Powertrain Calibration Optimisation

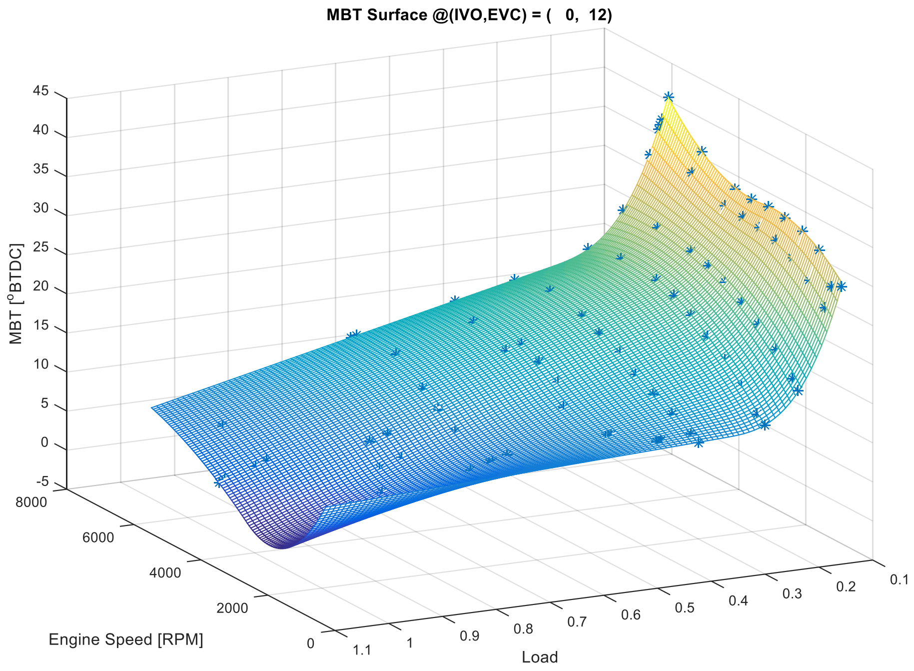
Introduction to Modelling

Overview

- What is a mathematical model?
- Classification of models
- Types of models:
  - Linear regression
  - Polynomial
  - Radial basis function
- Model evaluation

What is mathematical model?

- A model is a mathematical representation of a real-life process/system
- A data model explicitly describes a relationship between *predictor* and *response* variables
- Models can take different forms depending on the complexity of the relationships
- Models will, in general, be generated from experimental data
- Models can be also be generated from an understanding of the physics but will also require some experimental data



pp-spline MBT model

What is mathematical model?

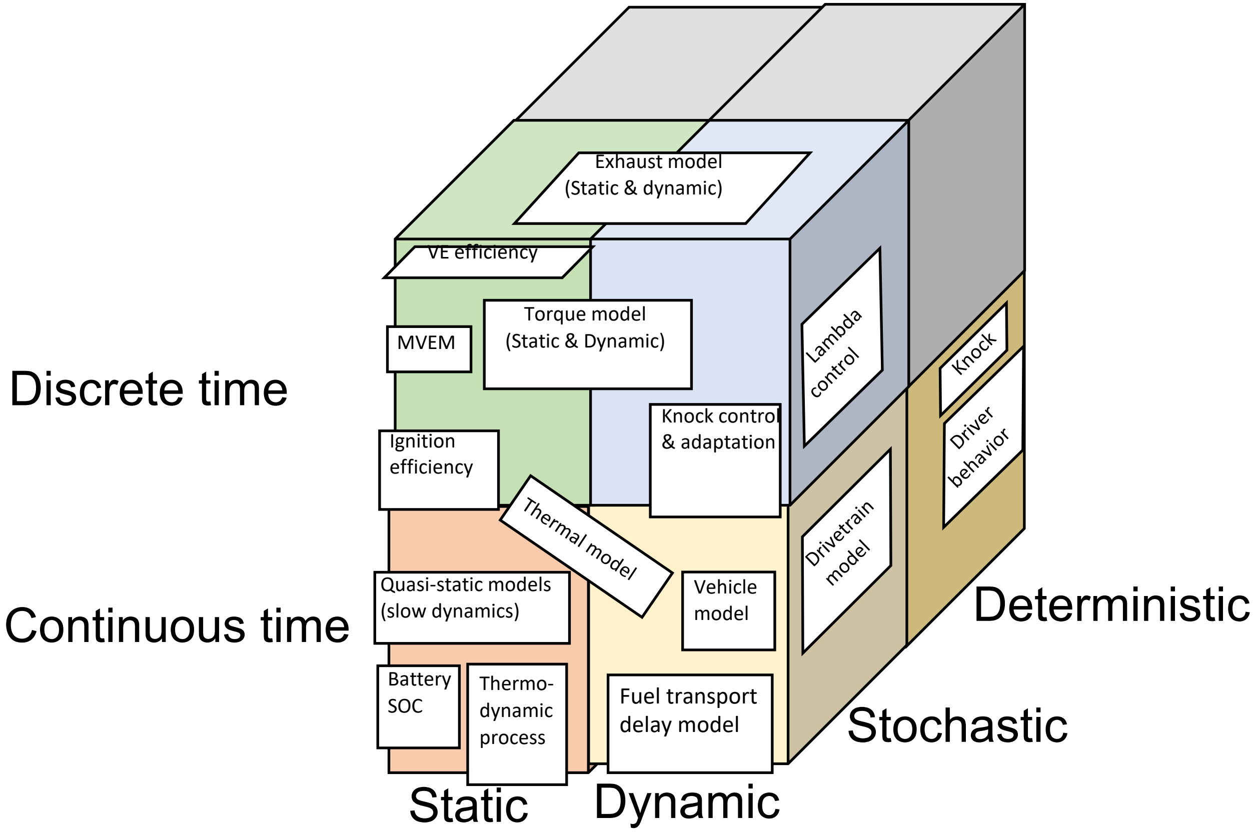
- A model is usually constructed as a functional relationship between the predictor variables and the response variable
- Consequently, the response can be calculated for arbitrary values of the predictors
  - Essential for optimisation for example
- The model becomes a means of <u>*rapid access*</u> to the data for the purpose of evaluation and then optimising the calibration

Types of model

- Black box
  - No knowledge of physics
  - Mathematical structure that doesn't relate to physics
- Grey box
  - Some knowledge of physics
  - Some of the mathematical structure may relate to physics
- White box
  - Complete knowledge of important/relevant physics
  - Mathematical structure is a description of the physics
  - Several possible implementations depending on the chosen topology.

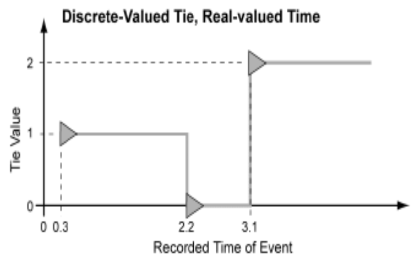
Classification of mathematical models

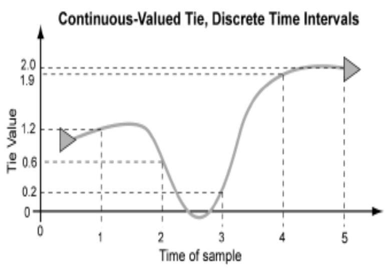
- Empirical
- Physical

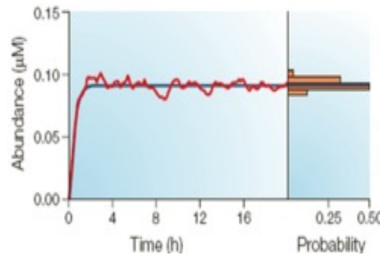


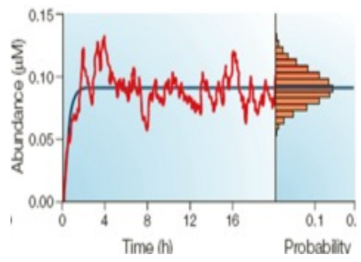
Classification of mathematical models

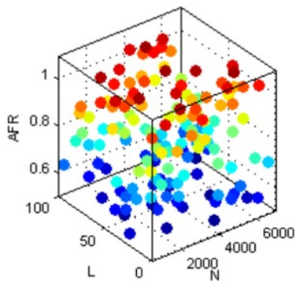
- **Discrete time**
  - Change of system state/output occurring at finite discreet time
- **Continuous time**
  - Change of system state/output between points in an infinite number of steps
- **Static**
  - System representation/description built using points in steady state or in system equilibrium
- **Dynamic**
  - Time variant system. System representation/description that changes with time
- **Deterministic**
  - Predictable. Fixed input gives fixed outputs
- **Stochastic**
  - Involves random variable

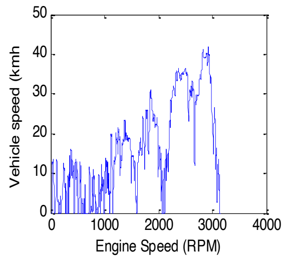












Types of models - Linear regression

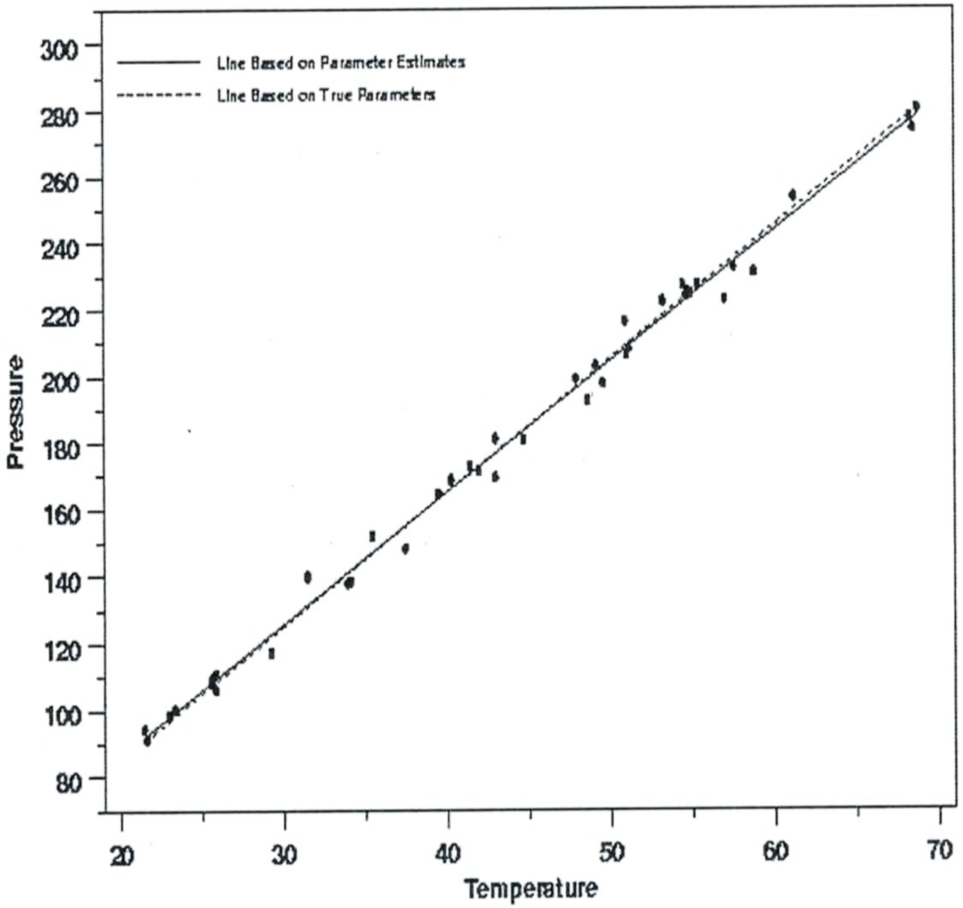
- Linear regression fits a data model that is linear in the model parameters

$$y_i = \eta(x_i, \beta) + \epsilon_i$$

$\beta = model\ parameters$
$x_i = input\ i$
$\epsilon_i = measurement\ noise\ on\ input\ i$

- The most common type of linear regression is a least-squares fit, based on reasonable assumptions about experimental error
  - We make $\eta(x_i, \beta) = \beta_0 + \beta_1 x$
  - To give the relationship $y = \beta_0 + \beta_1 x + \epsilon$

Types of models - Linear regression – an example



- How can this model be checked?
- R^2
- Structure of residuals
- Drift
- Missing terms

Types of models – Linear regression - Least squares fitting of polynomial functions

Finding the parameters requires minimizing the cost function.

$$J = \sum_{i=1}^{n} [y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)]^2$$

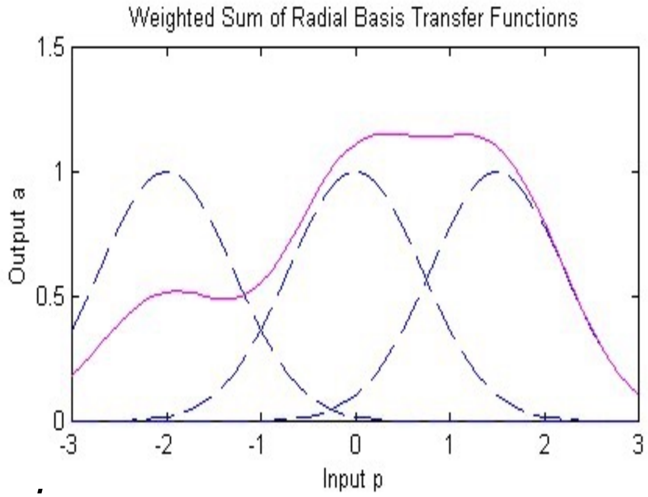
Differentiating with respect to each parameter gives the following expression for the estimate of each.

$$\hat{\beta}_1 = \frac{\sum y_i (x_i - \bar{x})}{\sum (x_i - \bar{x})^2}$$

Types of models - Polynomial - Advantages of polynomial functions

Advantages	Disadvantages
Well known and understood	Poor interpolation (of high order models)
Computationally easy to use	Poor extrapolation
Simple in form	Poor asymptotic behaviour – finite values of x -> finite value
	Can contain many hundreds of terms for physical (engine) behaviour

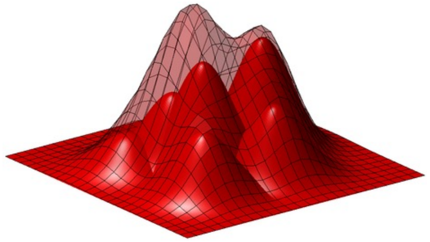
Types of models - Radial basis functions



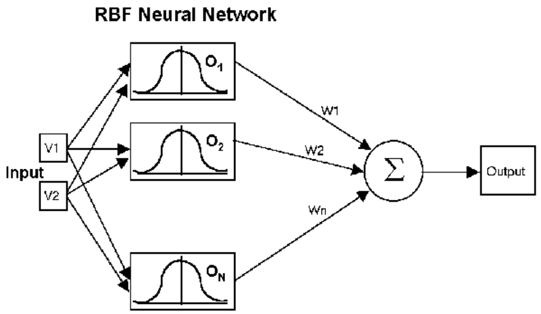
- The value of a *radial function,* $f(x-c)$ *depends only on its distance from the origin/center i.e. point* x *from center* c.
- An RBF model is made up of basis functions which must be shaped and weighted.

$$y(x) = \sum_{i=1}^{N} w_i \phi(\|x - x_i\|)$$

Where w_i *is the weighting of basis* i
x_i *the location of center* i *and*
N *the number of basis functions*



- The basis is usually a Gaussian
- The N Gaussian's have different centers and amplitudes
- The RBF architecture is that of a network and layers of RBFs are used to represent complex functions.



http://www.it.uu.se/research/project/rbf

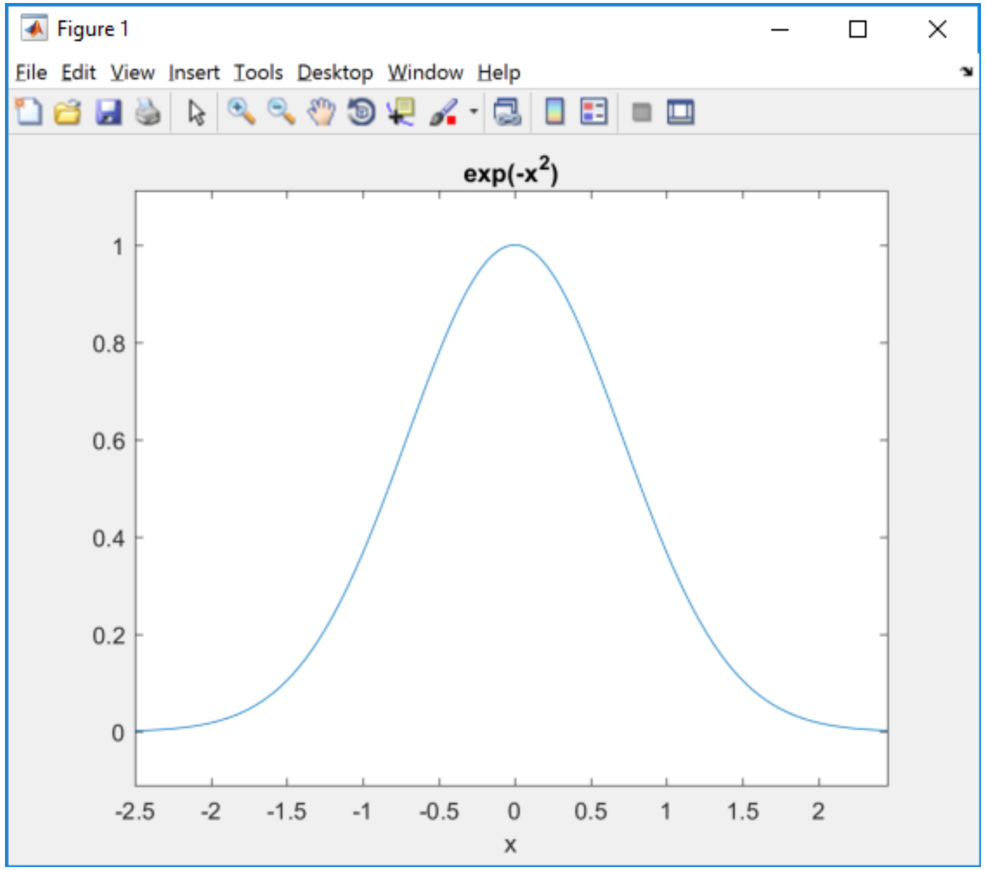


Figure 1
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exp(-x^2)
1
0.8
0.6
0.4
0.2
0
-2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2
x

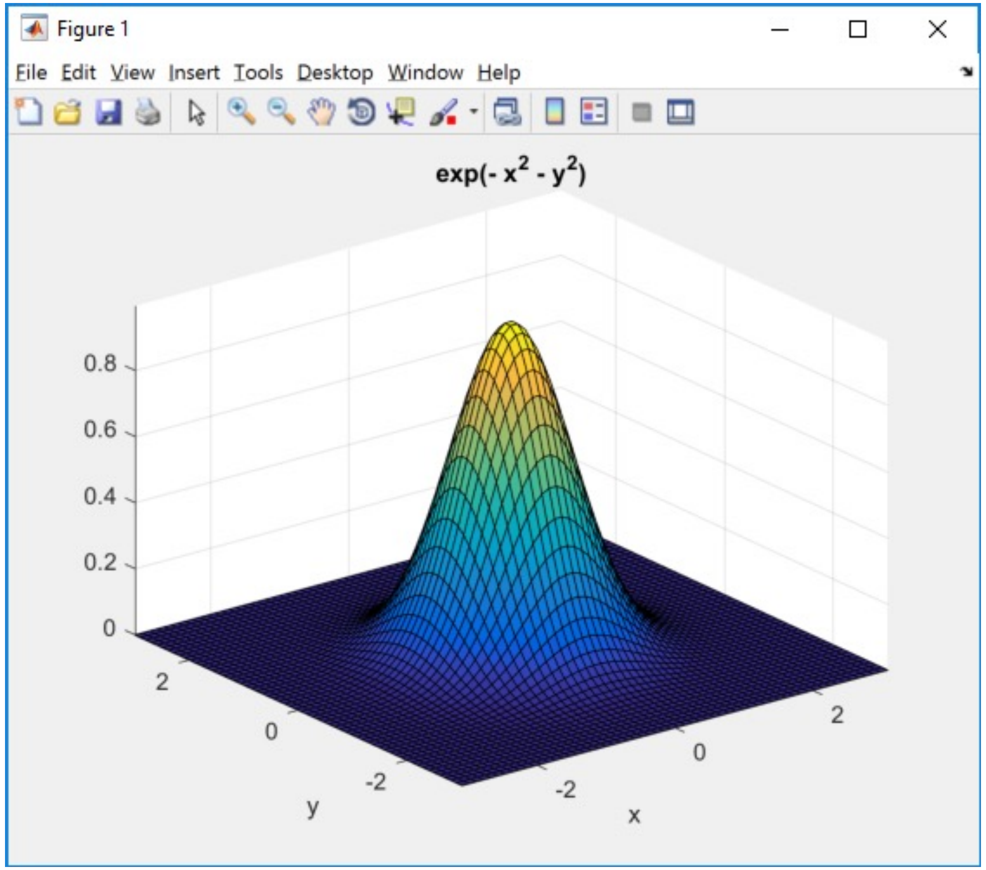
Figure 1
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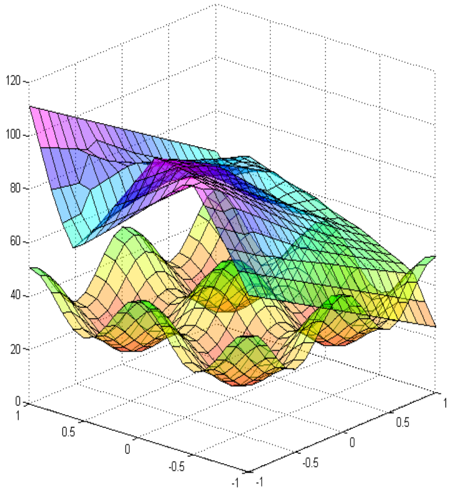
exp(- x^2 - y^2)
0.8
0.6
0.4
0.2
0
2
0
-2
y
-2
0
2
x

Types of models - Radial basis function

A BMEP response surface model using RBF with two inputs (torque and speed): Parameters requiring training:

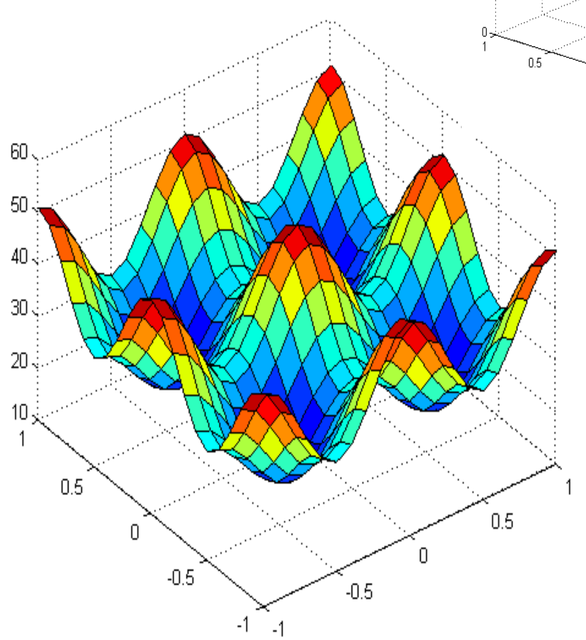
1. Weights w_i
2. Centers $c_{i,1}$ $c_{i,2}$
3. Widths σ_1^2 σ_2^2

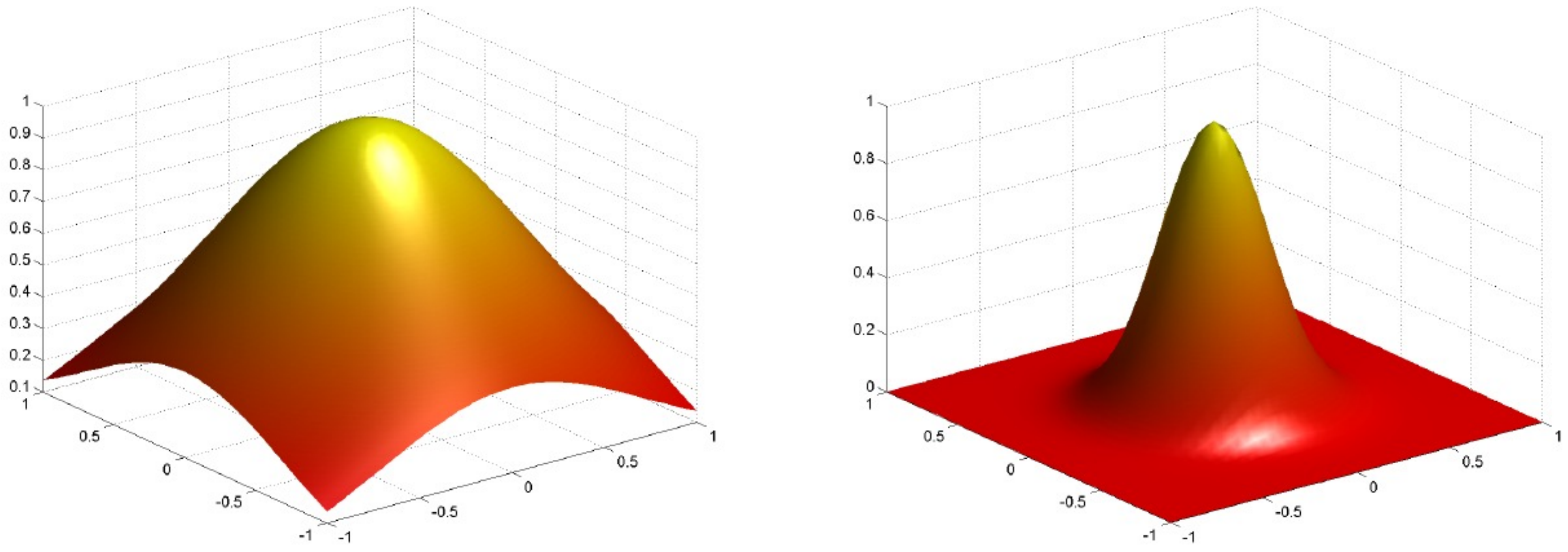
$$f_{BMEP}(x) = \sum_{i=1}^{9} w_i e^{-\frac{1}{2}\left(\frac{\left\|x_{torque}-c_{i,1}\right\|^2}{\sigma_1^2}+\frac{\left\|y_{speed}-c_{i,2}\right\|^2}{\sigma_2^2}\right)}$$



In Model Based Calibration Toolbox (MBC) the training is done automatically. It only needs training data and the maximum number of centers to use.

x_{torque}

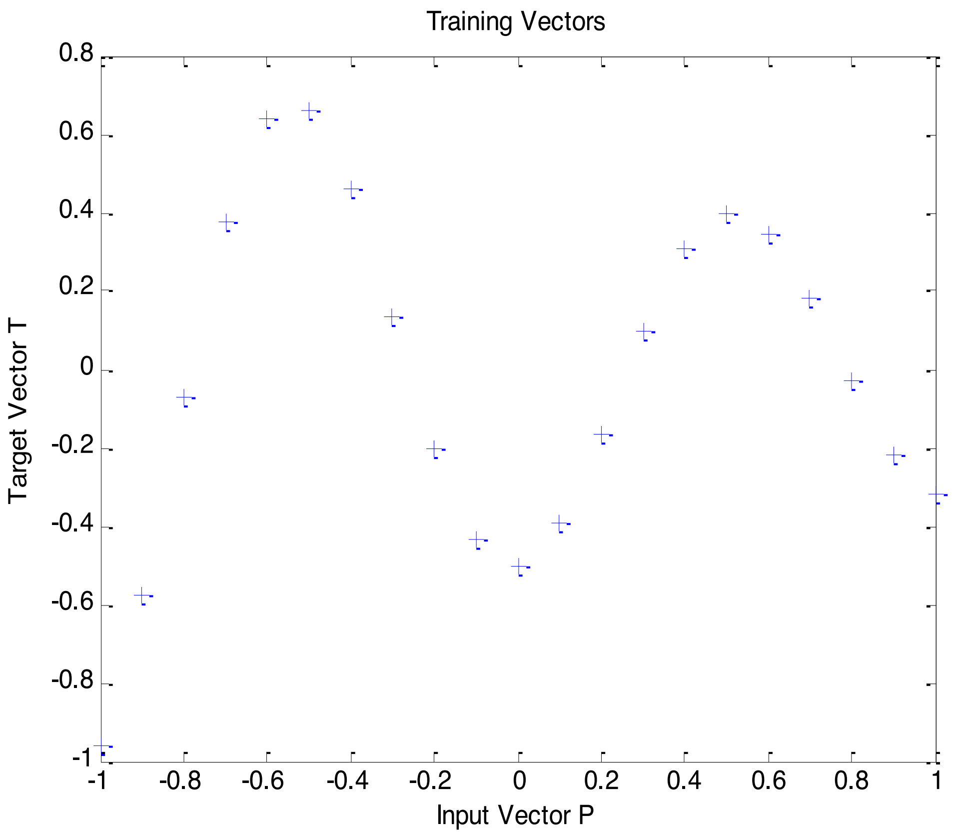




Larger σ^2

Smaller σ^2

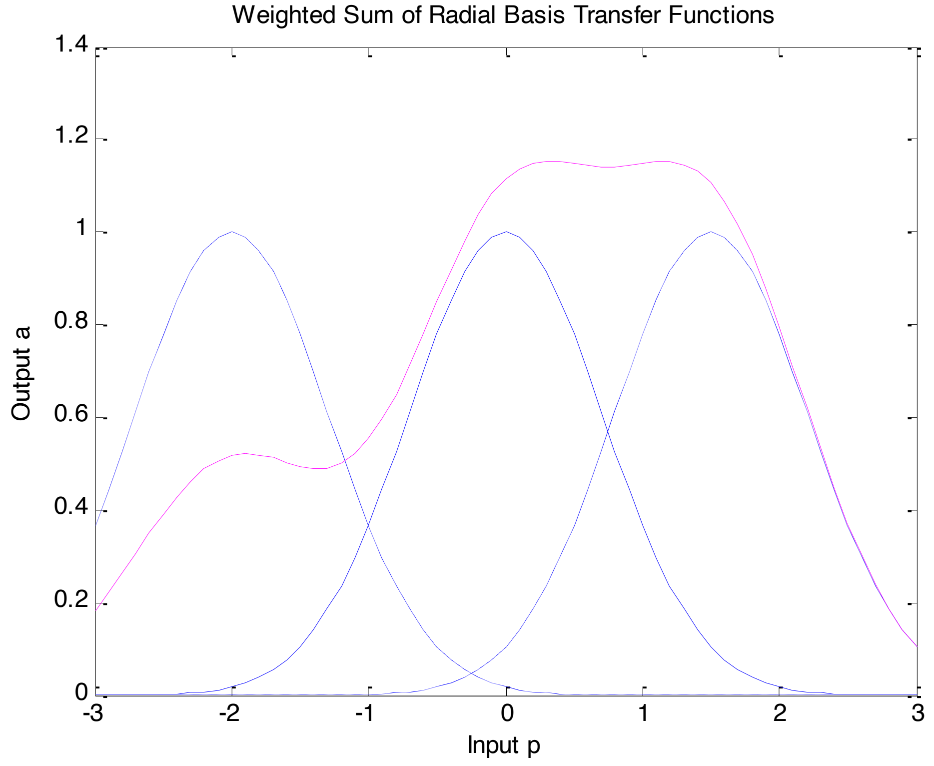
Types of models - radial basis functions – an example



- This is the data represented graphically.
- It is a series of data points but must be represented as a continuous function.

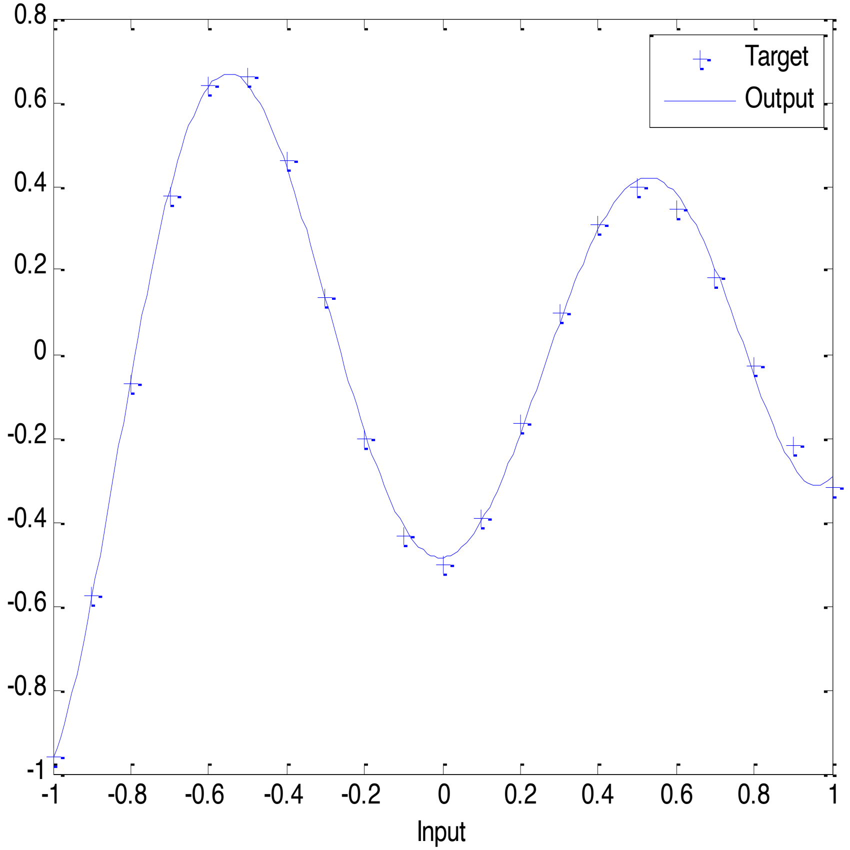
```
P = -1:.1:1;
T = [-.9602 -.5770 -.0729  .3771  .6405  .6600  .4609
...
     .1336 -.2013 -.4344 -.5000 -.3930 -.1647  .0988
...
     .3072  .3960  .3449  .1816 -.0312 -.2189 -
.3201];
```

Types of models - Radial basis functions - How the basis functions may be adjusted



We take the original basis function and two versions at centres (-2, 0 and 1.5) scaled differently.

Types of models - Radial basis functions - The result …



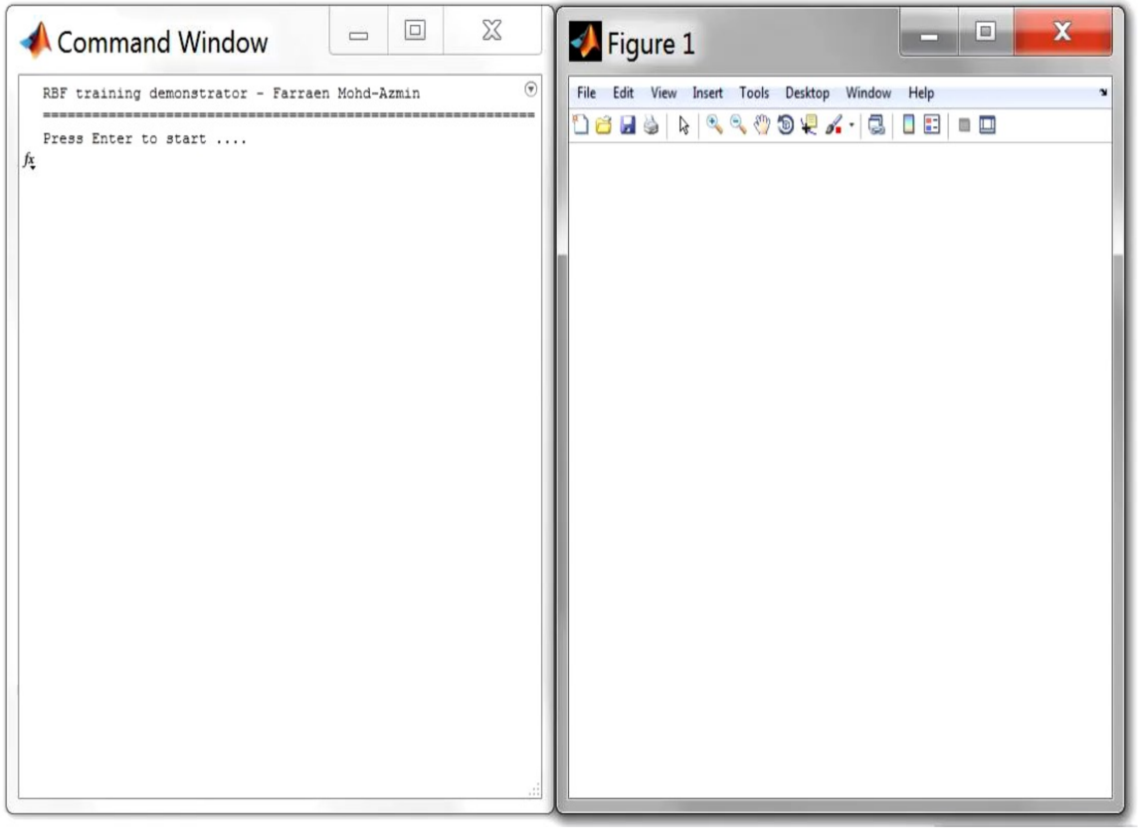
The fitted function is generated using a MATLAB function call, newrb() which allows the ‘quality’ of fit to be specified.

Types of models - radial basis functions – a working example (2D video)

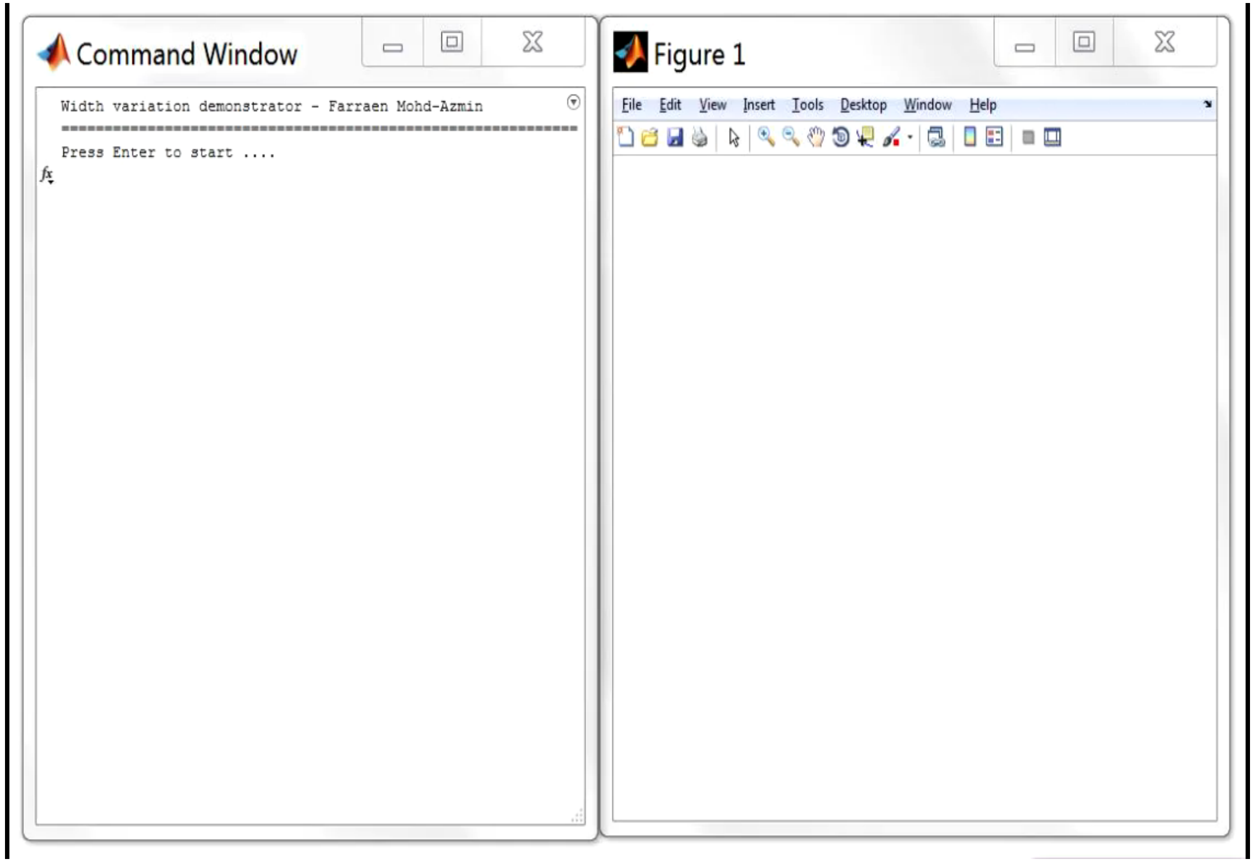
- The model output is the sum of **6 RBFs**.

- A target data set is introduced to the training algorithm. The algorithm will parameterise the model to make sure the model is able to output (Blue line) same value as the target data set (Red dots).

- The widths are optimised using Maximum Likelihood Estimation (MLE) and the weights are iteratively optimised using gradient descent optimisation.



Types of models - Radial basis functions – Width variations (2D video)

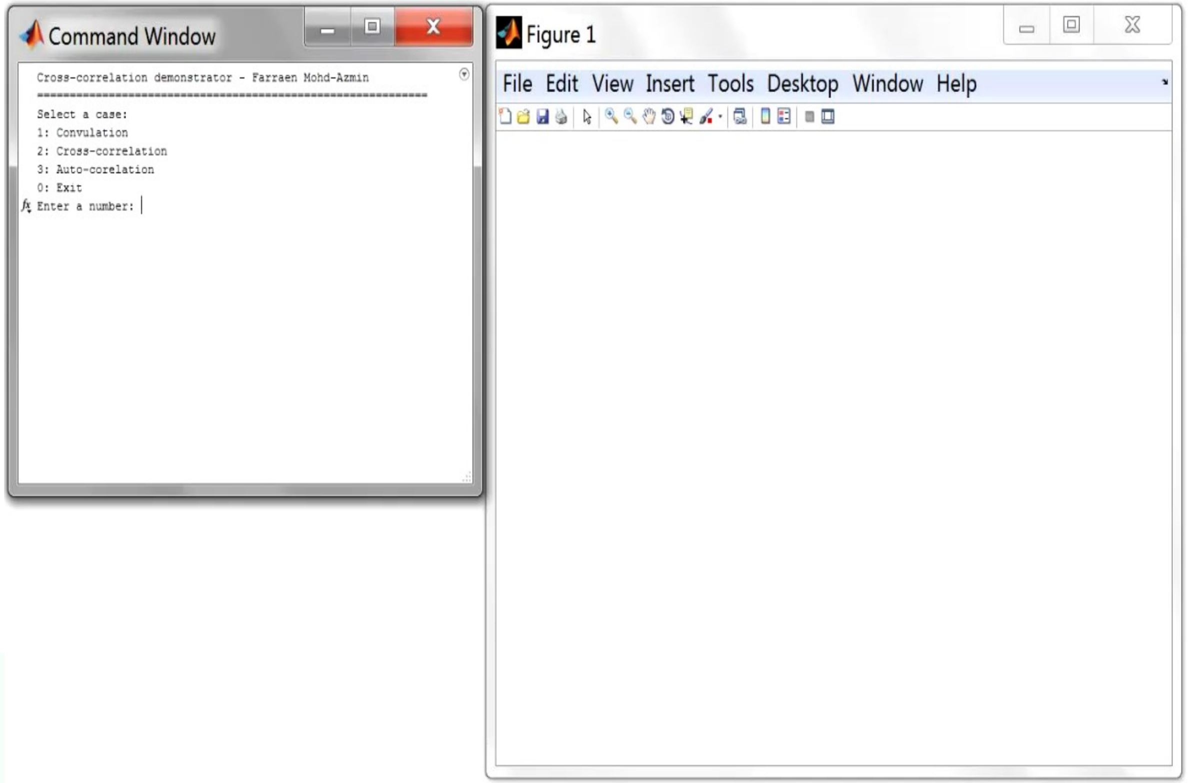


Model evaluation

- Cross-correlation functions
- Correlation plot
- Error metrics used for evaluating and selecting models:
  - Residuals
  - RMSE
  - R^2 (Coefficient of Determination)
  - PRESS RMSE/R2 (leave-one-out method)
- Outliers
- Types of validation

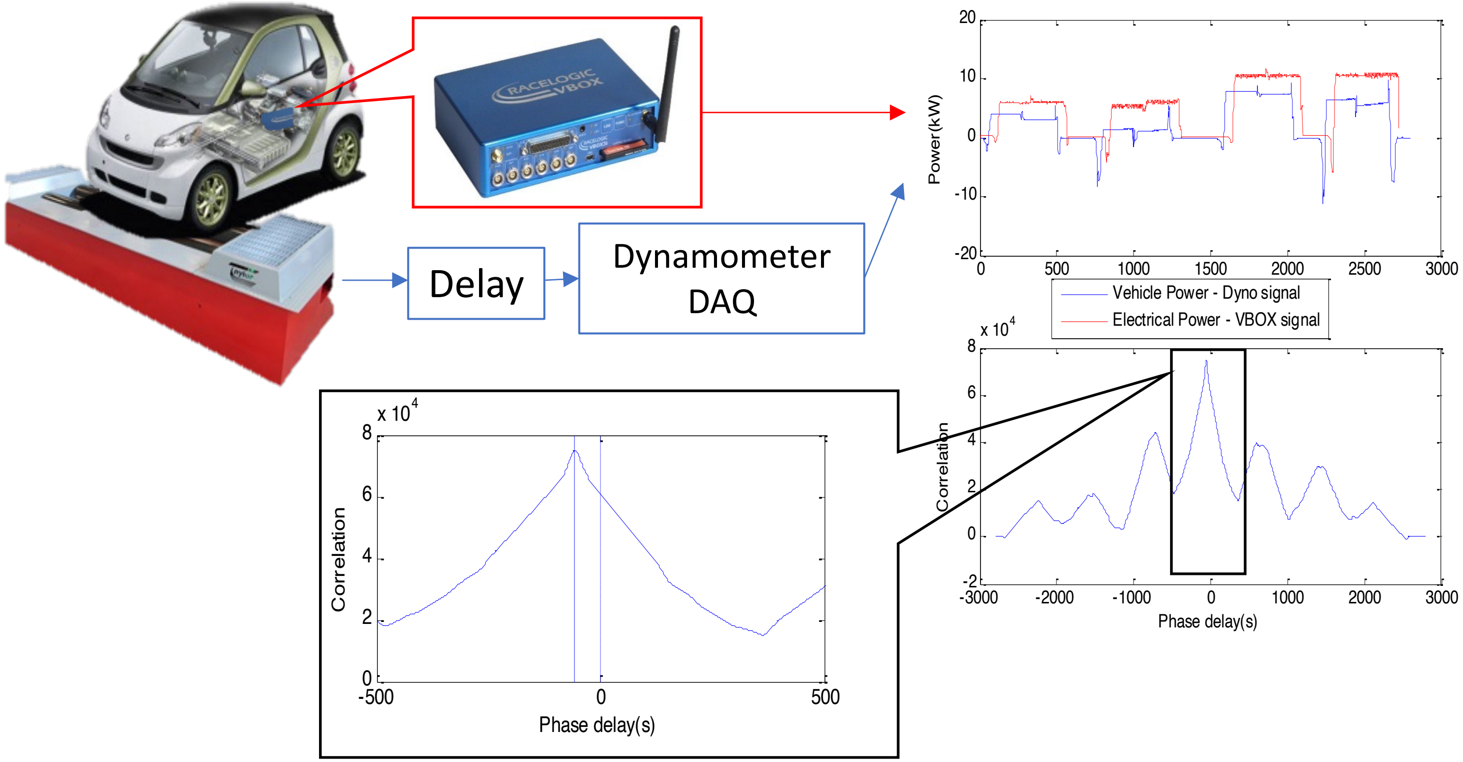
Model evaluation - Correlation functions

- Cross-correlation is a measure of similarity.
- Known as sliding dot product or area under both signals.
- Useful to determine the lag between two signals.
- Used in signal processing, image recognition, economics, etc.



$$(f \star g)(\tau) \triangleq \int_{-\infty}^{\infty} \overline{f(t)} g(t+\tau)\, dt$$

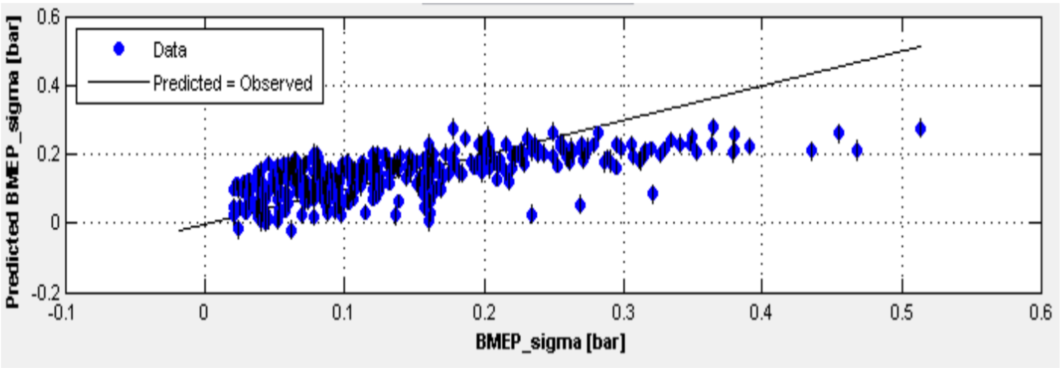
Model evaluation - Correlation functions

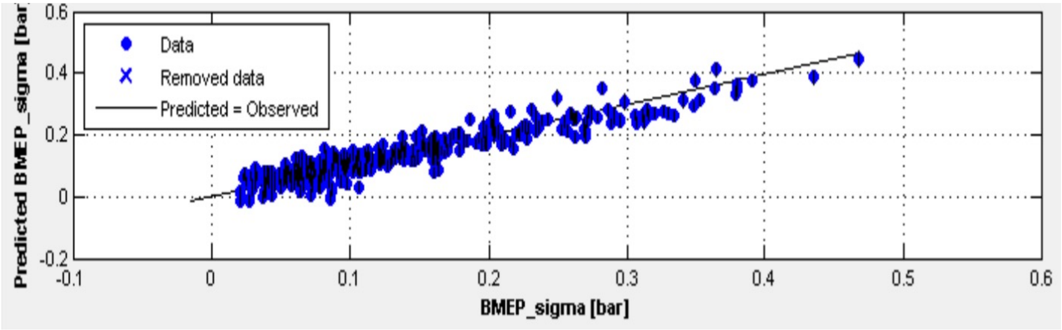


An example used in powertrain application at Loughborough EV lab

Model evaluation - Correlation plot

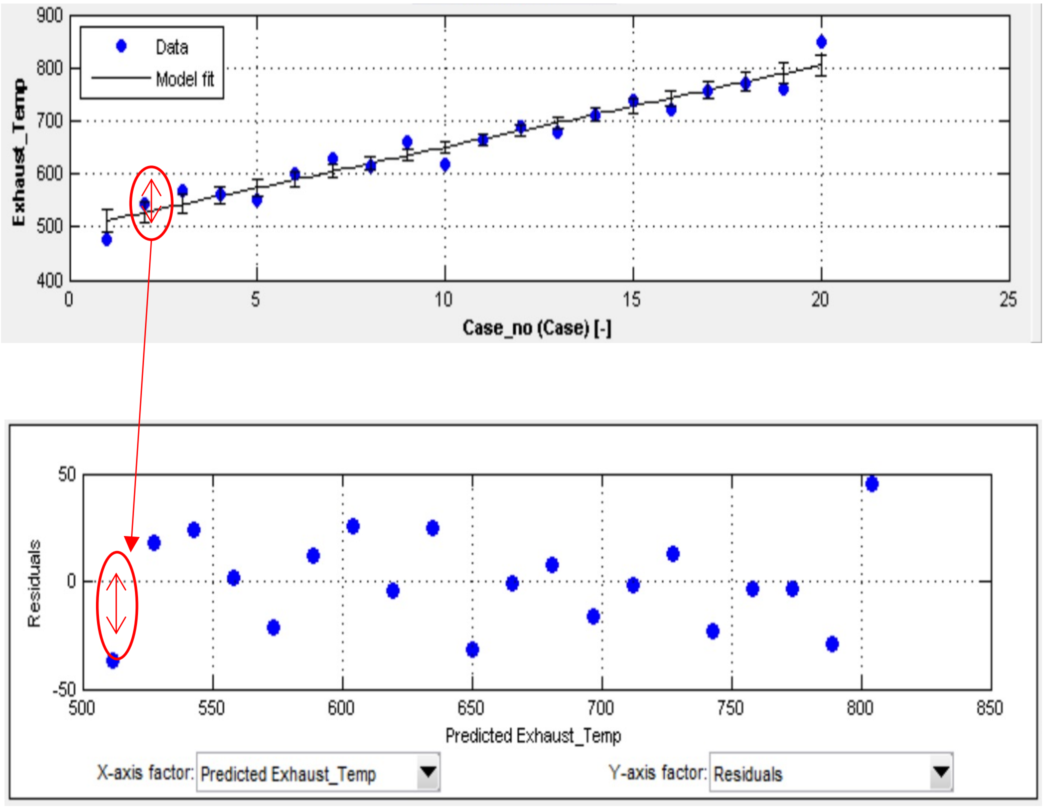
- Bad correlation. High model error. For high BMEP_sigma the predicted is lower than measured BMEP_sigma
- Good correlation between model and measured data.





Model evaluation - Error - Residuals

- Assuming a model is trained using a linear fit. A correlation between the measured dataset and model fit is plotted.
- Residuals are simply what is left after the model has been fitted - the unexplained variation
- Residuals are the difference between the data and the model output at each predictor value:
- **Residual = data – fit(model)**
- For a "good" model we would normally expect to see normally distributed residuals i.e. randomly distributed. Why??



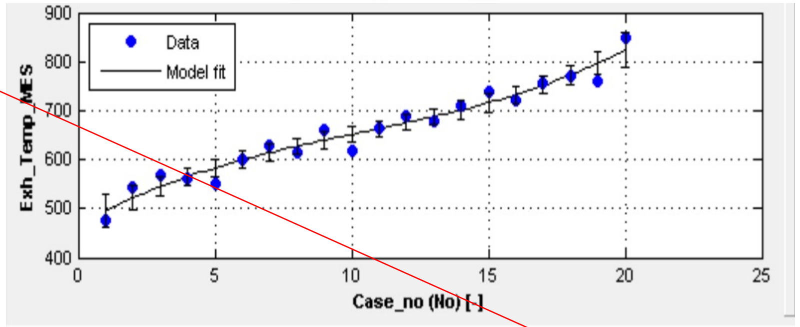
Model evaluation - Error - RMSE

- RMSE is the root square of the mean(norm) of all squares of error

- Model with smallest RMSE is usually selected. But RMSE alone doesn't guarantee model fitness because that is training RMSE.

- Typically, RMSE (between model and validation data) or PRESS RMSE are used as an indicator for model selection.

For example, an exhaust temp dataset was modeled using 4 types of model: linear, quadratic, cubic and 4th order polynomial. The table below shows the difference of RMSE between the models.



Name	Observations	Parameters	Box-Cox	PRESS RMSE	RMSE
Linear	20	2	1	24.668	22.532
Quadratic	20	3	1	28.661	23.169
Cubic	20	4	1	27.745	21.58
Poly_4	20	5	1	35.341	22.278

Model evaluation - Error - R^2

- R^2 (coefficient of determination) gives a measure of the goodness of fit by comparing the explained component of the data with the unexplained (ratio).

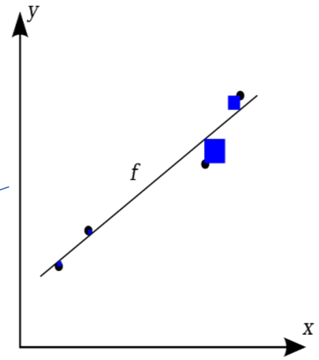
$$R^2 = 1 - \frac{SS_{err}}{SS_{tot}}$$

Unexplained variation

Total variation

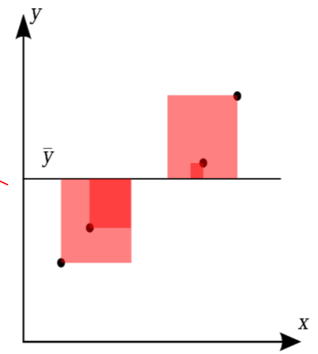
Model evaluation - Error - R^2

$$R^2 = 1 - \frac{\boxed{SS_{err}}}{\boxed{SS_{tot}}}$$



The squared residuals with respect to the linear regression

$$SS_{\text{res}} = \sum_i (y_i - f_i)^2$$



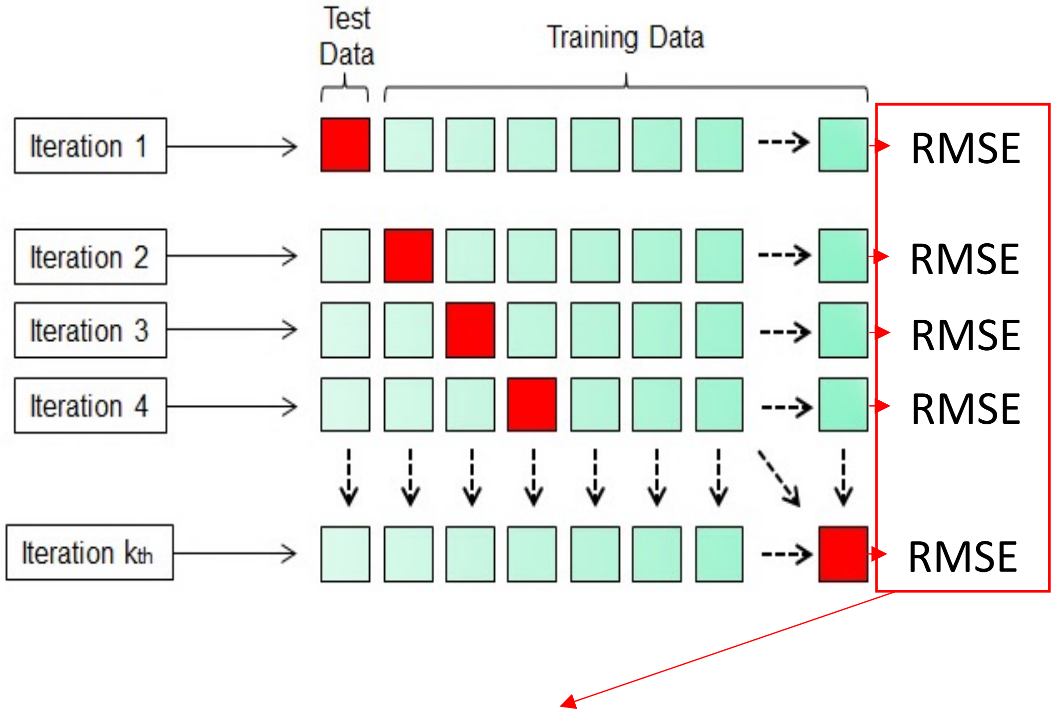
The areas of the red squares represent the squared residuals with respect to the average value.

$$SS_{\text{tot}} = \sum_i (y_i - \bar{y})^2,$$

http://en.wikipedia.org/wiki/File:Coefficient_of_Determination.svg

Model evaluation - Error – PRESS RMSE

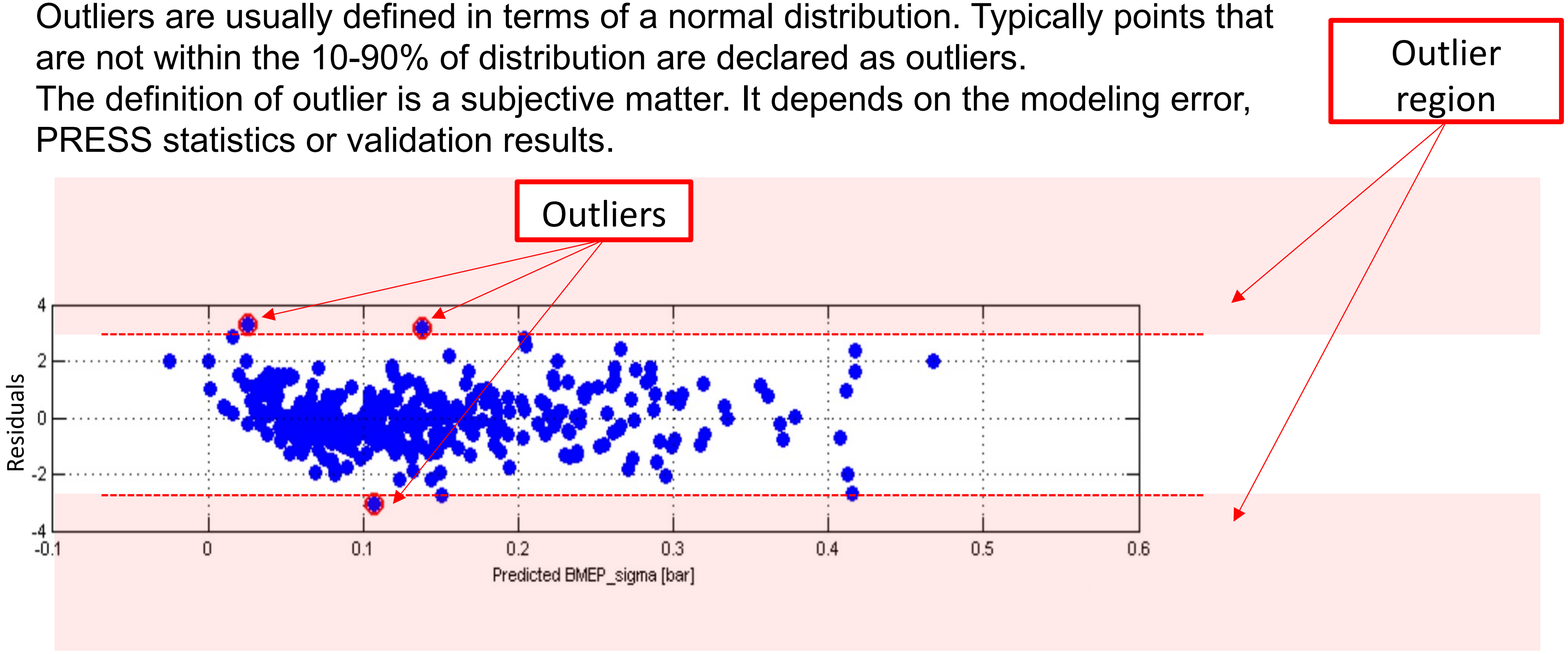
- The PRESS statistic gives a good indication of the predictive power of your model
- PRESS statistics is a method to validate a model without having to use validation data.
- The same training data is used to validate the model.
- Iteratively, a fraction of training data use as validation data and the rest use as training data. The cycle is iterated until all fraction of training data have been once used for validation data. The RMSE are calculated for each iteration. Finally, the average of RMSE is calculated.
- PRESS R^2 can be calculated using the same procedure



PRESS RMSE = SUM(RMSE)/k

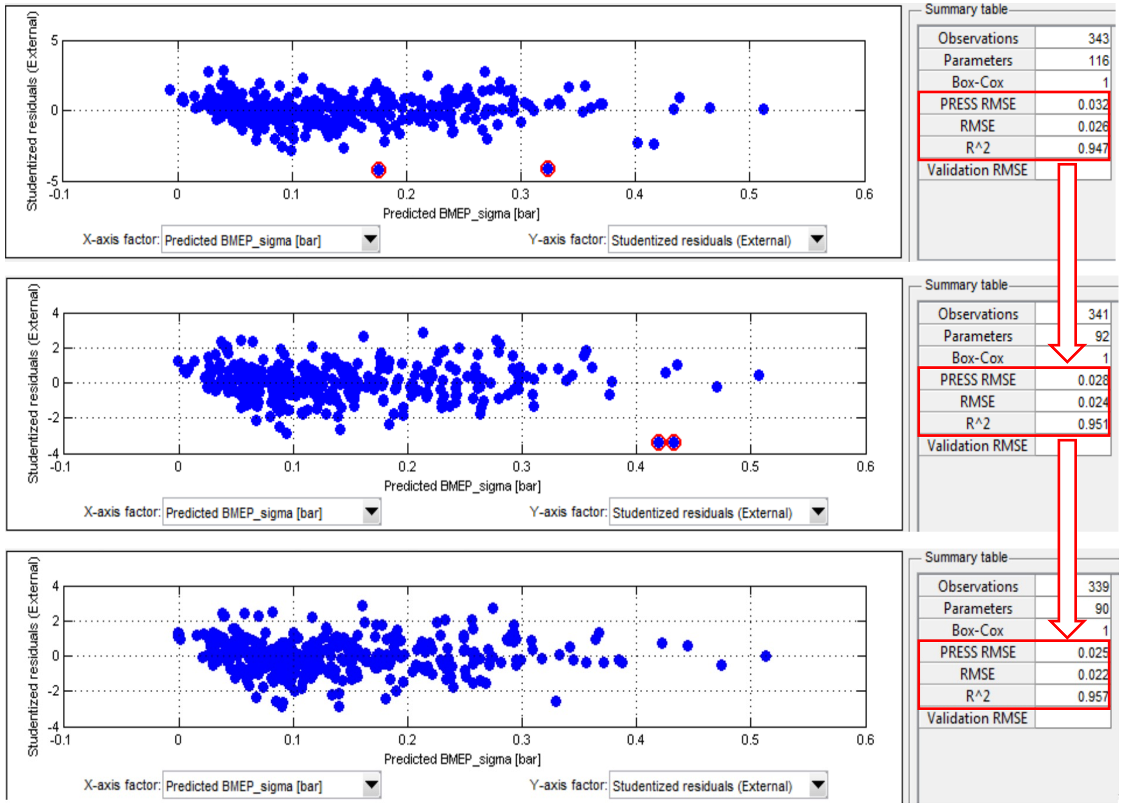
Model evaluation - Outliers

- Outliers are usually defined in terms of a normal distribution. Typically points that are not within the 10-90% of distribution are declared as outliers.
- The definition of outlier is a subjective matter. It depends on the modeling error, PRESS statistics or validation results.



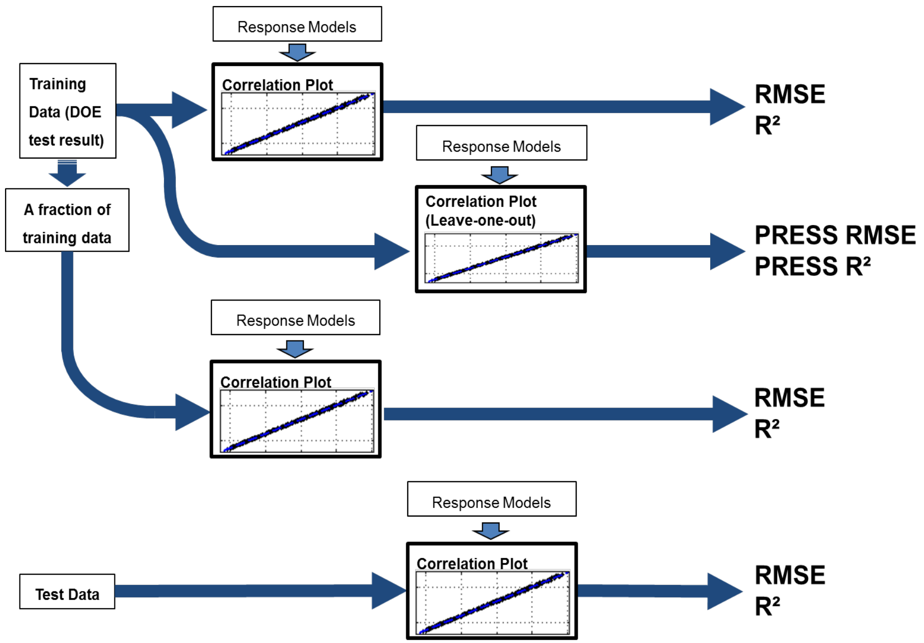
Model evaluation - Outliers

- Removing outliers can easily reduce RMSE (training) including PRESS RMSE of a model.
- But PRESS RMSE doesn't necessarily improve. In most cases, PRESS RMSE will start to increase, because the model is not able to capture the overall system behavior due to loss of information.
- **Data shouldn't normally be removed without good reason.**



Model evaluation – Types of validation

- Validation is the process of taking an independent set of data and checking (using RMSE or R^2 as a measure) of the quality of the model.



Type 1: Calculated purely using all training data → An inappropriate way to validate and select a model based error calculated purely on training data

Type 2: Iteratively using a fraction of the training data → Best method to validate and select a model. Doesn't require extra tests. (Explained on next slide – PRESS statistics)

Type 3: Calculated using test data set 1 → The fraction of data use for validation might miss critical part of the model. The fraction of data have points scattered across the operating region to ensure validation data represent the whole operating region.

Type 4: Calculated using test data set 2 → Using a separate validation data is recommended unless resources are limited. Then, use type 2 for validation.

Model validation with training data will lead to overconfidence in the model

Interaction order

- With an interaction level of 1, there are no terms in the model involving more than one factor. For example, for a four-factor cubic, for factor L, you see the terms for L, L^2, and L^3, but no terms involving L *and/with* other factors.
- If you increase the interaction level to 2, under second-order terms you see L^2 and also L multiplied by each of the other factors; that is, second-order cross-terms (for example, LN, LA, and LS).
- Increase the interaction to 3, and under third-order terms you see L^2 multiplied by each of the other factors (L^2A ,L^2N, L^2S), L multiplied by other pairs of factors (LNA, LNS, LAS), and L multiplied by each of the other factors squared (LN^2 ,LA^2 , LN^2). Interaction level 3 includes all third-order cross-terms.
- For example:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \boxed{\beta_3 X_{i1} X_{i2}} + \epsilon_i$$