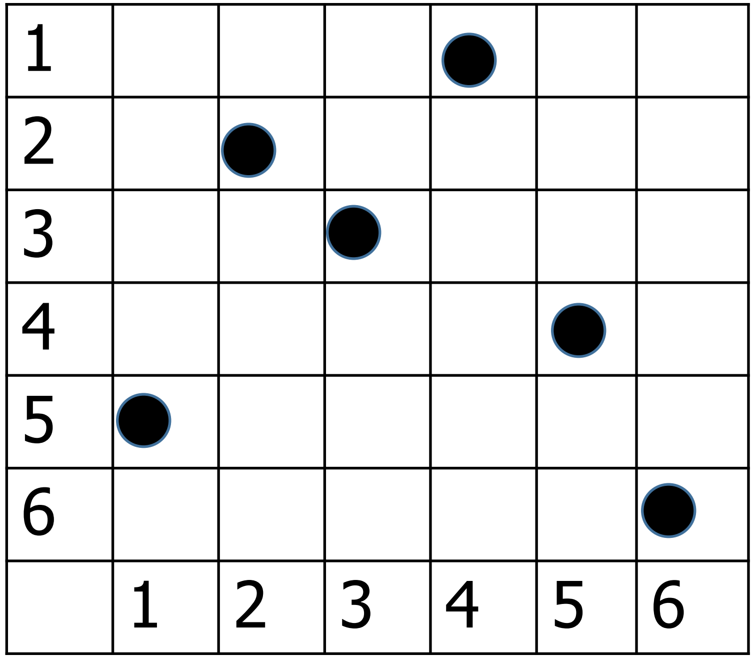
Space filling: Latin Hypercube

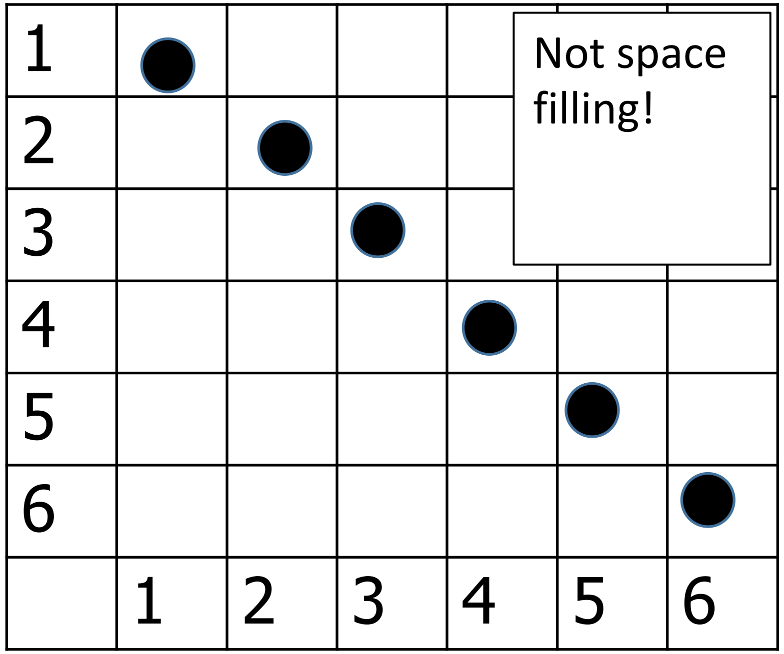
- □ Scheme for generating design points
- □ Efficient algorithm – same number of points for increased dimensions (factors)
- □ Generate sets of design points that, for an N point design, project onto M different levels in each factor.
- □ Try several such sets of randomly generated points and choose the one that best satisfies user-specified criteria (augment??)

Latin Hypercube design: construct an LHD

- Partition experimental region into a square with M^2 cells (M along each dimension)
- Label the cells with integers from {1,…M} such that a Latin square is obtained, each integer occurs exactly once in each row and column
- Select one of the integers, say i, at random
- Sample one point from each cell labelled with I

LHD generated may not be space filling. Design requirements need to be assessed.





Both LHD but not both space filling i.e. distributed evenly over the space

Space filling: Latin Hypercube design
Use measure of spread to assess quality of design

Examples:

- Maxmin distance design: design D that maximises smallest distance between any 2 points in D
- Minmax distance design: design D that minimises the largest distance between any point in the experimental region and the design
- Optimal average distance design: design D that minimises average distance between pairs of points in D

Optimal Designs

Good where;

- Factorial or fractional factorial require too many runs
- The design space is constrained

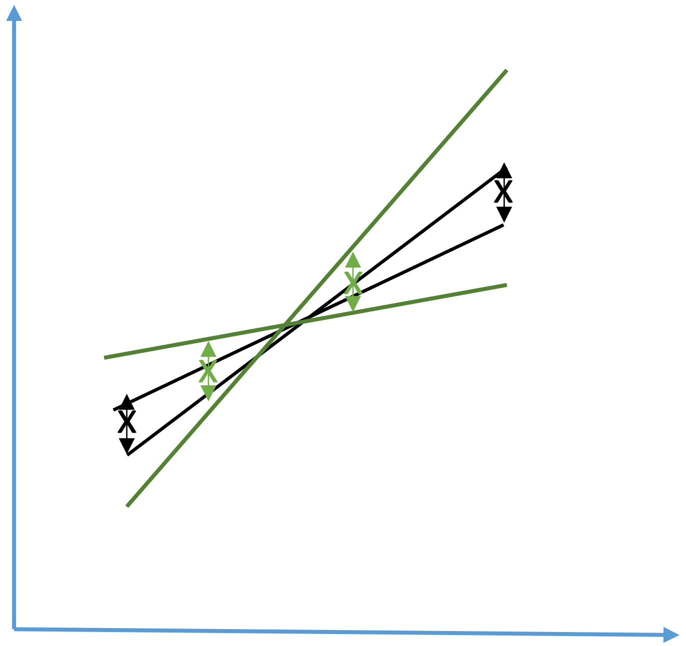
Require good knowledge about the model type -> hence system

- Formulate purpose of experiment in terms of optimising an objective
- Select design such that the design (i.e., set of points from experimental region) optimises some objective

Example:

- Fit straight line to given data $x=[x_1,x_2,\ldots x_n]$, response variable y
- Goal: select design to give most precise (minimum variance) estimate of slope

D-optimal



Optimal design: D-Optimality Criterion

- Linear system

$$y = \beta_1 X_1 + \beta_2 X_2$$

- The fitted model will be

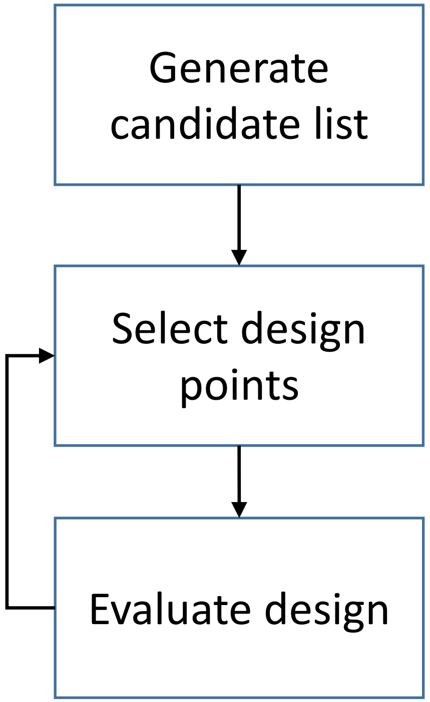
$$\hat{y} = \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2$$

D-optimal design minimises the **<u>covariance of the parameter</u>** estimates for a specified model

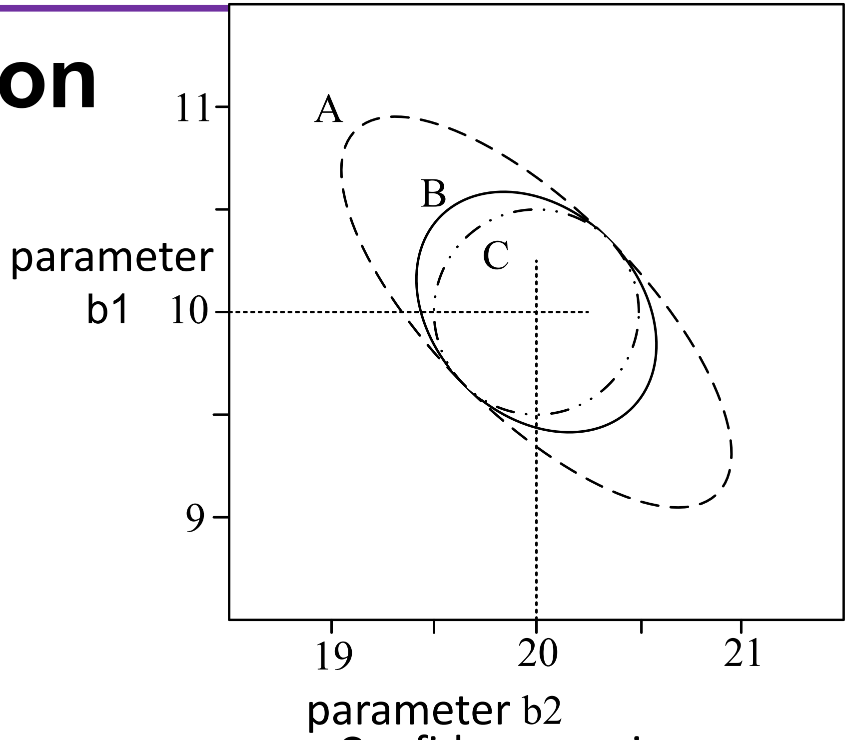
Where $\hat{\beta}_1$ and $\hat{\beta}_2$ are sample based estimates of β_1 and β_2 *(true value)*

Optimal design: D-Optimality Criterion

Designs A, B and C.
All two-factor, six-point designs



Design					
A		B		C	
0.75	0.25	1.0	0.0	1.0	0.0
0.75	0.25	1.0	0.0	1.0	0.0
0.50	0.50	0.5	0.5	1.0	0.0
0.50	0.50	0.5	0.5	0.0	1.0
0.25	0.75	0.0	1.0	0.0	1.0
0.25	0.75	0.0	1.0	0.0	1.0
X_A		X_B		X_C	



Confidence region

D-optimal design minimises the covariance of the parameter estimates for a specified model i.e. main$(D = |X^T X|^{-1})$

Optimal design: D-Optimality Criterion

- Table displays three possible six-point designs
- Figure displays joint confidence region for parameters b_1 and b_2 on the assumption that $b_1=10$, $b_2=20$, and $s=0.25$
- The largest ellipse is a 95% joint confidence region for b_1 and b_2 based on design A.
- The middle-sized ellipse is the corresponding region based on design B, while the smallest ellipse is for design C.
- The joint confidence region gets smaller and smaller, and estimates of b_1 and b_2 have become more and more precise

Confidence intervals for parameters

linear regression model

$$y_i = b_0 + b_1 x_{i,1} + b_2 x_{i,2} + \cdots + b_K x_{i,K} + e_i$$

Confidence interval for estimated coefficients

$$\hat{b}_1 \pm t_{\alpha/2} \sqrt{\mathrm{var}(\hat{b}_1)}$$

$t_{\alpha/2}$ is obtained from a t distribution with $n-2$ degrees of freedom

$$\mathrm{var}(\hat{\boldsymbol{\beta}}) = s_e^2 (\mathbf{X}^T\mathbf{X})^{-1}$$

Need to maximise this

variances of the parameters along the diagonal,
and the covariances as the off-diagonal elements

degrees of freedom of the t distribution is

$$df = n - K$$

where K is the number of predictors in the model,
and n is the sample size.

t table

df	0.95	0.99
2	4.303	9.925
3	3.182	5.841
4	2.776	4.604
5	2.571	4.032
8	2.306	3.355
10	2.228	3.169
20	2.086	2.845
50	2.009	2.678
100	1.984	2.626

	b1		b2	
Design	Low limit	High limit	Low limit	High limit
A	9.25	10.75	19.25	20.75
B	9.55	10.45	19.55	20.45
C	9.60	10.4	19.60	20.4

Optimal design: D-Optimality Criterion

Design A is put in the form of a design matrix and covariance matrix

Design					
A		B		C	
0.75	0.25	1.0	0.0	1.0	0.0
0.75	0.25	1.0	0.0	1.0	0.0
0.50	0.50	0.5	0.5	1.0	0.0
0.50	0.50	0.5	0.5	0.0	1.0
0.25	0.75	0.0	1.0	0.0	1.0
0.25	0.75	0.0	1.0	0.0	1.0

Determinant of the information matrix

$$\left|X'X\right| = \begin{vmatrix} 1.75 & 1.25 \\ 1.25 & 1.75 \end{vmatrix} = (1.75)^2 - (1.25)^2 = 1.5$$

Information matrix

$$X'X = \begin{bmatrix} 0.75 & 0.75 & 0.50 & 0.50 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.50 & 0.50 & 0.75 & 0.75 \end{bmatrix} \begin{bmatrix} 0.75 & 0.25 \\ 0.75 & 0.25 \\ 0.50 & 0.50 \\ 0.50 & 0.50 \\ 0.25 & 0.75 \\ 0.25 & 0.75 \end{bmatrix}$$

$$= \begin{bmatrix} 1.75 & 1.25 \\ 1.25 & 1.75 \end{bmatrix}$$

Thus for design A

Optimal design: D-Optimality Criterion

The relative areas of ellipses A, B, and C in Figure, are:

$$\frac{1}{\sqrt{1.5}} : \frac{1}{\sqrt{6}} : \frac{1}{\sqrt{9}} \equiv 1.0 : 0.50 : 0.41$$

Designs A, B, C X′X and |X′X| determinants:

Design	X'X	|X'X|
A	$\begin{bmatrix} 1.75 & 1.25 \\ 1.25 & 1.75 \end{bmatrix}$	1.5
B	$\begin{bmatrix} 2.5 & 0.5 \\ 0.5 & 2.5 \end{bmatrix}$	6.0
C	$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$	9.0

Best design – not necessarily optimal

Other Optimal designs

- D-optimal designs minimise the covariance estimates of the model parameters
- A-optimal designs minimises the average variance of the estimates of the model parameters
- V-optimal designs: minimise average prediction variance over specific points

$$\frac{(y_1 - m) + (y_2 - m) + \ldots (y_i - m) + \ldots + (y_n - m)}{n}$$

Summary

- Classical design:
  - Linear model: full factorial, fractional factorial
  - Quadratic model : central composite, Box-Behnken design
- Space filling: low knowledge about the system
- Optimal design: know system well and know the model

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